Research on Seismic Signals Denoising Method based on Multi-Threshold Wavelet Packet

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Abstract

For the distortion problem of traditional denoising threshold function exists, a seismic signal denoising method based on wavelet packet multi-threshold function was present. Seismic wave signals were done wavelet packet decomposition and wavelet packet decomposition coefficients were arranged in the order of magnitude of the frequency, the appropriate threshold criteria was selected on different frequency bands and threshold processing was done on different frequency bands. Synthetic seismic signals and real seismic data were done multi-threshold wavelet packet decomposition, which better remove high frequency random noise and retain useful signals. Experimental results showed that multi-threshold wavelet packet decomposition method can effectively remove noises and improve the resolution of seismic data, with better denoising effect.

Keywords: seismic signals; denoising ; wavelet packet; multi-threshold;

1. Introduction

In seismic exploration, seismic signals inevitably were affected by a variety of interference and noises, making the signal to noise ratio and resolution seismic data reduced, so the denoising is critical in seismic data processing. To this end, many methods were designed to remove a variety of noises and to improve the signal to noise ratio according to the differences of the various features of the signal and noises. For example, the frequency-domain high-pass filter to suppress surface waves, f-k filtering, k-l filtering, the Radon transform for multiple attenuation and focus filter, f-x prediction filtering for suppressing random interference *etc*. These methods in practice have had very good application effect, and were continuously improved. Wavelet transform as a new method of technique, because of its time-frequency analysis, multi-resolution, and other characteristics, has been widely applied in the signal de-noising. Wavelet analyses have been used to extract seismic signals and noise cancellation processing by many researchers home and abroad [1-3]. Wavelet packet transform is the promotion of the wavelet transform, which is a more detailed analysis and reconstruction methods. In wavelet packet transform the bands were done multi-level division, the high frequency portions were not decomposed in wavelet analysis could be further decomposed, and could adaptively select the appropriate frequency bands according to the analyzed characteristics of the signals, so as to match the signal spectrum, thereby enhancing the time-frequency resolution. But the traditional wavelet packet denoising did not conduct adequate research.

A seismic data wavelet packet denoising method based on different frequency components to select a different threshold function was presented, in which wavelet packet decomposition coefficients were arranged on the order of magnitude of frequency, and the appropriate threshold criteria was selected for each band according to the type of information segments. Synthetic seismogram denoising was implemented by simulation and real seismic signals were done denoising. Theory and examples showed that this method is superior to conventional methods denoising ability, and could effectively

extract the useful information in all frequency bands. Studies showed that the seismic wavelet packet multi-threshold denoising method is very effective in signal denoising, with better denoising effect.

2. Wavelet Packet Transforms

2.1. Wavelet Packet Definition

Wavelet packet transform generalized wavelet analysis, by a group of low-pass orthogonal filter H and high-pass orthogonal filter G, for a given signal multi-level high-frequency and low-frequency decomposition were done, making the signal time and frequency features decomposition more sophisticated, suitable for describing and characterization of non-stationary signals. In the multi-resolution analysis, $L^2(R) = \underset{j \in Z}{\oplus} W_j$ show multi-resolution analysis is that the Hilbert space $L^2(R)$ is decomposed to subspaces $W_j (j \in Z)$ orthogonal according to a different scale factor j. In which, W_j is the closures for the wavelet function $\varphi(t)$ (wavelet subspace). The wavelet subspace W_j was done frequency subdivision as binary fractions, in order to achieve the purpose of improving the frequency resolution. The scale subspace V_j and wavelet subspace W_j characterize uniformly with a new subspace U_j^n, and order

$$\begin{cases} U_j^0 = V_j \\ U_j^1 = W_j \end{cases}, (j \in Z) \tag{1}$$

The Hilbert space orthogonal decomposition $V_{j+1} = V_j \oplus W_j$ could unify using U_j^n decomposition:

$$U_{j+1}^0 = U_j^0 \oplus U_j^1, j \in Z \tag{2}$$

Define subspace U_j^n is the closure space of function $u_n(t)$, U_j^{2n} is the closure space of $u_{2n}(t)$ and $u_n(t)$ satisfies the following two-scale equation:

$$\begin{cases} u_{2n}(t) = \sqrt{2} \sum_{k \in Z} h(k) u_n(2t-k) \\ u_{2n+1}(t) = \sqrt{2} \sum_{k \in Z} g(k) u_n(2t-k) \end{cases} \tag{3}$$

