

## Extensive Experimental Analysis of Image Statistical Measures for Image Processing Appliances

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### Abstract

*Focal intention of this paper is to emphasize the appliance of the basic image statistics for image restoration, de-blurring, de-noising, enhancement, edge detection, edge sharpening, finding edge position and many more root level appliances of image processing and computer vision. For the evaluation and study of restoration, de-blurring and de-noising, some noise molds are discussed and then for the estimation of the results of statistics, some amount of different types of noise have been added to the image and then the process of filtration is performed for scrutiny the effect. For scrutiny of enhancement some de-enhanced or low contrast image are used. Some image quality measures i.e., Mean Square Error, Peak Signal to Noise Ratio Discussion and conclusions drawn from the experimental results are the comparative study of these appliances of image processing. This paper provide the detail study of selected appliances of image processing and computer vision experimentally, evaluate the performance, compare result on literature and give trend what can be done for new and better loom.*

**Keywords:** *Digital Image, Restoration, De-noising, Enhancement, Edge Detection, Gaussian, Gamma, Exponential, Salt & Pepper, Uniform, Min, Max, Mean, Midpoint, Median, Standard Deviation, Variance, Covariance*

### 1. Introduction

The studies to gather, systematize, scrutinize, and deduce numerical information from data is known as statistics. Min, max, mean, mode, midpoint, median, variance, standard deviation, covariance, histogram *etc* are the important image statistics used in various root level field of image processing and computer vision like Image enhancement [1] [2] [3], image restoration [2] [3], image de-blurring [2], image de-noising [2], edge detection *etc*.

The main causes of noise in digital images arise during image acquisition and/or transmission. The recital of imaging feeler is affected by a miscellany of causes, such as environmental circumstances during image gaining, or by the excellence of the sensing elements themselves. For occasion, in gaining images with a Charge-Coupled Devices (CCD) camera, light echelon and feeler temperature are main causes affecting the amount of noise in the resultant image. Images are degraded during transmission mainly due to intrusion in the channel used for transmission. For example, an image broadcast using wireless network might perhaps be degraded as a consequence of lightning or other atmospheric disorder.

In image Enhancement, the main objective of the restoration method is to recover an image in some predefined sense. Although there is vicinity of overlie, image enhancement is principally a subjective procedure, while image restoration is for the majority part an objective procedure. Restoration endeavors to recuperate an image that has been corrupted by using a previous acquaintance of the degradation occurrence. By distinction, enhancement methods principally are heuristic methods designed to maneuver an image

in order to take advantage of the physiological facets of the human visual system. For example, contrast stretching is measured as an enhancement method because it is based principally on the pleasing feature it might present to the spectator, whereas elimination of image blurs by applying a de-blurring function is measured as a restoration method.

The rest of the paper is arranged as second section includes the important image noise molds which are used in experiments, third section includes the definition and explanation of basic image statistics, fourth section include experimental results and discussion, fifth section gives conclusions and references are included in the last section of the paper.

## 2. Typical Image Noise Model [2, 3, 4, 5, 6, 7]

This section discuss some noise models that are added in images for analyzing and comparing the effect of different restoration filters based on image statics. The noise models gives probability density function (PDF), mean and variance of the considered noises.

### 2.1. Gaussian Noise

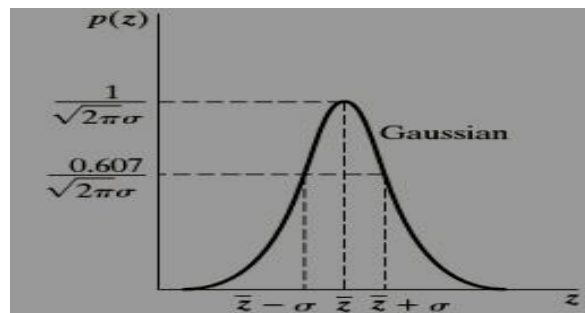
Gaussian noise is also named as amplifier noise, normal noise and random variation impulse noise. Primary cause of Gaussian noise in digital images occurs at some stage in acquisition. It is additive noise in nature. The noise probability density function (PDF) of Gaussian noise is defined as:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\bar{z})^2}{2\sigma^2}} \quad (2.1)$$

Where

$z = \text{greylevel}$ ,  $\bar{z} = \mu = \text{Mean of } z$  and  $\sigma = \text{standard deviation}$

A plot of this function is shown in Figure 1 (a).



