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Zero-Input Compensation Mechanism-Based Linear Estimation for Systems with Multiple Packet Dropouts and Multiplicative Noises

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Abstract

This paper studies the optimal linear estimation problem for systems with multiple packet dropouts and multiplicative noises. When the current measurement is lost, the popular zero-input compensation mechanism is used for compensation. Based on the zero-input compensator, the optimal linear estimators including filter, predictor and smoother in the linear minimum variance sense are given by innovation analysis approach. The proposed estimators can reduce the computational cost compared to the existing augmented estimators based on the hold-input mechanism. The performances of the two kinds of estimators are compared in terms of two simulation examples. The conclusion is that neither of the two compensation mechanisms can be claimed to be superior to the other.

Keywords: *Packet dropout; Compensation; Linear estimation; Multiplicative noise; Innovation analysis approach*

1. Introduction

Recently, the control and estimation problem for networked control system have received much attention due to their wide applications in target tracking, environmental monitoring, and communication [1-4]. Because of the communication noise, interference or congestion, random packet dropouts could occur in data transmissions, which is challenging for filtering of the system.

For systems with packet dropouts, from the literatures, there are two popular compensation mechanisms: the hold-input and the zero-input mechanism. The former means that the latest measurement or control signal received is used whereas the latter adopts zero value whenever the current signal is lost. The two compensators are straight forward and easy to implement. Based on the hold-input compensator, the stabilization problem of networked controls by a switched system approach [5], the optimal H_2 filtering problem by LIM approach [6], the optimal linear estimation and steady-state estimation problem by innovation analysis approach [7-8] are investigated, respectively. Based on the zero-input compensator, the optimal filtering problem for systems with multi-step random delays and multiple packet dropouts are studied in [9]. Further, the possible packet dropouts for a rehabilitation system [10] and for T-S fuzzy dynamic systems [11] are tackled, respectively.

In many practical systems, there often exist various uncertainties due to the unknown or partially unknown parameters and environmental disturbances. The

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uncertainties can be approximated mathematically by an additive noise or a multiplicative noise [12-16]. These systems are widely used in target tracking, detection, signal processing and other areas. Thus the research on systems with multiple packet dropouts and various stochastic uncertainties such as additive noise and multiplicative noise has the important practical significance. For such system, many results have been proposed, including the robust filter [13] for system with missing measurements, the distributed fusion filter [14] for multi-rate system, the linear augmented filter [15] and full-order filter [16] based on the hold-input compensator.

Motivated by the above literatures, in this paper, we investigate the optimal linear estimation problem for systems with multiple packet dropouts and stochastic multiplicative noise. First, we derive the optimal linear one-step predictor based on the zero-input mechanism by applying the innovation analysis approach. Then, we give the optimal linear filter and smoother based on the proposed one-step predictor. Compared to the augmented estimators based on the hold-input mechanism [15], the proposed filter can reduce the computational burden. Finally, we use two examples to compare the estimation performance of the above two estimators. The results show that neither of the two compensators always outperforms the other.

2. Problem Formulation

Consider the following linear discrete-time system modeled by

$$x(t+1) = (\Phi + \xi(t)\Phi_1)x(t) + \Gamma w(t) \tag{1}$$

$$y(t) = (H + \lambda(t)H_1)x(t) + v(t) \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^m$ is the sensor output. Φ, Γ and H are constant matrices. $w(t) \in \mathbb{R}^r$ and $v(t) \in \mathbb{R}^m$ are correlated zero-mean white noises with covariance matrices $\mathrm{E}[w(t)w^{\mathrm{T}}(t)]=Q_w$, $\mathrm{E}[v(t)v^{\mathrm{T}}(t)]=Q_v$ and $\mathrm{E}[w(t)v^{\mathrm{T}}(t)]=S$. Multiplicative noises $\xi(t)$ and $\lambda(t)$ are scalar white noise sequences that are uncorrelated with other random variables and are introduced for the structured perturbation in system and measurement matrices. They are of zero-mean with variance matrices Q_ξ and Q_λ. The initial state $x(0)$ is independent of $w(t)$ and $v(t)$, and has $\mathrm{E}\{x(0)\}=\sigma_0$ and $\mathrm{E}\{(x(0)-\sigma_0)(x(0)-\sigma_0)^{\mathrm{T}}\}=\Sigma_0$.

