

An Improved Super-resolution Image Reconstruction Algorithm

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Abstract

The paper introduces the Keren registration method and points out its disadvantage which means it will become inaccuracy on the large scale parameters. To reduce the error on large scale parameters of Keren registration, a two step method is proposed, which the phase correlation algorithm is used to estimate the large translation and rotation angle roughly and the improved Keren algorithm is used to estimate accurately the small translation and rotation angle. The experimental results show that the two step method makes less absolute error of angle than Keren method in the situation of large translation and rotation angle. A new method of estimating the standard deviation of noise is introduced to the robust certainty function, which reduces the impact of noise in the process of interpolation using normalized convolution algorithm. By the edge detection of fusion image in the first stage of the interpolation process of normalized convolution algorithm, a calculation method of long axis and short axis of the structure self-adaptive function is improved. The experimental results show that the proposed interpolation method can improve the performance of the original algorithm and enhance the effect of image super-resolution reconstruction.

Keywords: *super-resolution image reconstruction, image registration, phase correlation, Keren algorithm, normalized convolution*

1. Introduction

Super-resolution image reconstruction is to use signal processing techniques to obtain an high-resolution(HR) image from observed multiple low-resolution(LR) images^[1, 2]. The technique has a greatly application in remote sensing monitoring, military reconnaissance, medical diagnosis, pattern recognition and other fields of image processing, as it can improve the image quality greatly without the improved hardware facilities.

Image super-resolution reconstruction technologies can be divided into single frame and multiple-frame sequence image reconstruction in terms of the object of study. Multi-frame image reconstruction mainly consists of LR image acquisition, image registration, interpolation and image restoration. Super-resolution reconstruction based on interpolation algorithm is simple, fast, and has been widely applied in image reconstruction, but it depends on the accuracy of image registration to a large extent.

Image registration mainly consists of spatial methods and frequency domain methods. The typical methods in frequency domain are Fourier transform, wavelet transform and Walsh transform. The phase correlation method is to registrant two images of translation motion of the most basic Fourier transform methods, which can make full use of properties of Fourier transformations, such as translation and rotation invariant^[3]. And it may accomplish fast algorithm by using the FFT operation, but only has pixel-level accuracy.

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Bergen and Keren registration are approaches in space domain^[4]. Keren registration method based on the idea of Taylor series expansion, using rigid body model, has a good estimation of the effect on smaller rotation; Fan adopted simplified affine transformation model with 4 parameters to improve the Keren registration algorithm, which avoid the angle error caused by Taylor series expansion^[5]. Reference [6] proposes an improved algorithm which introduces weight factor and threshold value in the process of iteration, combines with Fan's affine transformation model of 4 parameters, but the complexity and calculation of algorithm is related to the size of translation and will greatly increase in the situation of large angle of rotation. And the complex pyramid can resolve the problem. Combined Reference [5] with [6], we propose a two step method based on phase correlation and Keren registration method for image registration in this paper. Firstly, the phase correlation algorithm is used to estimate the large translation and rotational angle roughly, and then the improved Keren algorithm is used to estimate accurately the small translation and rotational angle, which achieved high performance for sub-pixel estimation with the situation of large translation and rotational angle.

Vandewalle proposes a non-uniform interpolation algorithm based on normalized convolution[7, 13]. The basic idea of normalized convolution is to expand a Taylor series and minimize the error between the values of observed and approximation at the neighborhood of the signal. However, the signal's pixel values of neighborhood may lost or become incredible, which will cause great errors for the NC algorithm. Tuan introduces a robust certainty function and a structure-adaptive applicability function to the polynomial facet model and applies it to fusion of irregularly sampled data. Reference [8] based on the bilateral filtering improves the results by assigning weights to the certainty function. Reference [9] computes the local structure information of fusion image with gradient structure tensor method. In this paper, we introduce a new method of estimating the standard deviation of noise into the robust certainty function for reducing the impact of noise in the process of interpolation using normalized convolution algorithm. And a calculation method of long axis and short axis of the structure self-adaptive function is improved, through the edge of the fusion image were detected in the first stage of the interpolation process of normalized convolution algorithm. Our image registration method is applied to the improve NC algorithm effectively to obtain the accuracy of image super-resolution reconstruction.