**Figure 1. Probability Density Function Plot of Gaussian Noise**

Plot of Figure 1 and equation (2.1) shows that for 70%, the value of  $z$  will be in the range of

$[(\bar{z} - \sigma)(\bar{z} + \sigma)]$  and for 95% it will be  $[(\bar{z} - 2\sigma)(\bar{z} + 2\sigma)]$ .

### 2.2. Gamma Noise

This noise is also named as Erlang noise. The probability density function (PDF) of this noise is defined as

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (2.2)$$

Where

$a > 0$  and  $b$  is a positive integer. Mean and Variance are defined as:  $\mu = \text{mean} = b/a$  and  $\sigma^2 = b/a^2$ .

A plot of this function is shown in Figure 2.

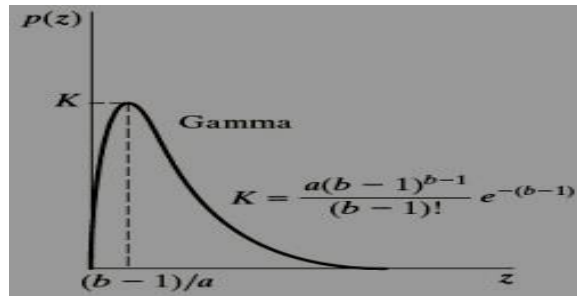


Figure 2. PDF Plot of Gamma Noise

### 2.3. Exponential Noise

The probability density function of exponential noise is defined as

$$p(z) = \begin{cases} ae^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (2.3)$$

Where  $a > 0$ , mean and variance for this PDF are  $\mu = 1/a$  and  $\sigma^2 = 1/a^2$  respectively.

A plot of this function is shown in Figure 3. This is a special case of Erlang probability density function, with  $b=1$ .

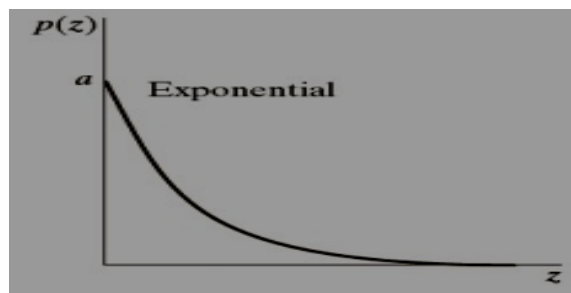


Figure 3. PDF Plot of Exponential Noise

### 2.4. Uniform

The probability density function of exponential noise is defined as

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

Mean and variance for this PDF are  $\mu = \frac{a+b}{2}$  and  $\sigma^2 = \frac{(b-a)^2}{12}$  respectively.

A plot of this function is shown in Figure 4.

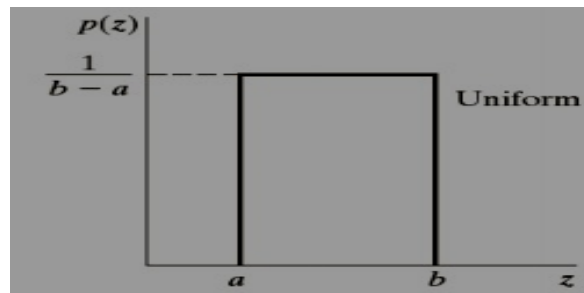


Figure 4. PDF Plot of Uniform Noise

## 2.5. Salt and Pepper Noise

Salt and pepper noise is also known as spike, random or independent noise. Black and white dots, which are black and white pixels, appear on the image as a result of this noise. The PDF of this function is defined as:

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \quad (2.5)$$

If  $b > a$ , then intensity  $b$  will appear as black dot in the image, otherwise it will be white. If  $P_a$  or  $P_b$  zero, then salt and pepper noise will be called unipolar. A plot of this function is shown in Fig.5.

