Anti-Distortion Image Contrast Enhancement Algorithm Based on Fuzzy Statistical Analysis of the Histogram Equalization

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Abstract

In order to solve such problems as excessive enhancement and chessboard effect, difficult image brightness keeping and distortion in the image enhancement algorithm based on histogram equalization, an anti-distortion image contrast enhancement algorithm based on fuzzy statistics and sub-histogram equalization is proposed in this article. Specifically, the fuzzy set theory is introduced therein to convert the image into fuzzy matrix; then, by virtue of the membership function and the probability of the image gradation, the weighting function is embedded to construct the weighted fuzzy histogram calculation model; then, the mid-value of the initial image is adopted to divide the fuzzy histogram into two sub-histograms, and the corresponding cumulative density functions are defined, and the transformation models thereof are also constructed; then, the inverse transformation function is established to realize defuzzification and output the enhanced image. The experimental data show: compared with the present image enhancement algorithm based on histogram equalization, this algorithm can significantly eliminate excessive enhancement and noise amplification, thus to not only have better visual enhancement quality and anti-distortion performance, but also have maximum AIC (Average Information Contents) value and minimum NIQE (Natural Image Quality Evaluator) value.

Keywords: Weighting function; Sub-histogram equalization; Fuzzy histogram; Cumulative density function

1. Introduction

As one of the main modes of multimedia technology, image is the most visual carrier in present economic life. However, due to such factors as weather, target surrounding conditions, camera hardware and manual operation, the acquired images can be damaged, for example, becoming too dark or too bright, and this seriously influences image information extraction and weakens the visual quality of the images [1-3]. In order to recover such degraded images, the scholars at home and abroad have designed the image contrast enhancement technology. As one of the most common and practical measures for image enhancement at present, histogram equalization [4] is widely researched by the scholars. Shanmugavadivu, et. al. [5] have designed the image enhancement algorithm based on threshold value and histogram equalization to divide the image into two Otsu threshold values, then they adopt PSO (Particle Swarm Optimization) technology to obtain the weight constraint set for equalization, and the corresponding experiment result shows that the algorithm has good enhancement performance, but such algorithm is difficult to
keep image brightness and accordingly causes unnatural enhancement and distortion in the output image. Kuldeeph, et. al. [6] have designed the contrast enhancement algorithm based on the mid-value subimage clipped histogram equalization, and the simulation data show that the algorithm has good enhancement quality and can well keep image brightness, but this algorithm cannot expand the area with minimum or maximum dynamic range intensity value in the sub-histogram, thus causing color fading and chessboard effect for the enhanced image. Chen Yongliang, et.al., [7] have designed the image enhancement technology based on adaptive dynamic clipped histogram equalization (ADCHE), and corresponding experiment has verified the algorithm effectiveness, but this algorithm can easily loss part of the detailed information and is difficult to eliminate noise amplification.

In order to solve such problems as excessive enhancement and noise amplification, an anti-distortion image contrast enhancement algorithm based on fuzzy set theory and sub-histogram equalization is proposed in this article. Specifically, the fuzzy histogram is calculated according to the fuzzy statistics of the images in order to generate smoother histogram; then, the fuzzy histogram is divided into two sub-histograms according to the mid-value of the initial image, and the cumulative density functions are defined and the transformation models thereof are constructed for the independent sub-histogram equalization; finally, the visual quality of the algorithm proposed in this article is verified.

2. Improved Histogram Equalization

An image \( f = f \{ f(i,j) \} \) includes \( N \) pixels and the gradation is in the range of \([0, L-1]\). For the given image \( f \), the probability density model \( P(f_k) \) thereof is as follows:

\[
P(f_k) = \frac{n_k}{n}, \quad k = 1, 2, 3...L-1
\]  

(1)

In the above model, \( k \) denotes the gradation; \( n_k \) is the occurrence number of gradation \( f_k \) in the input image; \( n \) is the number of the total samples in the input image.

According to model (1), \( P(f_k) \) is related to the histogram of the input image and denotes the number of the pixels with the specific intensity \( f_k \). The curve determined by \( f_k \) and \( n_k \) is the histogram of the input image. Then, the corresponding cumulative probability density function is as follows:

\[
C(f_k) = \sum_{j=0}^{k} P(f_j), \quad k = 1, 2, 3...L-1
\]  

(2)

In the above model, \( C(f_k) \) denotes the cumulative probability density function of \( P(f_k) \), and \( C(f_{L-1}) = L \) is defined in this article.