In which $g(k) = (-1)^k h(1-k)$, that is two coefficients have orthogonal relationships. When $n = 0$, the above equation becomes

$$\begin{cases} u_0(t) = \sum_{k \in Z} h_k u_0(2t-k) \\ u_1(t) = \sum_{k \in Z} g_k u_0(2t-k) \end{cases} \tag{4}$$

$u_0(t)$ and $u_1(t)$ are respectively scale function $\phi(t)$ and wavelet function $\varphi(t)$, Eq.(4) is the equivalent represention of Eq.(2). This equivalent representation were extended to $n \in Z_+$ (non-negative integer), that is equivalent representation of Eq.(3)

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1} \qquad j \in Z, n \in Z_+ \tag{5}$$

This allow the wavelet subspace further subdivided as binary:

$$W_j = U_j^1 = U_{j-1}^2 \oplus U_{j-1}^3$$
$$U_{j-1}^2 = U_{j-2}^4 \oplus U_{j-2}^5, U_{j-2}^5 = U_{j-2}^6 \oplus U_{j-2}^7$$
$$\dots$$
$$W_j = U_{j-k}^{2^k} \oplus U_{j-k}^{2^k+1} \oplus \dots \oplus U_{j-k}^{2^k+m} \oplus \dots \oplus U_{j-k}^{2^{k+1}-1} \tag{6}$$

In which orthonormal basis $\left\{ 2^{(j-k)/2} \varphi_{2^k+m}(2^{j-k}t - l); l \in z \right\}$ are called wavelet packet corresponding to $k = 1, 2, \dots j; j = 1, 2 \dots$ and subspaces $U_{j-k}^{2^k+m}$.

Assume $g_j^n(t) \in U_j^n$, then $g_j^n(t)$ can be expressed as $g_j^n(t) = \sum_l d_l^{j,n} u_n(2^j t - l)$. Wavelet packet decomposition are that $\left\{ d_l^{j,2n} \right\}$ and $\left\{ d_l^{j,2n+1} \right\}$ are obtained by $\left\{ d_l^{j+1,n} \right\}$, that is

$$d_l^{j,2n} = \sum_k a_{k-2l} d_k^{j+1,n}$$
$$d_l^{j,2n+1} = \sum_k b_{k-2l} d_k^{j+1,n} \tag{7}$$

Wavelet packet reconstruction are that $\left\{ d_l^{j+1,n} \right\}$ are obtain by $\left\{ d_l^{j,2n} \right\}$ and $\left\{ d_l^{j,2n+1} \right\}$, that is

$$d_l^{j+1,n} = \sum_k [h_{l-2k} d_k^{j,2n} + g_{l-2k} d_k^{j,2n+1}] \tag{8}$$

2.2. Denoising

In seismic exploration, the general wave interference include: surface waves, high frequency random noises, side waves, multiples, *etc*, in which the high-frequency surface waves and random interference waves have more serious impact on the effective waves, therefore it need to reduce noise processing to extract the useful signals. For non-stationary seismic signals, the wavelet packet decomposition technique is an effective noise reduction method.

In the wavelet analysis, the signal is decomposed into low frequency portion and high frequency detail rough. However, only the low-frequency portion is used for the second layer decompose while the high-frequency part without treatment, When the scale increases frequency resolution of wavelet analysis became lower and the resolution in high frequency became poor.