We assume that there exist possible consecutive packet dropouts during the data transmission from the sensor to the estimator over a network. Then, a compensation measurement is needed at the estimator when the current measurement is not available. Here, we consider the zero-input compensation method:

$$y_z(t) = \eta(t)y(t) \tag{3}$$

where $\eta(t)$ is an i.i.d Bernoulli process with $P\{\eta(t)=1\}=\bar{\eta}$ and $P\{\eta(t)=0\}=1-\bar{\eta}$. The subscripts z in the compensation measurements indicates the zero-input mechanism.

Remark 1: From (3), we see that if $\eta(t)=0$, *i.e.*, the current measurement is lost, zero is used for compensation.

3. Optimal Linear Estimators Based on the Zero-Input Compensator

In this section, we will give the optimal linear estimators including filter, predictor and smoother based on the zero-input compensation mechanism by the innovation analysis approach.

3.1. Linear One-Step Predictor

In this subsection, we shall derive the optimal linear one-step predictor.

Theorem 1. For system (1)-(3), the optimal linear one-step predictor is given by

$$\hat{x}_z(t+1|t)=\Phi\hat{x}_z(t|t-1)+L_z(t)\varepsilon_z(t) \tag{4}$$

$$\varepsilon_z(t)=y_z(t)-\bar{\eta}H\hat{x}_z(t|t-1) \tag{5}$$

$$L_z(t)=\bar{\eta}(\Phi P_z(t|t-1)H^{\mathrm{T}}+\Gamma S)Q_{\varepsilon_z}^{-1}(t) \tag{6}$$

$$Q_{\varepsilon_z}(t)=\bar{\eta}(1-\bar{\eta})Hq(t)H^{\mathrm{T}}+\bar{\eta}Q_v+\bar{\eta}^2HP_z(t|t-1)H^{\mathrm{T}}+\bar{\eta}Q_\lambda H_1q(t)H_1^{\mathrm{T}} \tag{7}$$

$$q(t+1)=\Phi q(t)\Phi^{\mathrm{T}}+Q_\xi\Phi_1q(t)\Phi_1^{\mathrm{T}}+\Gamma Q_w\Gamma^{\mathrm{T}} \tag{8}$$

$$P_z(t+1|t)=(\Phi-\bar{\eta}L_z(t)H)P_z(t|t-1)(\Phi-\bar{\eta}L_z(t)H)^{\mathrm{T}}+Q_\xi\Phi_1q(t)\Phi_1^{\mathrm{T}}+\bar{\eta}(1-\bar{\eta})L_z(t)Hq(t)H^{\mathrm{T}}L_z^{\mathrm{T}}(t)$$

$$+\bar{\eta}(1-\bar{\eta})Q_\lambda L_z(t)H_1q(t)H_1^{\mathrm{T}}L_z^{\mathrm{T}}(t)+\bar{\eta}L_z(t)Q_vL_z^{\mathrm{T}}(t)+\Gamma Q_w\Gamma^{\mathrm{T}}-\bar{\eta}L_z(t)S^{\mathrm{T}}\Gamma^{\mathrm{T}}-\bar{\eta}\Gamma SL_z^{\mathrm{T}}(t) \tag{9}$$

where $\varepsilon_z(t)$ is the innovation sequence with variance $Q_{\varepsilon_z}(t)$, $L_z(t)$ is prediction gain matrix, $P_z(t|t-1)$ is the one-step prediction error variance matrix, $q(t)$ is the state second-order moment matrix of the system state $x(t)$ .The initial values are $\hat{x}_z(0|-1)=\sigma_0$, $P_z(0|-1)=\Sigma_0$ and $q(0)=\sigma_0\sigma_0^{\mathrm{T}}+\Sigma_0$.