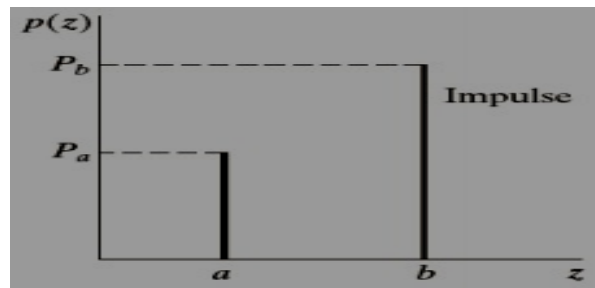


Figure 5. PDF Plot of Salt and Pepper Noise

## 3. Image Statistical Measures as Filter [8]

If RI is the restored image, CI is the corrupted image, W is the sliding window,  $N \times M$  is the size of sliding window,  $(x, y)$  represents the coordinates of RI image and  $(i, j)$  are the coordinates of the sliding window then basic image statistical measures can be defined and explain as following:

### 2.1. Min [8]

Min is the darkest point or pixel value in the image. In min filter, the minimum value from the sliding window W of size  $N \times M$  of the input corrupted image CI will be placed in central location of the sliding window of the restored image RI. Mathematically it is defined as:

$$RI(x, y) = \min_{(i,j) \in W} CI(i, j) \quad (2.6)$$

## 2.2. Max [8]

Max is the brightest point or pixel value in the image. In max filter, the maximum value from the sliding window  $W$  of size  $N \times M$  of the input corrupted image  $CI$  will be placed in central location of the sliding window of the restored image  $RI$ . Mathematically this can be expressed as:

$$RI(x, y) = \underset{(i,j) \in W}{\text{Max}} CI(i, j) \quad (2.7)$$

## 2.3. Midpoint

The midpoint is the average of highest and lowest value of the image. While filtering, it computes the average of max and min value of the sliding window and put it in the resultant location. Its mathematical definition is:

$$RI(x, y) = \frac{1}{2} \{ \underset{(i,j) \in W}{\text{Max}} CI(i, j) + \underset{(i,j) \in W}{\text{Min}} CI(i, j) \} \quad (2.8)$$

## 2.4. Mean

It is the most common and basic of all the statistical measures. Extensive series of this appraisal has been urbanized for this reason. It includes the following:

**2.4.1. Arithmetic Mean:** Arithmetic mean is simply the averaging of all the pixel values. In arithmetic mean filter simply the average value of the sliding window  $W$  of size  $N \times M$  of the image is replaced with the pixel value of the restored image. Mathematically this can be expressed as:

$$RI(x, y) = \frac{1}{N \times M} \sum_{(i,j) \in W} CI(i, j) \quad (2.9)$$

**2.4.2. Geometric Mean:** This is a variation of variation of arithmetic mean filter. Mathematically it is expressed as:

$$RI(x, y) = \left[ \prod_{(i,j) \in W} CI(i, j) \right]^{1/N \times M} \quad (2.10)$$

**2.4.3 Harmonic Mean:** This is just another variation of the arithmetic mean filter defined as:

$$RI(x, y) = \frac{N \times M}{\sum_{(i,j) \in W} \left( \frac{1}{CI(i, j)} \right)} \quad (2.11)$$

**2.4.4. Contra-Harmonic Mean:** This filter yields a restored image  $RI$  by the expression:

$$RI(x, y) = \frac{\sum_{(i,j) \in W} (CI(i, j))^{Q+1}}{\sum_{(i,j) \in W} (CI(i, j))^Q} \quad (2.12)$$

In equation (2.3),  $Q$  is known as the order of the filter. It could be positive, negative or zero.

## 2.5. Median [9, 10, 11, 12]

It is a ranked based filter. It simply replaces the value of pixel by the median of the neighborhood picture element. Its mathematical form is:

$$RI(x, y) = \underset{(i,j) \in W}{\text{Median}}\{CI(i, j)\} \quad (2.13)$$

## 2.6. Alpha Trimmed

Assume that if  $d/2$  lowest and  $d/2$  highest pixel intensity values of the corrupted image are deleted, then  $CI_r$  represents the remaining pixels of the corrupted image. A filter by averaging the  $CI_r$  is what we call the alpha trimmed filter. Mathematically defines as:

$$RI(x, y) = \frac{1}{NxM - d} \sum_{(i,j) \in W} CI_r(i, j) \quad (2.14)$$

## 2.7. Standard Deviation [13, 14]

Standard deviation is the most widely used measure for computing the variation from the mean or expected value in image processing. A low value of standard deviation means that the set of point are very close to the mean and inverse is true for the fact that data set will be far from the mean value. Its mathematical form is as:

$$RI(x, y) = \sqrt{\frac{1}{NxM} \sum_{(i,j) \in W} \left( CI(i, j) - \frac{1}{NxM - 1} \sum_{(i,j) \in W} CI(i, j) \right)^2} \quad (2.15)$$