Wavelet packet analysis is further promotion of the wavelet analysis, which provides a more complex and flexible analysis tools. Wavelet packet analysis subdivide the low-frequency part and high-frequency portion of last layer while the high frequency part of the signal can be described more detailed, with a more precise local analysis capabilities. Typically, wavelet packet noise reduction steps [5-6]:

The first was wavelet packet decomposition of the signal: Choose a wavelet and determine a wavelet decomposition level N, then do N-layer wavelet packet decomposition on the seismic waves signal S. Described with a three-tier decomposition, the wavelet packet decomposition tree were shown in Figure 1. In Figure 1 A represented the low frequency, D represented high frequency, the end serial number represented wavelet packet decomposition layers, namely the scale number. The original signal S is equivalent to:

$$S = AAA3 + DAA3 + ADA3 + DDA3 + AAD3 + DAD3 + ADD3 + DDD3 \tag{9}$$

The second was to calculate the optimal tree: Compute the best tree for a given entropy criteria, commonly used entropy criteria were Shannon, threshold, norm, log energy, sure and user and so on.

The third was threshold quantization of wavelet packet decomposition coefficients: Select a threshold and do coefficients threshold quantization for each wavelet packet decomposition coefficients. Wavelet transform coefficient values are compared with a threshold, it is believed that the values smaller than the threshold value were generated by the noise and set to zero, the values greater than the threshold values were corresponding to the signal mutation point and retained in order to achieve the purpose of denoising. In the process of denoising there were usually three treatments: force denoising, the default threshold denoising, given soft (or hard) threshold denoising. Soft (or hard) threshold denoising [7-8] is most commonly used, but there is distortion.
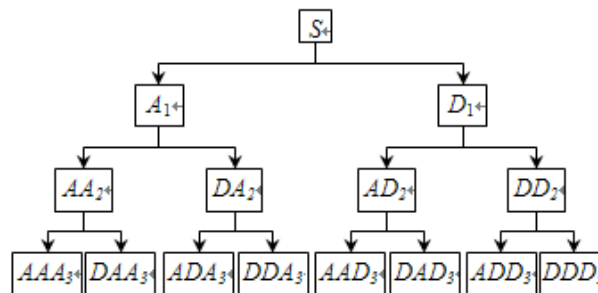

Figure 1. Three Layer Wavelet Packet Decomposition Diagram

The fourth was signal reconstruction: Do wavelet packet reconstruction on signals according to the L layer wavelet packet decomposition low-frequency coefficients of the original signal and high frequency coefficients after threshold quantization processing.

2.3. Wavelet Packet Frequency Sequences

With the example N = 3, the signal S was decomposed by three layers wavelet packet, the original signal S was equivalent to Eq.(9). In Eq.(9) A represented low-frequency, D represented the high frequency, the end serial number three indicated three-layer wavelet packet decomposition (logarithmic scale was three), the decomposition structure was shown in Figure 1. The noischir signals was done sym6 three layers wavelet decomposition, each wavelet packet decomposition coefficients were shown in Figure 2. From Figure 2 it can be found that the natural orders of wavelet packet tree node were inconsistencies with the frequency order. The lowest frequency portion is corresponding to AAA3; most high frequency portion corresponds to AAD3. From the literature [3] showed that any wavelet packet frequency sequences would produce dislocation, and the orders of frequency are different with the natural order. Each layer of wavelet packet decomposition, the low-frequency decomposition parts were sorted by frequency, high frequency part in descending order according to the frequency. The reason for this phenomenon is due to the wavelet packet decomposition, high-pass filter would conduct a "flip" operation. By the nature of wavelet packet can prove that any wavelet packet decomposition will produce the natural order and the order of frequency inconsistency and the inconsistency of the situation is the same.
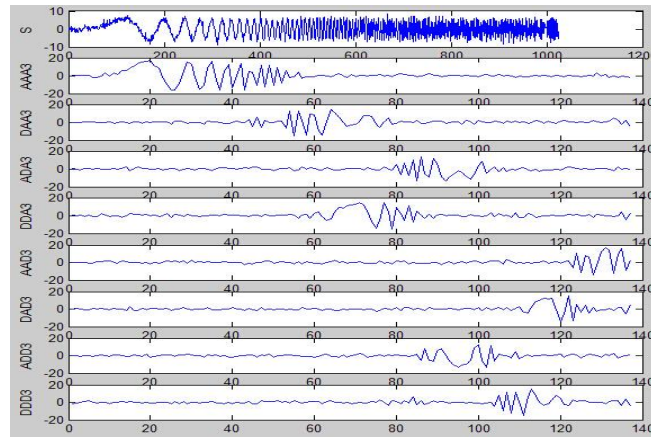
Figure 2. Three Layers Wavelet Packet Decomposition Coefficients of Noischir Signals