Proof: By projection [17], we have (4) and (5). The gain matrix $L_z(t)$ is defined as:

$$L_z(t)=\mathrm{E}[x(t+1)\varepsilon_z^{\mathrm{T}}(t)]Q_{\varepsilon_z}^{-1}(t) \tag{10}$$

Substituting (3) into (5), $\varepsilon_z(t)$ can be rewrite as

$$\varepsilon_z(t)=[\eta(t)-\bar{\eta}]Hx(t)+\eta(t)\lambda(t)H_1x(t)+\bar{\eta}H\tilde{x}_z(t|t-1)+\eta(t)v(t) \tag{11}$$

where the error is denoted by $\tilde{x}_z(t|t-1)=x(t)-\hat{x}_z(t|t-1)$. From (1) and $\mathrm{E}[\xi(t)]=0$, we have

$$\mathrm{E}[x(t+1)\varepsilon_z^{\mathrm{T}}(t)]=\Phi\,\mathrm{E}[x(t)\varepsilon_z^{\mathrm{T}}(t)]+\Gamma\,\mathrm{E}[w(t)\varepsilon_z^{\mathrm{T}}(t)]$$

From (11), and using $\mathrm{E}[\eta(t)-\bar{\eta}]=0$, $\mathrm{E}[\lambda(t)]=0$, $x(t)\perp v(t)$, $\tilde{x}_z(t|t-1)\perp\hat{x}_z(t|t-1)$, $\tilde{x}_z(t|t-1)\perp v(t)$, and $\tilde{x}_z(t|t-1)\perp w(t)$, where the symbol "$\perp$"denotes orthogonality, we have

$$\mathrm{E}[x(t)\varepsilon_z^{\mathrm{T}}(t)]=\bar{\eta}\,\mathrm{E}[\tilde{x}_z(t|t-1)\tilde{x}_z^{\mathrm{T}}(t|t-1)]H^{\mathrm{T}}=\bar{\eta}P_z(t|t-1)H^{\mathrm{T}},\ \mathrm{E}[w(t)\varepsilon_z^{\mathrm{T}}(t)]=\bar{\eta}S \tag{12}$$

Substituting (12) into (10) yields (6). By substituting (11) into $Q_{\varepsilon_z}(t)=\mathrm{E}[\varepsilon_z(t)\varepsilon_z^{\mathrm{T}}(t)]$ and using $\mathrm{E}[(\eta(t)-\bar{\eta})^2]=\bar{\eta}(1-\bar{\eta})$, we obtain (7). From state Equation (1), $\mathrm{E}[\xi(t)]=0$ and $x(t)\perp w(t)$, we obtain the state second-order moment matrix $q(t)=\mathrm{E}[x(t)x^{\mathrm{T}}(t)]$ of state $x(t)$ as follows:

$$q(t+1)=\mathrm{E}[(\Phi+\xi(t)\Phi_1)x(t)x^{\mathrm{T}}(t)(\Phi+\xi(t)\Phi_1)^{\mathrm{T}}]+\Gamma Q_w\Gamma^{\mathrm{T}} \tag{13}$$

From (13), and noting $\mathrm{E}[\xi(t)]=0$, we obtain (8).

Next, we derive the one-step prediction error variance matrix $P_z(t+1|t)=\mathrm{E}[\tilde{x}_z(t+1|t)\tilde{x}_z^{\mathrm{T}}(t+1|t)]$. By subtracting (4) from $x(t+1)$ yields the error equation

$$\tilde{x}_z(t+1|t) = x(t+1) - \hat{x}_z(t+1|t) = (\Phi - \bar{\eta}L_z(t)H)\tilde{x}_z(t|t-1)$$

$$+(\eta(t)-\bar{\eta})L_z(t)Hx(t) + \eta(t)\lambda(t)L_z(t)Hx(t) + \xi(t)\Phi_1 x(t) + \Gamma w(t) - L_z(t)\eta(t)v(t) \tag{14}$$

From (14), we have (9).