## 2.8. Variance [13, 14]

Variance computes that how far the set of numbers spread out. It is expressed mathematically as:

$$RI(x, y) = \frac{1}{NxM} \sum_{(i,j) \in W} \left( CI(i, j) - \frac{1}{NxM - 1} \sum_{(i,j) \in W} CI(i, j) \right)^2 \quad (2.16)$$

## 2.9. Covariance [13, 14]

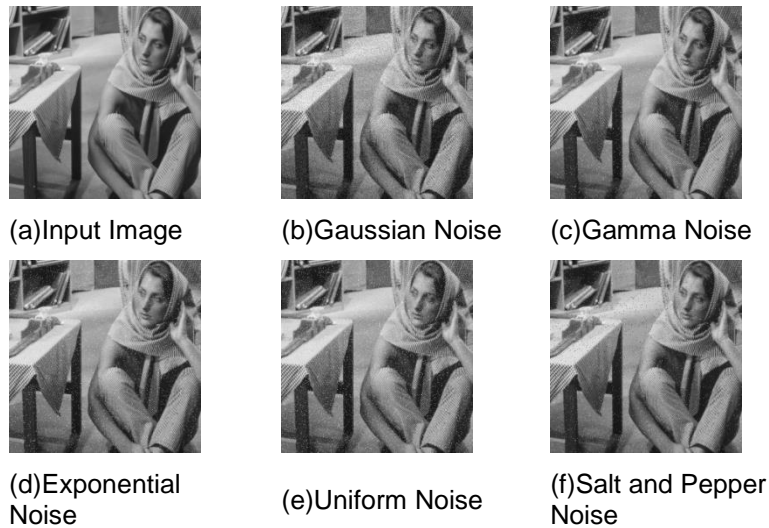
This is a measure in statistics that evaluate that how the two random variables change together. This shows the linear relationship between two variables. It is defined as:

$$\text{Covariance}(RI, CI) = \frac{1}{NM} \sum_{i=1}^N \sum_{j=1}^M (RI_{(i,y)} - \mu_{RI})(CI_{(i,j)} - \mu_{CI}) \quad (2.17)$$

In equation (2.3),  $\mu$  represent the mean value

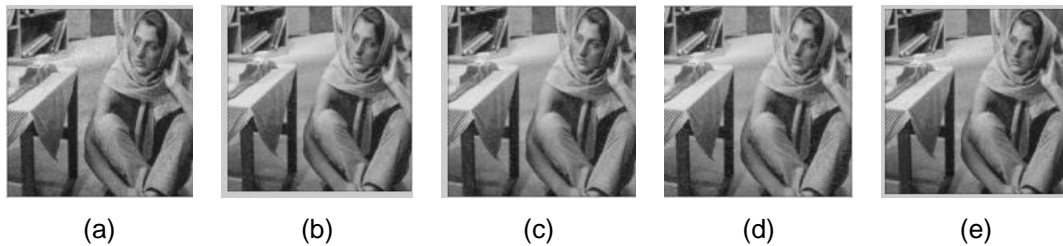
## 4. Experimental Results and Discussion

Figure 6 shows the Barbara original image and Barbara image after adding different type noise. Figure 6 (a) is the original Barbara grayscale image; (b) is the image with Gaussian noise. Mean of the Gaussian noise is 0 and variance is 1, (c) Shows the Barbara image with Gamma noise having mean 2 and variance 5, (d) shows the Barbara image with exponential noise having mean and variance 1, (e) shows the image with uniform noise having mean 0 and variance 1. (f) Shows the image with salt and pepper noise having mean and variance 0.05.