For the histogram equalization, \( C(f_k) \) is regarded as one transformation to map the input image into the whole dynamic range \((f_0, f_{L-1})\). In other words, an intensity transformation function \( T \) is defined on the basis of \( C(f_k) \).

\[
T(f) = \{ f_0 + (f_{L-1} - f_0) \times C(f_k) \}
\]  

(3)
According to model (3), the enhancement image $g = g(i,j)$ based on histogram equalization is as follows:

$$g(i,j) = T(f) = \left\{ T\left[f(i,j)\right] \right\} \text{if } f(i,j) \in f$$

(4)

In the above model, $g, f$ respectively denote the enhanced image and the initial image; $(i,j)$ denotes 2D coordinates of the image.

Figure 1(a) is taken as an object, and then the above technology is adopted for image enhancement. The result is as shown in Figure 1(b). Although this algorithm can improve image contrast and visual effect, the enhanced image output by this histogram equalization technology has chessboard effect, as shown in Figure 1(b), and it is difficult to keep image brightness.

![Image](image.png)

(a) Low Contrast Image  
(b) Enhanced Image

**Figure 1. Image Enhanced By Histogram Equalization Technology**

In order to solve above problem, the mid-value of the image $f$ is adopted to improve the histogram equalization technology. $f_m \in [0,L-1]$ is set as the mid-value of image $f$, and then the image is divided into two subimages $f_L$ and $f_U$ according to $f_m$:

$$f = f_L \cup f_U$$

(5)

$$f_L = \left\{ f(i,j) \mid f(i,j) \leq f_m, \forall f(i,j) \in f \right\}$$

(6)

$$f_U = \left\{ f(i,j) \mid f(i,j) > f_m, \forall f(i,j) \in f \right\}$$

(7)

In the above models, subimage $f_L$ is composed of $\{f_0,f_1,f_2,...,f_m\}$ and subimage $f_U$ is composed of $\{f_{m+1},f_{m+2},f_{m+3},...,f_{L-1}\}$. Afterwards, the probability density functions of $f_L$ and $f_U$ are defined as follows:

$$P_L(f_k) = \frac{n^L_k}{n_L}, k = 1,2,3...m$$

(8)

$$P_U(f_k) = \frac{n^U_k}{n_U}, k = m,m+1,m+2...L-1$$

(9)

In the above models, $n^L_k, n^U_k$ denote the occurrence number of gradation $f_k$ respectively in subimages $f_L$ and $f_U$. 
The cumulative density models of subimages $f_L$ and $f_U$ are as follows:

\[
C_L(f_k) = \sum_{j=0}^{m} P(f_j)k = 1, 2, 3...m
\]  

(10)

\[
C_U(f_k) = \sum_{j=0}^{k-m} P(f_j)k = m, m+1, m+2...L-1
\]  

(11)

According to models (10) and (11), the intensity transformation models are constructed as follows:

\[
T_L(f_k) = f_o + (f_m - f_o)C_L(f_k)
\]  

(12)

\[
T_U(f_k) = f_m + 1 + (f_{L-1} - f_m + 1)C_U(f_k)
\]  

(13)

According to models (12) and (13), the output image $g = g(i, j)$ is as follows:

\[
g(i, j) = T_L(f_L) \cup T_U(f_U)
\]  

(14)

\[
T_L(f_L) = \{T_L(f(i, j)) \forall f(i, j) \in f_L\}
\]  

(15)

\[
T_U(f_U) = \{T_U(f(i, j)) \forall f(i, j) \in f_U\}
\]  

(16)

The improved histogram equalization technology is adopted to enhance Figure.1(a), and the result is as shown in Figure.2. According to the figure, the output image does not have obvious chessboard effect and can well keep image brightness and detail information.