2. Theory of Keren Image Registration

Image registration can be defined as a mapping between two images both spatially and with respect to intensity[10]. For a series LR images, we denoted the $f(x, y)$ and $g(x, y)$ as their respective intensity values, then the mapping between images can be expressed as:

$$g(x, y) = I(f(H(x, y))) \quad (1)$$

where H is a 2D spatial coordinate transformation and I is 1D intensity or radiometric transformation.

For a super-resolution image, the precise subpixel image registration is a basic requirement for a good reconstruction. The Keren image registration algorithm has been found to be the most accurate and robust for super-resolution. Fan Chong replaces the rigid body transformation model with the special affine transformation model with four parameters. The model avoids the Taylor expansion of rotation angle.

$$\begin{cases} x' = x + a_1x + a_2y + a_3 \\ y' = y + a_1y - a_2x + a_4 \end{cases} \quad (2)$$

where a_3 is a horizontal translation and a_4 is a vertical translation. The relation of $f(x, y)$ and $g(x, y)$ can expressed as:

$$g(x, y) = f(x + a_1x + a_2y + a_3, y + a_1y - a_2x + a_4) \quad (3)$$

Expanding f to the first term of its own Taylor series gives the following first order equation:

$$g(x, y) \approx f(x, y) + (a_1x + a_2y + a_3) \frac{\partial f}{\partial x} + (a_1y - a_2x + a_4) \frac{\partial f}{\partial y} \quad (4)$$

The error functions between g and f can then be approximated by:

$$E(a_1, a_2, a_3, a_4) = \sum [f(x, y) + (a_1x + a_2y + a_3) \frac{\partial f}{\partial x} + (a_1y - a_2x + a_4) \frac{\partial f}{\partial y} - g(x, y)]^2 \quad (5)$$

To minimum the $E(a_1, a_2, a_3, a_4)$ by computing its derivatives by a_1, a_2, a_3, a_4 and comparing them to zero, then we can get formula (6) by ignoring the nonlinear term:

$$X = C^{-1}V \quad (6)$$

$$\text{Where } C = \begin{bmatrix} \sum R_1^2 & \sum RR_1 & \sum R_1 \frac{\partial f}{\partial x} & \sum R_1 \frac{\partial f}{\partial y} \\ \sum RR_1 & \sum R^2 & \sum R \frac{\partial f}{\partial x} & \sum R \frac{\partial f}{\partial y} \\ \sum R_1 \frac{\partial f}{\partial x} & \sum R \frac{\partial f}{\partial x} & \sum (\frac{\partial f}{\partial x})^2 & \sum \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} \\ \sum R_1 \frac{\partial f}{\partial y} & \sum R \frac{\partial f}{\partial y} & \sum \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} & \sum (\frac{\partial f}{\partial y})^2 \end{bmatrix}, \quad X = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix},$$

$$V = \begin{bmatrix} \sum R_1 (g - f) \\ \sum R (g - f) \\ \sum \frac{\partial f}{\partial x} (g - f) \\ \sum \frac{\partial f}{\partial y} (g - f) \end{bmatrix}, \quad R = y \frac{\partial f}{\partial x} - x \frac{\partial f}{\partial y}, \quad R_1 = x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y}$$

Then, the relationship between a, b, θ and a_1, a_2, a_3, a_4 can be draw as:

$$\begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} a_3 \\ a_4 \\ -\arcsin(a_2) \times 180 / \pi \end{bmatrix} \quad (7)$$

To make the image registration results stability and robustness, we adopt the iteration algorithm and introduce the weight factor and threshold value for speed and efficient of algorithm.

$$I_{k+1} = I_k + \delta C_k^{-1} V_k, \quad (|\arcsin(a_2) \times 180 / \pi| + |a_3| + |a_4| < T) \quad (8)$$

where δ is the weight factor, T is threshold value, k is the number of iteration. We can define $\delta = 0.8$, $T = 0.2$, $k = 50$.

In order to increase speed and robustness, we use a coarse-to-fine structure of the image in this paper, called a gaussian image pyramid. In this scheme, the original image of size $M \times N$ is filtered by a gaussian and sub-sampled to give an image of size $M/2 \times N/2$. This process is repeated, we get four images of size $M/4 \times N/4$, thus get the gaussian image pyramid. Firstly we get the value of X from the first layer and correct the second layer by estimated shift and rotation parameters, then we interpolate the image to get the second layer image. And by this analogy, we get the image registration parameters accurately, while it does not fit the large rotation.