In the following text, we shall derive the N-step predictor, filter and N-smoother based on the obtained one-step predictor.

3.2. Optimal Linear N-Step (N>1) Predictor

Theorem 2. For system (1)-(3), the optimal linear N-step predictor is given by

$$\hat{x}_z(t+N|t) = \Phi\hat{x}_z(t+N|t-1) \tag{15}$$

The N-step prediction error covariance is given by

$$P_z(t+N|t) = \Phi P_z(t+N-1|t)\Phi^{\mathrm{T}} + \Gamma Q_w \Gamma^{\mathrm{T}} + Q_\xi \Phi_1 q(t+N-1)\Phi_1^{\mathrm{T}} \tag{16}$$

where the initial values $\hat{x}_z(t+1|t)$, $P_z(t+1|t)$ and $q(t+N-1)$ are computed by Theorem 1.

Proof: This proof is analogous to [7] and is omitted here.

Next, we will give the optimal linear filter and N-step (N > 1) smoother.

3.3. Optimal Linear Filter and Smoother

Theorem 3. For system (1)-(3), the optimal N-step smoother (N>0) and linear filter (N=0) are given by

$$\hat{x}_z(t|t+N) = \hat{x}_z(t|t+N-1) + K_z(t+N)\varepsilon_z(t+N) \tag{17}$$

where the corresponding gain matrices are given by

$$K_z(t+N) = \bar{\eta}M(t+N)H^{\mathrm{T}}Q_{\varepsilon_z}^{-1}(t+N) \tag{18}$$

$$M(t+N) = M(t+N-1)(\Phi - \bar{\eta}L_z(t+N-1)H)^{\mathrm{T}} \tag{19}$$

and the estimation error covariance matrices are computed by

$$P_z(t|t+N) = P_z(t|t+N-1) - K_z(t+N)Q_{\varepsilon_z}(t+N)K_z^{\mathrm{T}}(t+N) \tag{20}$$

where the initial values $\hat{x}_z(t|t-1)$ and $P_z(t|t-1)$ are computed by Theorem 1.

Proof: From projection theory [17], we obtain (17), where the gain matrices $K_z(t+N)$ are defined as

$$K_z(t+N) = \mathrm{E}[x(t)\varepsilon_z^{\mathrm{T}}(t+N)]Q_{\varepsilon_z}^{-1}(t+N) \tag{21}$$

By replacing t of (11) with $t+N$, we have the innovation at moment $t+N$ as

$$\varepsilon_z(t+N) = [\eta(t+N)-\bar{\eta}]Hx(t+N) + \eta(t+N)\lambda(t+N)H_1 x(t+N)$$

$$+\bar{\eta}H\tilde{x}_z(t+N|t+N-1) + \eta(t+N)v(t+N) \tag{22}$$

By defining $M(t+N) = \mathrm{E}[x(t)\tilde{x}^{\mathrm{T}}(t+N|t+N-1)]$ and using $x(t) \perp v(t+N)$, $\mathrm{E}[\eta(t+N)-\bar{\eta}] = 0$ and $\mathrm{E}[\lambda(t+N)] = 0$, we obtain $\mathrm{E}[x(t)\varepsilon^{\mathrm{T}}(t+N)] = \bar{\eta}M(t+N)H^{\mathrm{T}}$. Substituting it into (21), the gain matrices (18) are obtained. Next, we compute $M(t+N)$. From (14), we have

$$\tilde{x}_z(t+N|t+N-1) = (\Phi - \bar{\eta}L_z(t+N-1)H)\tilde{x}_z(t+N-1|t+N-2)$$

$$+(\eta(t+N-1)-\bar{\eta})L_z(t+N-1)Hx(t+N-1)+\eta(t+N-1)\lambda(t+N-1)L_z(t+N-1)Hx(t+N-1)$$

$$+\xi(t+N-1)\Phi_1 x(t+N-1)+\Gamma w(t+N-1)-L_z(t+N-1)\eta(t+N-1)v(t+N-1) \tag{23}$$

By applying $\mathrm{E}[\eta(t+N-1)-\bar{\eta}]=0$, $\mathrm{E}[\lambda(t+N-1)]=0$, $x(t)\perp w(t+N-1)$ and $x(t)\perp v(t+N-1)$ ($N\geq 1$), we get (19).