![Image](image1.png)

**Figure 2. Image Enhanced by Improved Histogram Equalization Technology**

### 3. Algorithm Design

The anti-distortion image contrast enhancement algorithm designed in this article is as shown in Figure.3. According to the figure, this algorithm includes the following four parts: (1) Fuzzy matrix solution; (2) Fuzzy histogram calculation; (3) Histogram division and equalization; (4) Defuzzification. Firstly, the fuzzy histogram is calculated through the fuzzy set theory in order to obtain smoother histogram; then, the fuzzy histogram is divided into two sub-histograms according to the mid-value of the initial image, and the corresponding cumulative density functions are defined and the transformation models thereof are also constructed for the independent sub-histogram equalization; finally, the inverse transformation function is constructed for defuzzification in order to output the enhanced image.
3.1. Generation of Fuzzy Matrix

The fuzzy image enhancement is realized through adopting the membership function to map the image gradation to the fuzzy plane, and aims at giving greater weight to the area with the gradation approaching to the average gradation of the image in order to generate and output the image with the contrast ratio more than that of the initial image. For this purpose, the fuzzy set theory [8] is introduced in this article to convert the image into a fuzzy matrix. If the size of image \( I \) is \( M \times N \) pixels, and \( L \) gradation levels included therein are regarded as the fuzzy array, and each gradation includes a relative brightness which is used to denote some brightness values related to intensity \( Q = (0, 1, 2, ..., L - 1) \), and the fuzzy membership function is modified to realize image enhancement, then image \( I \) can be converted into the following fuzzy matrix \( F \):

\[
F = \bigcup_{m=1}^{M} \bigcup_{n=1}^{N} \left\{ \frac{u_{mn}}{Q_{mn}}, u_{mn} \subseteq [0, 1] \right\}
\]

(17)

\[
F = \begin{bmatrix}
\frac{u_{11}}{Q_{11}} & \frac{u_{12}}{Q_{12}} & \frac{u_{13}}{Q_{13}} & \cdots & \frac{u_{1N}}{Q_{1N}} \\
\frac{u_{21}}{Q_{21}} & \frac{u_{22}}{Q_{22}} & \frac{u_{23}}{Q_{23}} & \cdots & \frac{u_{2N}}{Q_{2N}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{u_{M1}}{Q_{M1}} & \frac{u_{M2}}{Q_{M2}} & \frac{u_{M3}}{Q_{M3}} & \cdots & \frac{u_{MN}}{Q_{MN}}
\end{bmatrix}
\]

(18)

In the above models, \( u_{mn} \) denotes the fuzzy membership function of the image, wherein If \( u_{mn} = 0 \) is true, then the image is dark; if \( u_{mn} = 1 \) is true, then the image is bright. \( Q \) is the intensity value.

In order to reduce image fuzzyness, the contrast intensity is introduced into the fuzzy set theory to generate new fuzzy set, and meanwhile the new membership function is also defined as follows:
\[ u_{F(i,j)} = \begin{cases} 2 \times (u_{ij})^2 & 0 \leq u_{ij} \leq 0.5 \\ 1 - 2 \times (1 - u_{ij})^2 & 0.5 < u_{ij} \leq 1 \end{cases} \]  

3.2. Calculation of Fuzzy Histogram

In order to well handle the uncertainty of the image gradation and obtain smoother histogram, the weighting function is embedded on the basis of the probability \( P(i) \) of the image gradation \( i \) according to model (19). The fuzzy histogram calculation model \( H \) constructed thereby is as follows:

\[
H_F \leftarrow \lambda(i) \left( P(i) + \sum_i \sum_j u_{F(i,j)} \right) \tag{20}
\]

\[
\lambda(i) = \begin{cases} \left( \frac{i}{T} \right)^\alpha & 0 \leq i < T \\ 1 & T \leq i \leq L - 1 \end{cases}
\tag{21}
\]

\[
T = \frac{1}{M \times N} \sum_{x=1}^{M} \sum_{y=1}^{N} f(x, y) \tag{22}
\]

In the above models, \( \lambda(i) \) denotes the weighting function; \( i \) denotes image gradation; \( \alpha \in [0,1] \) denotes noise suppression coefficient.