3. Image Registration based on Phase Correlation and Keren Registration Method

In order to reduce the error of large rotation for Keren registration method, we proposed a new algorithm based on the phase correlation and Keren registration method.

3.1. The Phase Correlation Algorithm

The phase correlation is a frequency domain approach allows us to estimate the horizontal and vertical shift and the rotation separately. Assume we have a reference signal $f(x, y)$ and its shifted and rotated version $g(x, y)$:

$$g(x, y) = f(x \cos \theta_0 + y \sin \theta_0 - x_0, -x \sin \theta_0 + y \cos \theta_0 - y_0) \quad (9)$$

If there is only translation between the reference image $f(x, y)$ and the image to be registered $g(x, y)$:

$$g(x, y) = f(x - x_0, y - y_0) \quad (10)$$

Transform the $g(x, y)$ into its Fourier domain, we get the normalized power spectral:

$$Corr(u, v) = \frac{G(u, v) F^*(u, v)}{|G(u, v) F^*(u, v)|} = \exp[-j(ux_0 + vy_0)] \quad (11)$$

where $G(u, v)$ and $F(u, v)$ are the Fourier transform of $g(x, y)$ and $f(x, y)$ respectively, $F^*(u, v)$ is the complex conjugate of $F(u, v)$. As we can see that the normalized power has the only relationship with x_0 and y_0 , a unit impulse function is derived via the inverse Fourier transform of equation(10). Find the largest coordinates of the impulse function, we get the shifted parameters between the two images.

The phase correlation method can also be used to obtain the rotation parameters between two images which have the rotation transform, which takes the advantage of rotation invariance of amplitude spectrum between two images. The formulas (12) can be obtained from the formulas (9) by Fourier transform.

$$G(u, v) = F(u \cos \theta_0 + v \sin \theta_0, -u \sin \theta_0 + v \cos \theta_0) \exp[-j(ux_0 + vy_0)] \quad (12)$$

Assume $M_g(\rho, \theta)$ and $M_f(\rho, \theta)$ are the polar form of the amplitude spectrum of $g(x, y)$ and $f(x, y)$ respectively.

$$M_g(\rho, \theta) = M_f(\rho, \theta - \theta_0) \quad (13)$$

Obviously, the rotation transform has been a shift transform in the polar form which can be solved by the phase correlation method.

3.2. Our Proposed Image Registration Algorithm

Our algorithm based on the phase correlation and Keren registration method is a two step method from coarseness to fine. Firstly, to estimate the motion parameters at pixel level by the phase correlation method, then compensation the images according to the estimation results, thus the image motion parameters will be limited to a small scale, lastly, the improved Keren algorithm be used to estimate at subpixel level. The algorithm process is summarized as Algorithm 1:

- 1) Input: reference image $f(x, y)$ and the image to be registered $g(x, y)$.
- 2) Compute the Fourier transforms of $f(x, y)$ and $g(x, y)$, get the amplitude spectrum $|F(u, v)|$ and $|G(u, v)|$ which are transformed in polar coordinates.
- 3) Compute the rough estimate rotation angle θ' by the phase correlation.
- 4) Angle compensation of θ' to image $g(x, y)$, we get $g'(x, y)$, then estimate the shifted parameters (x'_0, y'_0) of $f(x, y)$ and $g'(x, y)$ by phase correlation method.
- 5) Compensating the image $g'(x, y)$ with estimated shift parameters. (θ'', x''_0, y''_0) is deserved by formulas (8), then the image registration parameters are obtained by the formulas(14).

$$\begin{cases} \Delta\theta = \theta' + \theta'' \\ \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x''_0 \\ y''_0 \end{bmatrix} + \begin{bmatrix} \cos \theta'' & \sin \theta'' \\ -\sin \theta'' & \cos \theta'' \end{bmatrix} \begin{bmatrix} x'_0 \\ y'_0 \end{bmatrix} \end{cases} \quad (14)$$

4. Reconstruction

Normalized convolution (NC) is a technique for local signal modeling from projections onto a set of basis functions, which is an algorithm to minimize the error between observed value and approximations value of local neighborhood for reconstruction images^[13].