Next, we derive the estimation error variance matrices $P_z(t\,|\,t+N)=\mathrm{E}[\tilde{x}_z(t\,|\,t+N)\tilde{x}_z^{\mathrm{T}}(t\,|\,t+N)]$. By subtracting (17) from $x(t)$ yields

$$\tilde{x}_z(t\,|\,t+N)=x(t)-\hat{x}_z(t\,|\,t+N)=\tilde{x}_z(t\,|\,t+N-1)-K_z(t+N)\varepsilon_z(t+N) \tag{24}$$

Rewrite (24) as

$$\tilde{x}_z(t\,|\,t+N)+K_z(t+N)\varepsilon_z(t+N)=\tilde{x}_z(t\,|\,t+N-1)$$

Using $\tilde{x}_z(t\,|\,t+N)\perp\varepsilon_z(t+N)$, we have

$$P_z(t\,|\,t+N)-K_z(t+N)Q_{\varepsilon_z}(t+N)K_z^{\mathrm{T}}(t+N)=P_z(t\,|\,t+N-1) \tag{25}$$

From (25), we have (20).

Remark 3. From Theorems 1-3, the computational cost of the proposed estimators under the zero-input mechanism has the order of magnitude $O((n)^3)$. Compared with the augmented estimators with the magnitude $O((n+m)^3)$ under the hold-input mechanism in [15], the computational cost can be reduced.

4. Simulation Example

In this section, we give two examples to show the effectiveness of the proposed estimators.

Example 1: Consider the numerical example in [15].

$$x(t+1)=\left(\begin{bmatrix}0.8 & 0\\ 0.9 & 0.2\end{bmatrix}+\xi(t)\begin{bmatrix}0.1 & 0.05\\ 0.2 & 0.1\end{bmatrix}\right)x(t)+\begin{bmatrix}1\\ 0.5\end{bmatrix}w(t) \tag{26}$$

$$z(t)=\left([1\quad 2]+\lambda(t)[1\quad 2]\right)x(t)+v(t) \tag{27}$$

where the white noise $w(t)$ of mean 0 and variance 1 is correlated with $v(t)$, and they satisfy the relation $v(t)=cw(t)+\varsigma(t)$ where white noise $\varsigma(t)$ of mean 0 and variance 1 is uncorrelated with $w(t)$ and the correlation coefficient $c=1$. In the simulation, we set $Q_\xi=2$ and $Q_\lambda=2$.

The tracking performances of the estimators with $\bar{\eta}=0.8$ are shown in Figures 1-3. The corresponding estimation error variances are shown in Figure 4. From Figures 1-4, we see that the smoother gives the best performance while the predictor is the worst.