3. Choice of Wavelet Base and Multi-Threshold

3.1. Determination of Best Wavelet Packet (Calculates the Optimum Tree).

A signal with length of $L = 2^N$ could have 2^N kinds of different signal decomposition method, while the number of full two forks trees with depth of N are 2^N. This number is too large to enumerate every situation, and the minimum entropy criterion can be obtained by an optimal signal decomposition method.

There are several types of traditional standards based on entropy: shannon entropy, threshold entropy, norm entropy, logenergy entropy, sure entropy and user entropy and so on. Shannon entropy is defined as follows:

$$El(s_i) = -s_i^2 \log(s_i^2) \quad \text{and} \quad 0\log 0 = 0 \tag{10}$$

The signals were decomposed layer by layer, the entropy of each decomposition node was calculated, entropy values of a node and its child nodes were compared, the base obtaining minimum entropy is the optimal wavelet packet basis.

3.2. Multi-Threshold

Wavelet threshold denoising was that high frequency coefficient of wavelet decomposition were done threshold processing based on the characteristics that noises manifests themself in the high frequency signals. The high frequency coefficients less than threshold were considered caused by the noise and were set zero, the high frequency coefficients greater than the threshold value corresponding to the desired signals were left, to achieve denoising purposes.

Wavelet packet decomposition is the further promotion of wavelet analysis, further decomposes the high frequencies which were not done in wavelet analysis, so high-frequency noises and high-frequency signals could be separated to result in more ideal de-noising effect. Because the decomposed wavelet packet coefficients contain useful signals and the noises, if the same thresholds were taken for all the wavelet packet coefficients, it would cause over-denoising or the effect is not obvious or denoising phenomenon and would affect the denoising precision, so the multi-threshold criteria will well overcome these shortcomings.

The basic idea of multi-threshold criterion is different threshold criterias were flexiblely used for each wavelet packet decomposition coefficients in order to maximizely retain the useful information and remove useless information according to certain rules. The wavelet packet coefficients could order in order of frequency in

ascending to perform band division while the frequency coefficients of wavelet packet decomposition were known. According to the different manifestations of wavelet packet decomposition coefficients of the noise and the signal on each band, multi- thresholds were used for denoising. Different threshold processing methods were taken in different frequency bands, finally were done signal reconstruction to achieve multi- threshold processing on different bands.

Four threshold criteria commonly used in wavelet packet analysis include fixed form threshold criterion (sqtwolog), adaptive threshold criterion (rigrsure), heuristic threshold criterion (heursure) and minimax threshold criterion (minimaxi), denoising effect is different due to the respective selection rule varies.

(1) Fixed form threshold criterion (sqtwolog). Fixed form thresholds were used and the threshold was sqrt(2*log(length(X))).

(2) Adaptive threshold criterion (rigrsure). It is adaptive threshold selection based onStein unbiased likelihood estimation principle.

(3) Heuristic threshold criterion (heursure). It is the best predictor threshold selection. If the SNR is small, SURE estimation would cause loud noises, in this case such a fixed threshold could be used.

(4) Minimax threshold criterion (minimaxi). It produces minimum mean square error extremes.