3.3. Histogram Division and Equalization

According to the mid-value \( M \) of image \( f \), the weighted fuzzy histogram obtained according to model (20) is divided into two sub-histograms \( H_{F(L)} \) and \( H_{F(U)} \). Afterwards, according to the “improved histogram equalization” in the first part of the article, the following models can be obtained:

\[
H_F = H_{F(L)} \cup H_{F(U)} \tag{23}
\]

\[
H_{F(L)} = \left\{ H_{F}(i,j) | H_{F}(i,j) \leq M, \forall H_{F}(i,j) \in H_{F} \right\} \tag{24}
\]

\[
H_{F(U)} = \left\{ H_{F}(i,j) | H_{F}(i,j) > M, \forall H_{F}(i,j) \in H_{F} \right\} \tag{25}
\]

The fuzzy sub-histograms \( H_{F(L)} \) and \( H_{F(U)} \) are put into models (8) and (9) to respectively obtain the probability density functions of \( H_{F(L)} \) and \( H_{F(U)} \):

\[
P_L(H_{F(i,k)}) = \frac{n_L^k}{n_L}, \quad k = 1, 2, 3...m \tag{26}
\]

\[
P_U(H_{F(i,k)}) = \frac{n_U^k}{n_U}, \quad k = m, m+1, m+2...L-1 \tag{27}
\]

The cumulative density function models of \( H_{F(L)} \) and \( H_{F(U)} \) are as follows:
\[ C_L \left( H_{F(i)} \right) = \sum_{j=0}^{n} P \left( H_{F(j)} \right) \]  \hspace{1cm} (28)

\[ C_U \left( H_{F(i)} \right) = \sum_{j=0}^{k-1} P \left( H_{F(j)} \right) \]  \hspace{1cm} (29)

Then, according to models (28) and (29), the following transformation functions are constructed:

\[ T_L \left( H_{F(i)} \right) = H_{F(0)} + \left( M - H_{F(0)} \right) C_L \left( H_{F(i)} \right) \]  \hspace{1cm} (30)

\[ T_U \left( H_{F(i)} \right) = M + 1 + \left( H_{F(L-1)} - M + 1 \right) C_U \left( H_{F(i)} \right) \]  \hspace{1cm} (32)

Models (30) and (31) are adopted for the independent equalization of \( H_{F(L)} \) and \( H_{F(U)} \), and then the equalized \( H_{F(L)} \) and \( H_{F(U)} \) are combined to obtain the output image \( g(i,j) \):

\[ g(i,j) = T_L \left( H_{F(L)} \right) \cup T_U \left( H_{F(U)} \right) \]  \hspace{1cm} (33)

\[ T_L \left( H_{F(L)} \right) = \left\{ T_L \left( H_{F(L)}(i,j) \right) \mid \forall H_{F(L)}(i,j) \in H_{F(L)} \right\} \]  \hspace{1cm} (34)

\[ T_U \left( H_{F(U)} \right) = \left\{ T_U \left( H_{F(U)}(i,j) \right) \mid \forall H_{F(U)}(i,j) \in H_{F(U)} \right\} \]  \hspace{1cm} (35)

Obviously, if \( 0 \leq C_L \left( H_{F(i)} \right), C_U \left( H_{F(i)} \right) \leq 1 \) is true, then the sub-histogram \( H_{F(L)} \) is equalized by \( T_L \left( H_{F(L)} \right) \) in the dynamic range \( H_{F(0)}, M \) and the sub-histogram \( H_{F(U)} \) is equalized by \( T_U \left( H_{F(U)} \right) \) in the dynamic range \( M + 1, H_{F(L-1)} \). As a result, the initial image is equalized in the dynamic range \( H_{F(0)}, H_{F(L-1)} \).

### 3.4. Image Defuzzification

The defuzzification is realized through mapping the fuzzy plane to the gradation, thus to output the final enhanced image \( G(i,j) \):

\[ G(i,j) = T^{-1} \left( g(i,j) \right) = \bigcup_{i,j} g(i,j) \times (L-1) \]  \hspace{1cm} (36)

In the above model, \( G(i,j) \) denotes the gradation of the pixel at the coordinates \( (i,j) \) in the image; \( T^{-1} \) denotes the inverse transformation of the transformation functions in models (30) and (31).
4. Simulation Result and Analysis

In order to test the enhancement performance of the algorithm proposed in this article, a simulation experiment is carried out in Matlab platform. Meanwhile, for the sake of presenting the advantages of the algorithm proposed in this article, the present advanced image enhancement algorithms based on histogram equalization are regarded as the comparison group: algorithms proposed in literatures [9] and [10]. Therein, $\alpha = 0.65$ and $T = 1.5$.