A second-order polynomial basis be used in our paper. Within a local neighborhood centered at $s_0 = \{x_0, y_0\}$, the intensity value at position $s = \{x + x_0, y + y_0\}$ is approximated by a polynomial expansion:

$$\hat{f}(s, s_0) = P(x, y) = p_0 + p_1x + p_2y + p_3x^2 + p_4xy + p_5y^2 + \dots \quad (15)$$

where $\{x, y\}$ are the local coordinates of sample s with respect to the center of analysis s_0 . $p(s_0) = [p_0 p_1 p_2 \dots p_m]^K(s_0)$ are the projection coefficients onto the corresponding polynomial basis functions at s_0 .

To solve for the projection coefficients p at an output position s_0 , the approximation error is minimized at s_0 :

$$\varepsilon(s_0) = \int (f(s) - \hat{f}(s, s_0))^2 c(s) a(s - s_0) ds \quad (16)$$

where the signal certainty $0 \leq c(s) \leq 1$ specifies the reliability of the measurement at s , with zero representing completely untrustworthy data and one representing very reliable data. Although both $c(s)$ and $a(s)$ act as scalar weights for the squared errors, they represent different properties, each of which can be made adaptive to the local image data as shown in the next two sections.

4.1. The Improved Structure-Adaptive Applicability Function

The adaptive applicability function is an anisotropic Gaussian function whose main axis is rotated to align with the local dominant orientation:

$$a(s, s_0) = \exp\left(-\left(\frac{(x \cos \theta + y \sin \theta)^2}{\sigma_u(s_0)} + \frac{(-x \sin \theta + y \cos \theta)^2}{\sigma_v(s_0)}\right)\right) \quad (17)$$

where $s_0 = \{x_0, y_0\}$ is the center of analysis, θ is directional gradient angular. σ_u and σ_v are the directional scales of the anisotropic Gaussian kernel.

In the first phase, we obtained initial images by make normalized convolution to non-uniform sample images according to the fixed structure of the adaptive function, then compute the local structural information of fusion image and determine directional gradient angular^[14].

In the second phase, image g was obtained by edge detection using Canny operator of the fusion image. The direction scale can be calculated from the grayscale values of image g . The gradient angular and the direction scale were applied to the structure adaptive function, which effectively improve the image reconstruction.

$$\sigma_u = \frac{\alpha}{\alpha + A} \sigma_c \quad \sigma_v = \begin{cases} \sigma_u & , g(i, j) = 1 \\ \sigma_v = \frac{\alpha + A}{\alpha} \sigma_c & , g(i, j) = 0 \end{cases} \quad (18)$$

where $\alpha = 0.5$, is to adjust the relation between σ_u and σ_v , σ_c is the local neighborhood-scale.

4.2. The New Robust Certainty Function

The robust certainty function was proposed with the idea of bilateral filtering^[8].

$$c(s, s_0) = \exp\left(-\frac{|f(s) - \hat{f}(s, s_0)|^2}{2\sigma_r^2}\right) \quad (19)$$

Where the photometric spread σ_r defines an acceptable range of the residual error. We select σ_r to be two times the standard deviation of input noise (σ_{noise} is estimated from low-gradient regions in the image). So that all samples within $\pm 2\sigma_{noise}$ deviation from the initial polynomial surface fit get a certainty close to one, the value of certainty function is set as 0.98 in our paper. We introduced a new method to estimate the standard deviation of input noise with reference [9]:

$$\sigma_{noise} = \sqrt{\frac{\pi}{2} \frac{\sum_{i,j=1}^{m/2,n/2} |HD^T * (HD * f(i, j))|}{6(m/2)(n/2)}} \quad (20)$$

Where the HD = (1,-2,1) is a highpass filter with the correlation of laplace, m and n represents the number of columns and rows of the image respectively.