Four kinds of threshold criterion have their own characteristics. Minimaxi threshold criteria and rigrsure criteria are more conservative, which not easy to lose useful signals components, but only remove less noise. Sqtwolog threshold criterion and heursure threshold criterion are similar, and all their factors have been processed strongly removing noise, easily cause over denoising correspondingly, which could be called "radical" noise guidelines. Therefore, minimaxi criteria and guidelines rigrsure could be used in in the low frequency band of decomposed signals, and sqtwolog threshold criterion and heursure threshold criteria could be used in the high frequency part.
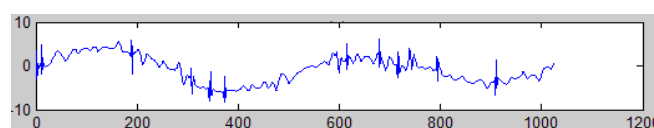
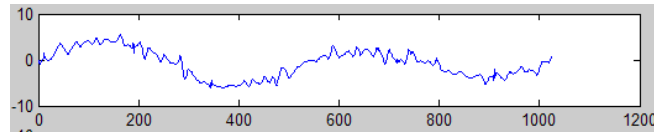## 4. Simulation

4.1. Simulation Signal Denoising.

Special test signal heavysine as signals of evaluation wavelet denoising quality was used with Gaussian white noise and the signal to noise ratio is 10dB, the signal length are one thousands and twenty four sampling points, the sampling frequency is 1Hz. Wavelet packet hard threshold method, wavelet packet soft threshold method, wavelet packet multi-threshold method were used for processing the signals with noises, the results were shown in Figure 3. Figure 3 (a) are the noisy signals, with the SNR 10dB, Figure 3 (b) are the signals after the wavelet packet hard threshold denoising, Figure 3 (c) are the signals after the wavelet packet soft threshold denoising, Figure 3 (d) are the signals after the wavelet packet multi-threshold denoising. The results showed that the wavelet packet multi-threshold denoising was best.
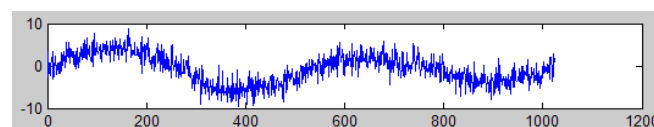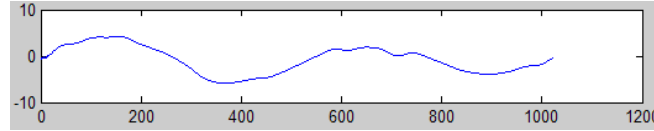

(a) the noisy signals

(b) The signals after the wavelet packet hard threshold denoising

(c) The signals after the wavelet packet soft threshold denoising

(d) The signals after the wavelet packet multi-threshold denoising

Figure 3. Simulation Signal Denoising

In order to compare noise reduction results of different threshold noise-reduction methods, the root mean square error (RMSE), signal to noise ratio (SNR)) were chosen to compare the denoising effect quantitatively. The original signal was as a standard signal f (n) and the noisy signal was s (n), the signal length was L, signal to noise ratio (SNR) is defined as formula:

$$SNR = 10 * \log \left[\frac{\sum_{i=1}^{L} f^2(i)}{\sum_{i=1}^{L} (s(i) - f(i))^2} \right] (db) \tag{11}$$

Root mean square error (RMSE) between the original signal and the estimated signals is defined as:

$$RMSE = \sqrt{\frac{1}{L} \sum_{i=1}^{L} (s(i) - f(i))^2} \tag{12}$$

The better the denoising, the greater the signal to noise ratio and the smaller the root mean square error. Noising effect of different methods were shown in Table 1. As known from Table 1, the signal to noise ratio after wavelet packet multi-threshold were maximum, the root mean square error was minimized, and denoising effect was best.

Table 1. Comparison of Various Methods of Denoising

	SNR/dB	RMSE/mm
the wavelet packet hard threshold denoising	23.0231	1.0632
the wavelet packet soft threshold denoising	23.8627	1.0616
the wavelet packet multi-threshold denoising	26.5822	0.9357