A color image with the size of $228 \times 228$ is taken as the test object in this article, because most images acquired by electronic equipment are color images. In order to more rationally evaluate the enhancement effect of these algorithms, AIC (Average Information Contents) and NIQE (Natural Image Quality Evaluator) engines are adopted in this article as the measurement indexes. If AIC value is larger and NIQE value is smaller, then the algorithm has better enhancement effect. Therein, AIC model is as follows:

$$AIC = -\sum_{k=0}^{L-1} P(k) \log P(k)$$

In the above model, $P(k)$ denotes the probability density function of the $k$th gradation and $L$ denotes image gradation.

4.1. Image Enhancement Effect Comparison

The above three algorithms are adopted to enhance Figure 4(a), and the results are as shown in Figure 4(b) ~ Figure 4(d). According to the figures, the algorithm proposed in this article has the best enhancement quality and can well reserve the initial image contents and details, without changing the brightness of the initial target, and the image output thereby is visually natural, as shown in Figure 4(b). Although the comparison algorithms can well improve image contrast ratio, the enhancement quality is not ideal; as shown in Figure 4(c), the image has color fading due to noise amplification; as shown in Figure 4(d), the image has excessive enhancement and is visually unnatural. The reason for the good enhancement quality of the algorithm proposed in this article is as follows: the membership function of the fuzzy set theory and the probability of the image gradation are adopted in this article, and the weighting function is embedded to construct the weighted fuzzy histogram calculation model in order to obtain smoother histogram and suppress noise, and meanwhile the histogram is divided into two sub-histograms according to the image mid-value for the equalization in the whole dynamic range, thus to significantly improve the image recovery quality. For the comparison group, only the histogram equalization technology is adopted for image enhancement, so the image brightness can be easily changed.
4.2. Objective Evaluation for Enhancement Quality

In order to test AIC values and NIQE values of the above three algorithms, four images (Figure 5(a)~Figure 5(d)) are randomly selected from TID2008 database \(^{[13]}\) and are enhanced by the above three algorithms. Specifically, AIC value and NIQE value of the enhanced images are estimated according to the calculation methods proposed in literatures \([11-12]\), and the results are as shown in Figure 5, Table 1 and Table 2. According to Figure 5 and the two tables, the algorithm proposed in this article has maximum AIC value, with the average value of 7.229, and has minimum NIQE value, namely 20.786, and AIC values and NIQE values of the comparison algorithms are respectively as (6.925, 23.010) and (7.064, 22.144). Obviously, compared with the comparison group, the algorithm proposed in this article has significantly enhanced visual quality.
Table 1. AIC Test Values of Different Algorithms

<table>
<thead>
<tr>
<th>Name</th>
<th>Algorithm in this Article</th>
<th>Algorithm in Literature [9]</th>
<th>Algorithm in Literature [10]</th>
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</thead>
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<tr>
<td>Image 1</td>
<td>7.183</td>
<td>7.024</td>
<td>6.895</td>
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<tr>
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<td>7.367</td>
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<td>Image 3</td>
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<td>6.946</td>
<td>7.033</td>
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<tr>
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<td>6.904</td>
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<td>Average</td>
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<td>6.925</td>
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</table>

Table 2. NIQE Test Values of Different Algorithms

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<th>Algorithm in Literature [9]</th>
<th>Algorithm in Literature [10]</th>
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<td>Average</td>
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</table>
5. Conclusion

In order to solve such problems as excessive enhancement and noise amplification of histogram equalization, an anti-distortion image contrast enhancement algorithm based on fuzzy statistics and sub-histogram equalization is proposed in this article on the basis of the fuzzy set theory and the histogram division thought. Firstly, the fuzzy histogram calculation model is constructed according to the fuzzy statistical property of the initial image, the membership function and the probability of the image gradation in order to obtain smoother histogram and significantly improve image color contents, brightness, etc.; then, the fuzzy histogram is divided into two sub-histograms, and the corresponding cumulative density functions are defined, and the transformation models thereof are also constructed for independent sub-histogram equalization. The experiment result has verified the feasibility and the advantages of the algorithm proposed in this article.

References


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YaoNan was born in Jiangsu, China, in 1976. He Received M.S. Degrees in Computer software engineering from NanJing University, JiangSu, China, in 2005, respectively. He is currently work in Jiangsu Electric Power Company Research Institute. His research direction includes the power image intelligent technology and information technology. He has received a number of scientific and Technological Progress Award.