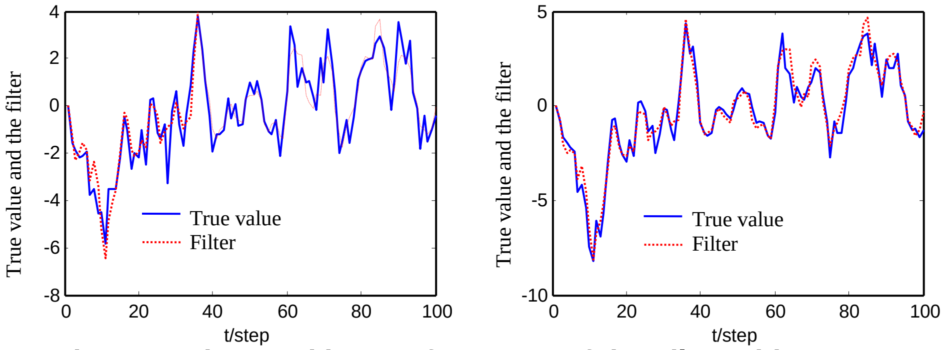

Figure 1. The Tracking Performance of the Filter with $\bar{\eta}=0.8$

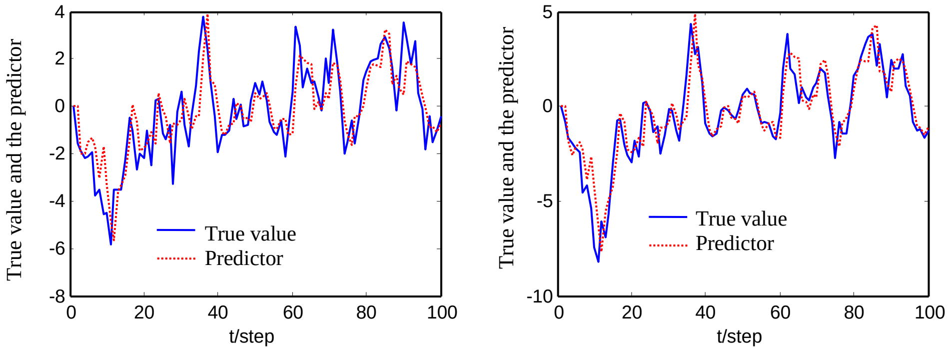

Figure 2. The Tracking Performance of the Predictor with $\bar{\eta}=0.8$

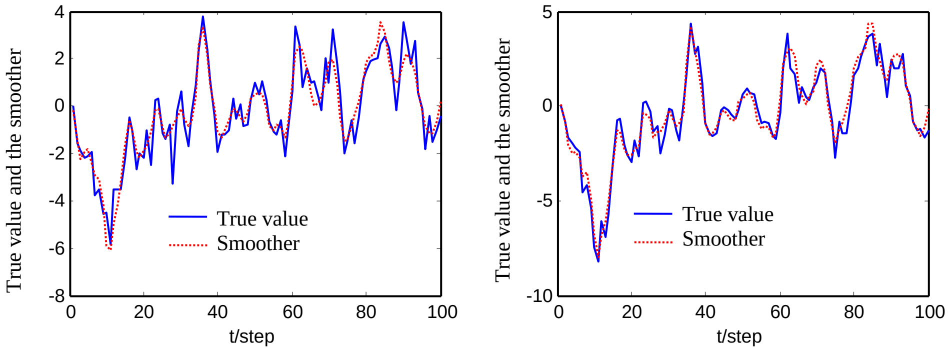

Figure 3. The Tracking Performance of the Smoother with $\bar{\eta}=0.8$

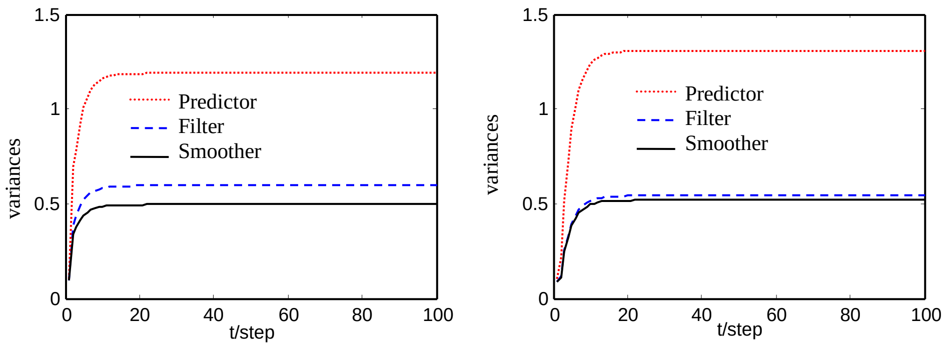

Figure 4. Estimation Error Variances with $\bar{\eta}=0.8$.

Figure 5, shows the estimation error variances against $\bar{\eta}$ ranging from 0.1 to 1. Figure 6, shows the filtering error variances against Q_ξ and Q_λ ranging from 0 to 2. It is clear that the proposed filter performs better for larger $\bar{\eta}$ and smaller Q_ξ and Q_λ.