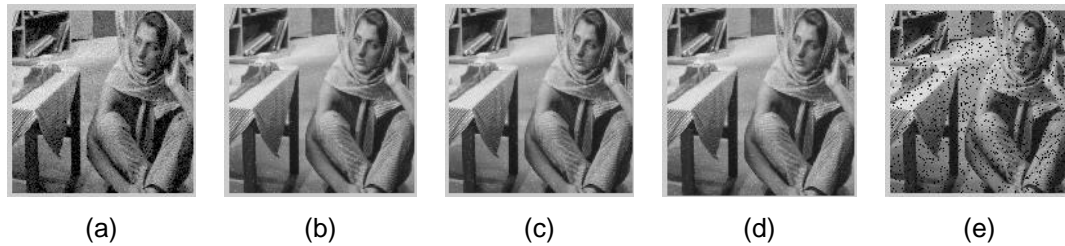
**Figure 6. Barbara Original Image and Image after Adding Noise**

Figure 7 shows the result of arithmetic mean filter. In Figure 7, (a) is the arithmetic mean filter of Figure 6(b), (b) is the arithmetic mean filter of Figure 6(c), (c) is the arithmetic mean filter of Figure 6(d), (d) is the arithmetic mean filter of Figure 6(e) and (e) is the arithmetic mean filter of Figure 6(f). For this filter the size of the sliding window is 3x3. From Figure 6 it is clear that although this filter removes the noise but it creates blurring effect in the output restored image. This blurring effect is proportional to the window size. As the size of the sliding window increased the blurring effect also increased.



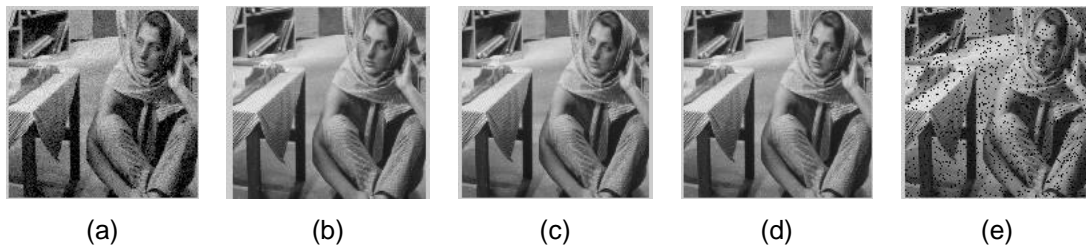
**Figure 7. Result of Arithmetic Mean Filter**

Figure 8 shows the result of Geometric mean filter. In Figure 8, (a) is the Geometric mean filter of Figure 6(b), (b) is the Geometric mean filter of Figure 6(c), (c) is the Geometric mean filter of Figure 6(d), (d) is the Geometric mean filter of Figure 6(e) and (e) is the Geometric mean filter of Figure 6(f). For this filter the size of the sliding window is 3x3. From Figure 8 it is clear that in the presence of Gaussian noise this filter does not gives good results, in the presence of gamma and exponential noise the result are good as compare to that of Gaussian noise, for uniform noise although some noise have been reduced but not all and in the presence of salt and pepper noise it shows a different variation in terms that this filter removes the salt noise and increase the pepper noise.



**Figure 8. Result of Geometric Mean Filter**

Figure 9 shows the result of harmonic mean filter. In Figure 9, (a) is the harmonic mean filter of Figure 6(b), (b) is the harmonic mean filter of Figure 6(c), (c) is the harmonic mean filter of Figure 6(d), (d) is the harmonic mean filter of Figure 6(e) and (e) is the harmonic mean filter of Figure 6(f). For this filter the size of the sliding window is  $3 \times 3$ . From Figure 9 it is clear that in the presence of Gaussian noise this filter does not gives good results, in the presence of gamma and exponential noise the result are good as compare to that of Gaussian noise, for uniform noise although some noise have been reduced but not all and in the presence of salt and pepper noise it shows a different variation in terms that this filter removes the salt noise and increase the pepper noise.



**Figure 9. Result of Harmonic Mean Filter**

Figure 10 shows the result of Contra-harmonic mean filter when  $Q=-ve$ . In Figure 10, (a) is the contra-harmonic mean filter of Figure 6(b), (b) is the contra-harmonic mean filter of Figure 6(c), (c) is the contra-harmonic mean filter of Figure 6(d), (d) is the contra-harmonic mean filter of Figure 6(e) and (e) is the contra-harmonic mean filter of Figure 6(f). For this filter the size of the sliding window is  $3 \times 3$ . From Figure 10 it is clear that in the presence of Gaussian noise this filter does not gives good results, in the presence of gamma and exponential noise the result are good as compare to that of Gaussian noise, for uniform noise although some noise have been reduced but not all and in the presence of salt and pepper noise it shows a different variation in terms that this filter removes the salt noise and increase the pepper noise.