4.2. Analog Seismic Record Noising.

In order to verify the effectiveness of the new threshold function wavelet packet denoising a period of analog seismic signals added random noise were tested, the length of the signal is1000, the signal to noise ratio added noise is 7 dB, "sym4" wavelets was selected to conduct

three layers wavelet packet decomposition, coefficients after decomposition were done denoising based on different frequency bands. Minimaxi criterion and rigrsure criterion could

be used in the low bands and middle bands, and sqtwolog threshold criterion and heursure threshold criterion could be used in the high frequency parts. Figure 4 showed

the processing to noisy signals, Figure 4 (a) was the original analog signals, Figure 4 (b) was the signals with noises of SNR7dB ,Figure 4(c) was the signals after conventional wavelet threshold denoising, Figure 4 (d) was the signals after multi-threshold denoising.
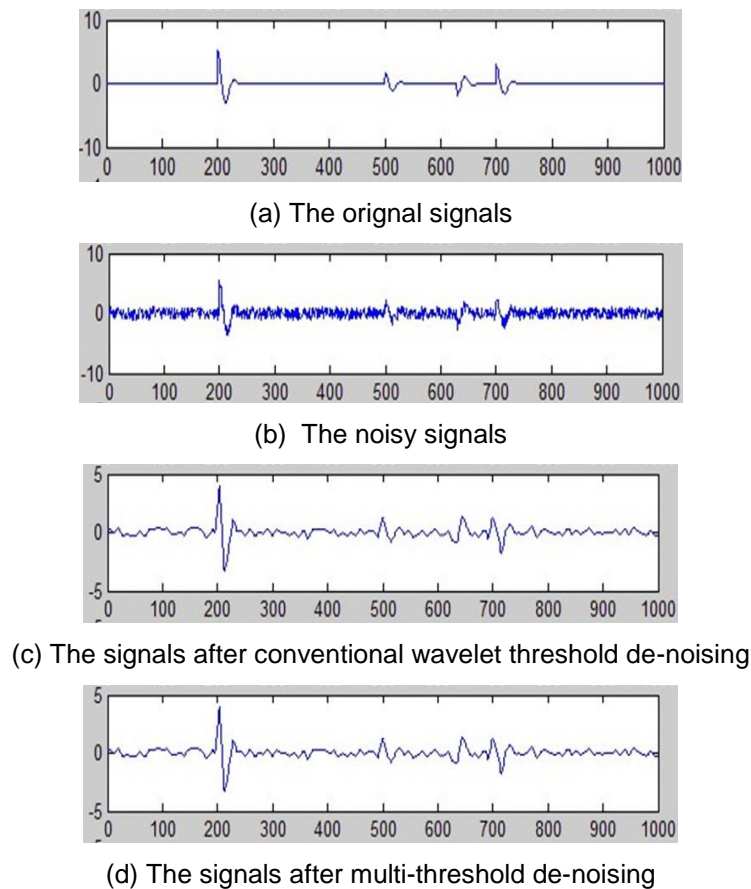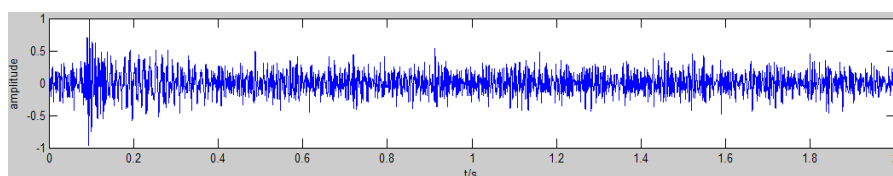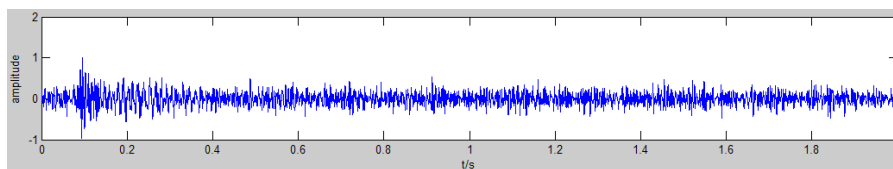

(a) The orignal signals

(b) The noisy signals

(c) The signals after conventional wavelet threshold de-noising

(d) The signals after multi-threshold de-noising

Figure 4. Analog Seismic Signals Denoising

As can be seen from Figure 4, the latter two methods have increased SNR denoising, most noise have been suppressed, the distortion is small and smoothness is better after using the new threshold, high-frequency useful information of signals is relatively well preserved, and the signal to noise ratio is higher than conventional wavelet threshold denoising,with good results.