5. Experiments

In this subsection, one experiment is carried out to demonstrate the effective of our registration method. First we translate and rotate the LR images which the parameters $(\Delta\theta, \Delta x_0, \Delta y_0)$ are $(2.5^\circ, 2, -2.1)$, $(-3^\circ, 15, -18)$, $(30.7^\circ, 3.2, -5.3)$, $(-32.7^\circ, 35.2, -37)$, then we compute the registration parameters by our registration method and the method in reference [6] respectively. The experiment shows that our approach makes less absolute error of angle than reference [6], especially in the situation of large angle and translation. The result is shown in Table 1. We can see that our algorithm is simple and efficient, and has a very strong noise immunity. And the method's computational complexity has no relation with the size of translation parameters.

Table 1. Results of Image Registration

Actual parameters ($\Delta\theta, \Delta x_0, \Delta y_0$)	Methods	Estimate parameters ($\Delta\theta, \Delta x_0, \Delta y_0$)	Error ($\Delta\theta, \Delta x_0, \Delta y_0$)
(2.5°, 2, -2.1)	Reference [6]	(2.3759°, 1.8364, -2.0021)	(0.1241°, 0.1636, 0.0979)
	Proposed algorithm	(2.4238°, 1.8404, -1.9463)	(0.0762°, 0.1596, 0.1537)
(-3°, 15, -18)	Reference [6]	(-2.9668°, 14.7685, 17.6660)	(0.0332°, 0.2315, 0.3340)
	Proposed algorithm	(-3.0035°, 14.7757, -17.5495)	(0.0035°, 0.2243, 0.4505)
(30.7°, 3.2, -5.3)	Reference [6]	(30.5779°, 2.7772, -5.5991)	(0.1221°, 0.4228, 0.2991)
	Proposed algorithm	(30.6314°, 2.8486, -5.5499)	(0.0686°, 0.3514, 0.2499)
(-32.7°, 35.2, -37)	Reference [6]	(2.2533°, -4.1375, -9.4637)	(34.9533°, 39.3375, 7.5363)
	Proposed algorithm	(-32.3026°, 34.8681, -36.5232)	(0.3974°, 0.3319, 0.4768)

In the second experiment, we started from two high-resolution images, which are Lena(256×256) and Baboon(512×512). In this simulation, four LR images are generated

by a decimation factor of four in both the horizontal and vertical directions from the HR images. $(0^\circ, 0, 0)$, $(1.2^\circ, 0.1, 1.7)$, $(0.8^\circ, 2.5, 1.5)$, $(2.1^\circ, 1.1, 0.6)$ are used for the shift(pixels) and rotation(degrees) parameters. Some visual results of Lena image are shown in Figure 1. Figure 1 (a) shows the original high-resolution image, (b) is the sequence of LR images which have the translation and rotation. (c) is the result of bicubic interpolation. (d) is the result of our algorithm. Note that the proposed algorithm performs visually much better than bicubic interpolation, having less visual artifacts and producing sharper results. Compared with NC, the proposed algorithm provides more image details with improved PSNR. The results using different algorithms are summarized in Table2.

Table 2. RMSE and PSNR of Reconstruction Algorithm

Reconstruction algorithm	Baboon		Lena	
	RMSE	PNSR	RMSE	PNSR
Bicubic(Keren image registration)	21.0115	21.6817	18.0008	23.0250
Bicubic(Proposed registration)	20.1435	22.0481	17.9787	23.0356
NC	20.5854	21.8596	17.6187	23.2113
Proposed algorithm	19.5914	22.2895	17.2281	23.4060



(a) Original image(256x256) (b) LR image sequences(64x64)



(c) Bicubic interpolation (d) NC algorithm (e) Proposed algorithm

Figure 1. Results of Lena Image Reconstruction

6. Conclusion

In order to reduce the sensitivity of the large-scale motion parameters of the Keren algorithm for image registration, a two step method is proposed, which the phase correlation algorithm is used to estimate the large translation and rotational angle roughly and the improved Keren algorithm is used to estimate accurately the small translation and rotation angle. The experimental results show that high performance for sub-pixel

estimation is achieved for the situation of large translation and rotation angle. A new method of estimating the standard deviation of noise is introduced to the robust certainty function for reducing the impact of noise in the process of interpolation using normalized convolution algorithm. By the edge detection of fusion image, a calculation method of long axis and short axis of the structure self-adaptive function is improved. The experimental results show that the proposed interpolation method can improve the performance of the original algorithm and enhance the effect of image super-resolution reconstruction.

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