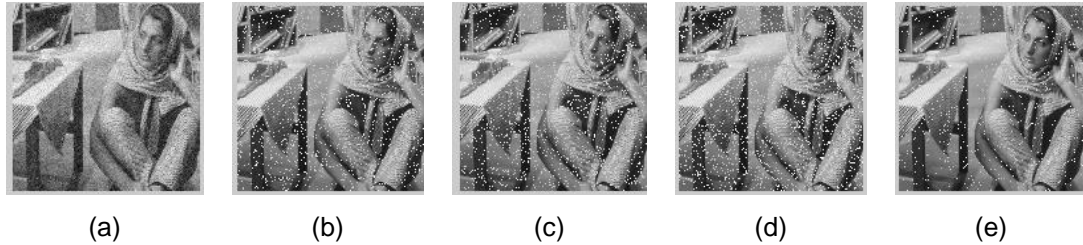


**Figure 10. Result of Contra-harmonic Mean Filter when  $Q=-ve$**

Figure 11 shows the result of Contra-harmonic mean filter when  $Q=+ve$ . In Figure 11, (a) is the contra-harmonic mean filter of Figure 6(b), (b) is the contra-harmonic mean filter of Figure 6(c), (c) is the contra-harmonic mean filter of Figure 6(d), (d) is the

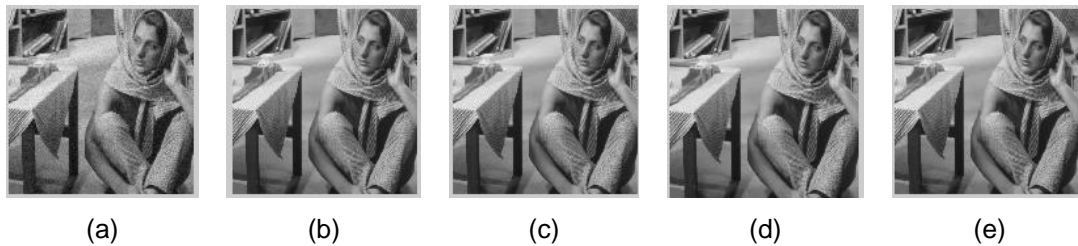


contra-harmonic mean filter of Figure 6(e) and (e) is the contra-harmonic mean filter of Figure 6(f). For this filter the size of the sliding window is 3x3. From Figure 10 it is clear that in the presence of Gaussian noise this filter does not give good results, in the presence of gamma, exponential and uniform noise the results are poor as it increases the white noise and in the presence of salt and pepper noise it shows a different variation in terms that this filter removes the pepper noise and increases the salt noise.



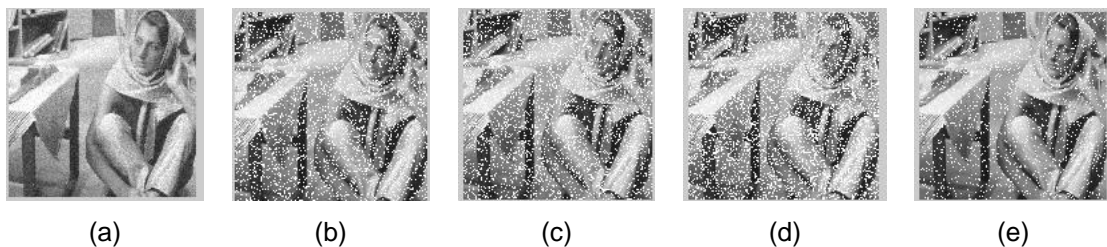
**Figure 11. Result of Contra-Harmonic Mean Filter when  $Q=+ve$**

Figure 12 shows the result of Median filter. In Figure 12, (a) is the median filter of Figure 6(b), (b) is the median filter of Figure 6(c), (c) is the median filter of Figure 6(d), (d) is the median filter of Figure 6(e) and (e) is the median filter of Figure 6(f). For this filter the size of the sliding window is 3x3. From Figure 12 it is clear that in the presence of Gaussian noise this filter does not give good results, in the presence of gamma and exponential noise it shows good results, for uniform noise the results are quite good and for salt and pepper it removes the noise.