5. The Seismic Data Denoising

The interference noises are inevitable in actual seismic data acquisition, data acquisited should been done denoising processing. Distributed seismographs were used for observation monitoring point seismic data in experiments with four hundreds and eighty channels, track pitch were one meters, offset were ten meters, gun pitch (distances between two shots) were two meters, seismograph sampling rate were eight thousands, recording time were four seconds. Sym4 wavelet was used to do wavelet decomposition to the acquired seismic waves for three times. Figure 5 (a) was parts data of the original data of one channel, and Figure 5 (b) was the signals after the conventional wavelet threshold denoising, and Figure 5 (c) was signals after mult-threshold denoising.

(a) The orignal signals

(b) The signals after the conventional wavelet threshold denoising
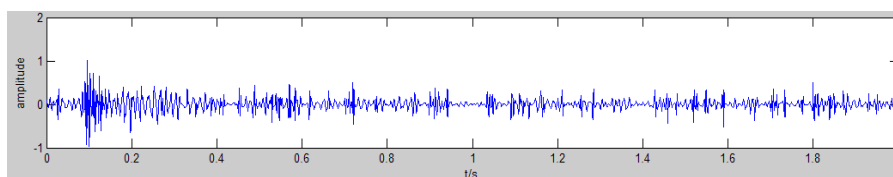
(c) Signals after mult- threshold denoising

Figure 5. Analog Seismic Signals Denoising

As can be seen from Figure 5 seismic signals after conventional threshold function denoising are filtered part noises, seismic signals after mult-threshold function denoising are filtered out most of the noises and components of useful signals are well preserved.Before doing wavelet packet multi-threshold denoising, twenty and four channels seismic profiles were previously done a fine processing, firstly the deconvolution was used to compress wavelet to improve seismic resolution, then the static correction, velocity analysis and NMO correction were done, the profile after procesing was showed in Figure 6 (a), and then the superimposed multi-channel seismic profile was done wavelet packet multi-threshold processing, as shown in Figure 6 (b). By observation the phase axis of processed seismic profile were more clearly which effectively improve the resolution of seismic data.
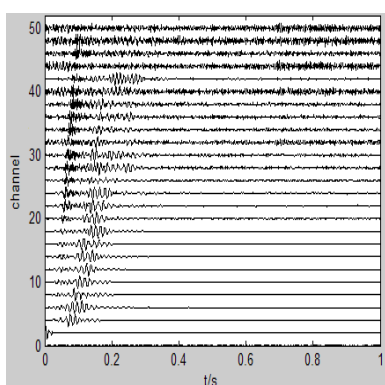
(a)The profile after procesing
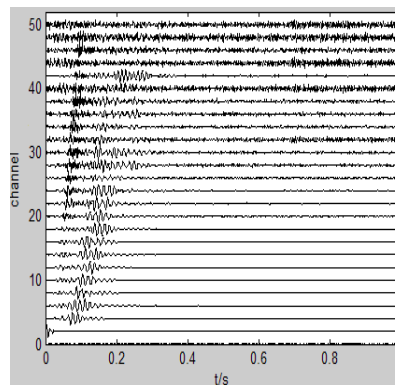
(b) the profile after wavelet packet multi-threshold processing

Figure 6. Multi-Threshold Wavelet Packet Denoising Common Shot Gathers

6. Conclusions

A multi-threshold seismic wavelet packet method was put forward, coefficients of the signals through wavelet packet decomposion are arranged in the order of magnitude of

frequency,in different bands an appropriate threshold rule was selected and threshold processing was done. The simulation signalsynthetic seismogram and real seismic record were processed, from theanalysis of SNR and RMSE it can be drawn that the method could effectively remove the seismic signal noises and improve signal to noise ratio and resolution of seismic data.

Acknowledgments

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