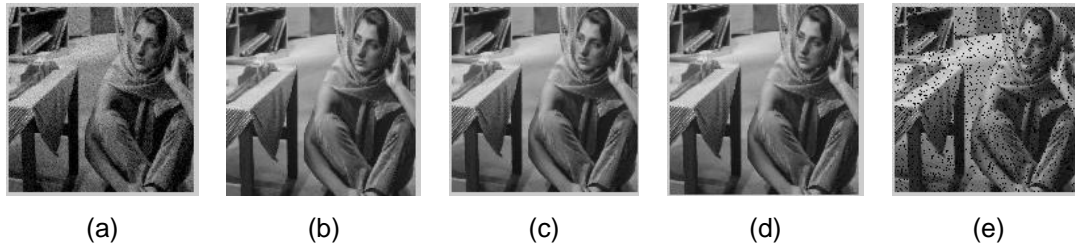
**Figure 12. Result of Median Filter**

Figure 13 shows the result of Max filter. In Figure 13, (a) is the max filter of Figure 6(b), (b) is the max filter of Figure 6(c), (c) is the max filter of Figure 6(d), (d) is the max filter of Figure 6(e) and (e) is the max filter of Figure 6(f). For this filter the size of the sliding window is 3x3. Figure 13 shows that in the presence of Gaussian noise the results are not good, for gamma, exponential and uniform the results are very poor as there is an increase in white pixels and for the salt and pepper noise the result shows the removal of pepper noise and an increase in salt noise.



**Figure 13. Result of Max Filter**

Figure 14 shows the result of Min filter. In Figure 14, (a) is the min filter of Figure 6(b), (b) is the min filter of Figure 6(c), (c) is the min filter of Figure 6(d), (d) is the min filter of Figure 6(e) and (e) is the min filter of Figure 6(f). For this filter the size of the sliding window is 3x3. Figure 14 shows that in the presence of Gaussian noise the results are not good, for gamma, exponential and uniform the results are quite good and for the salt and pepper noise the result shows the removal of salt noise and increase in pepper noise.



**Figure 14. Result of Min Filter**

Figure 15 shows the result of Midpoint filter. In Figure 15, (a) is the midpoint filter of Figure 6(b), (b) is the midpoint filter of Figure 6(c), (c) is the midpoint filter of Figure 6(d), (d) is the midpoint filter of Figure 6(e) and (e) is the midpoint filter of Figure 6(f). For this filter the size of the sliding window is 3x3. Figure 15 shows that the results are very poor in all the cases.



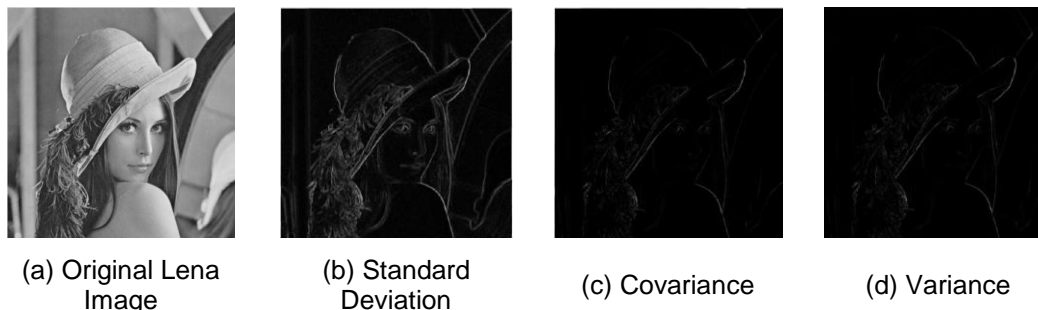
**Figure 15. Result of Midpoint Filter**

Figure 16 shows the result of Alpha-trimmed filter. In Figure 16, (a) is the alpha-trimmed filter of Figure 6(b), (b) is the alpha-trimmed filter of Figure 6(c), (c) is the alpha-trimmed filter of Figure 6(d), (d) is the alpha-trimmed filter of Figure 6(e) and (e) is the alpha-trimmed filter of Figure 6(f). For this filter the size of the sliding window is 3x3. Figure 16 shows that the results are very poor in all the cases. The reason is that this filter works well in the presence of multiple types of noise.



**Figure 16. Result of Alpha-Trimmed Filter**

Figure 17 shows the effect of standard deviation, covariance and variance. Figure 17(a) is the original Lena image; (b) is the Lena image after applying standard deviation filter and it detect the edges; (c) is the Lena image after covariance filter and it sharpen the edges and (d) is the Lena image after variance filter and it shows the edge position.



**Figure 17. Result of Standard Deviation, Covariance and Variance Filter**

## 5. Conclusion and Future Work

This paper provides the detail study and experimental scrutiny of common image noise molds, important image statistical measures, use of image statistical measures in image processing and computer vision applications and demonstrate the effect of all the filter experimentally. Conclusions are based on the experimental results which are performed on almost 50 standard images. Image statistical measures are very important for the image restoration, image de-noising, image de-blurring, image enhancement, find edge position, edge sharpening, edge detection *etc.* Experimental results shows that: Mean filter reduce many types of noise but it works most excellent for image restoration in the presence of Gaussian, uniform or Erlang noise and it creates blurring effect in the image which is proportional to the window size. Geometric mean filter works well for the Gaussian noise. Restoration process by harmonic mean filter works well in the presence of Gaussian and salt noise. Contra-harmonic mean filter is best for salt or pepper noise. Median filter works best for the salt and pepper noise. Min filter works well for salt noise and max filter gives good result for pepper noise. Standard deviation, covariance and variance are very important and useful for edge detection, edge sharpening and to find edge position. Image restoration, enhancement, de-noising and de-blurring are the fundamental and preprocessing step in almost all the applications of image processing and computer vision. All this analysis is a very supportive and useful guide for all those who want to work in the field of digital image processing and computer vision. This experimental scrutiny will also provide the guidelines to the researcher to project new and better approaches in this field.

The future work of this contribution towards the scientific community will be the explanation of the noise appears during different acquisition process in real time images. Moreover some quantitative measures will be introduced for the evaluation of the effect of noise reduction.

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## References

- [1] P. Kaushik and Y. Sharma, "Comparison Of Different Image Enhancement Techniques Based Upon Psnr & Mse", International Journal of Applied Engineering Research, ISSN 0973-4562 Vol.7 No.11 (2012)
- [2] P.Kamboj1 and V.Rani, A BRIEF STUDY OF VARIOUS NOISE MODEL AND FILTERING TECHNIQUES , Journal of Global Research in Computer Science, Volume 4, No. 4, April (2013)
- [3] E. J.Leavline, S. Sutha, "Gaussian noise removal in gray scale images using fast Multiscale Directional Filter Banks", Proceedings of International Conference on Recent Trends in Information Technology, Chennai, India, (2011), 3-5.
- [4] E. J. Leavline, S. Sutha, D. A. A.G. Singh, International Journal of Computer Applications, vol. 33, no. 10, (2011).
- [5] S. Sutha, E. J. Leavline, D. A.A.G.Singh, European Journal of Scientific Research, vol. 86, no. 4, (2012).
- [6] E. J. Leavline, D. A. A. G. Singh, International Journal of Imaging & Robotics, vol. 11, no. 3, (2013). [6] H. Hwang and R. A. Haddad, IEEE Transactions on Image Processing, vol. 4, no. 4, (1995).
- [7] E. J. Leavline and D. A. A. G. Singh, International Journal of Imaging & Robotics, vol. 11, no. 3, (2013).
- [8] Mr. S. S. Al-amri, Dr.N.V.Kalyankar, Dr.S.D.Khamitkar, Linear and Non-linear Contrast Enhancement Image, IJCSNS International Journal of Computer Science and Network Security, VOL.10 No.2, February (2010)
- [9] S. Zhang and M. A. Karim, IEEE Signal Processing Letters, vol. 9, no. 11, (2002).
- [10] Z. Wang and D. Zhang, IEEE Transactions on Circuits and Systems II, vol. 46, no. 1, (1999).
- [11] P.-E. Ng and M. A. Kai-Kuang, IEEE Transactions on Image Processing, vol. 15, no. 6, (2006).
- [12] H. Hwang and R. A. Haddad, IEEE Transactions on Image Processing, vol. 4, no. 4, (1995).
- [13] J. Robinson, Covariance matrix estimation for appearance-based face image processing. In Proc. BMVC05, page 389-398, (2005)
- [14] I. Motoyoshi, "Image statics and the perception of surface quality", Nature, vol 447,pp 206-209, may (2007)

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