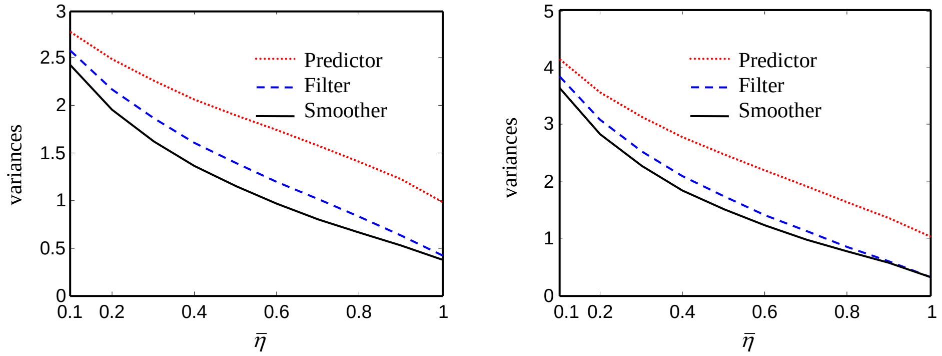

Figure 5. Estimation Error Variances with $Q_\xi = 2$, $Q_\lambda = 2$, $0.1 \le \bar{\eta} \le 1$

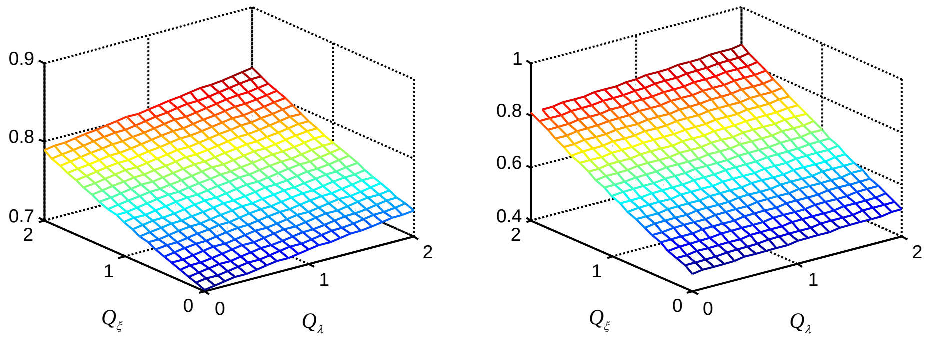

Figure 6. Filtering Error Variances with $0 \le Q_\xi, Q_\lambda \le 2$, $\bar{\eta} = 0.8$

To compare the performance of the proposed filter under the zero-input mechanism and the augmented filter under the hold-input mechanism, the filtering error variances of the two filters with $\bar{\eta} = 0.8$ are shown in Figure 7. It can be seen that the hold-input mechanism performs better than the zero-input mechanism.

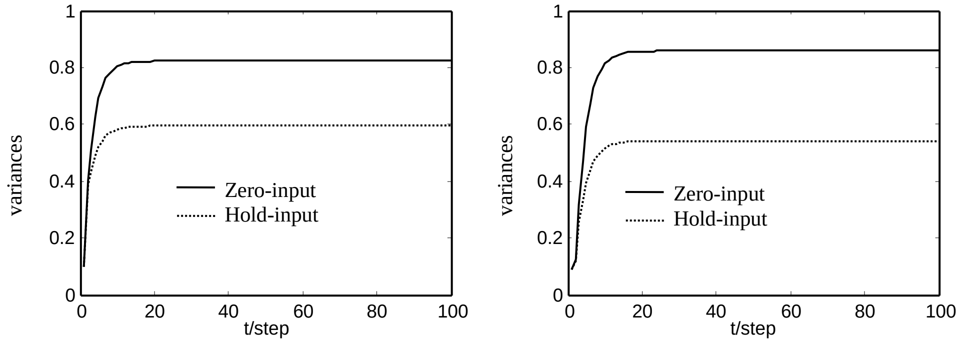

Figure 7. Filtering Error Variances of the Two Filters

To further compare the performance of the two filters, we use the following example, the corresponding parameter matrices are given as follows:

$$\Phi = \begin{bmatrix} 0.9226 & -0.6330 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad \Phi_1 = \begin{bmatrix} 0.1 & 0.05 & 0.1 \\ 0.2 & 0.1 & 0.25 \\ 0 & 0.3 & 0.2 \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0.5 \\ 0 \\ 0.2 \end{bmatrix},$$

$$H = \begin{bmatrix} 23.738 & 20.287 & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.2 & 0.15 & 0.1 \end{bmatrix} \tag{28}$$

Other parameters are the same as example 1. The filtering error variances of the two mechanisms with respect to arrival rate $\bar{\eta} = 0.8$ are shown in Figure 8. It can be seen that the zero-input mechanism performs better than the hold-input mechanism. Hence, the conclusion is that neither of the two compensators always outperforms the other. On the other hand, the proposed estimators can reduce the computational cost.

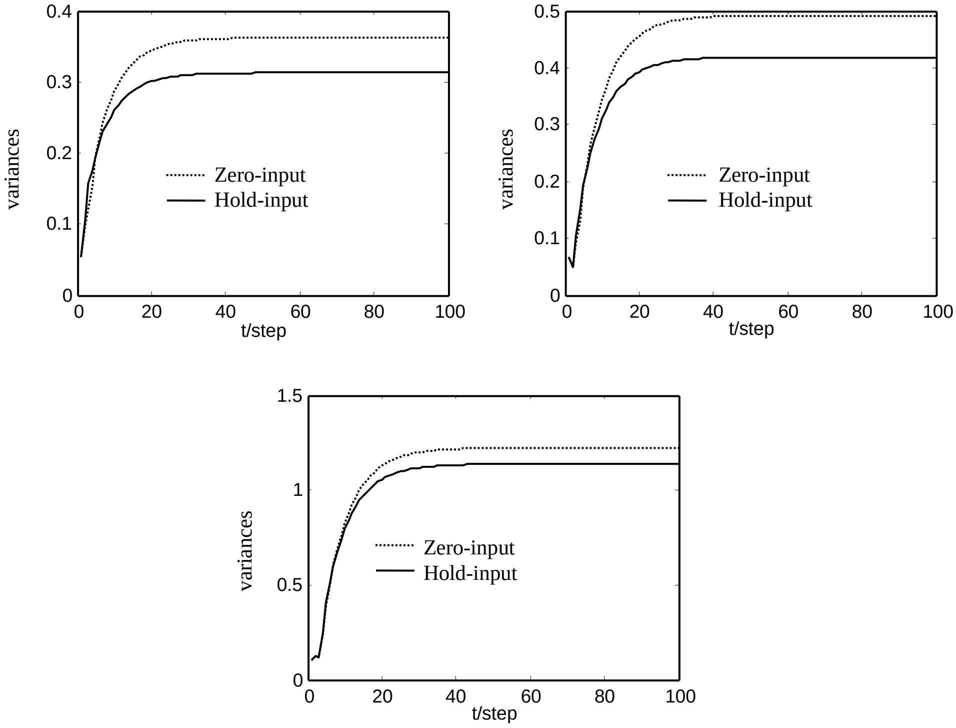

Figure 8. Comparison Curves of Filtering Error Variance of the Two Compensation Mechanisms

5. Conclusion

In this article, the optimal linear estimation problem for system with random packet dropouts and stochastic uncertainties of multiplicative noises is investigated. The zero-input compensator is used to design the estimators in the linear minimum variance sense. The comparison of the estimation performance of the proposed estimators under the zero-input compensator and the augmented estimators under the hold-input compensator is given in simulation example section. It can be concluded from the simulation results that neither of the two compensators always outperforms the other. Hence, it is necessary to design an optimal compensator. This is also a research topic in the future.

Acknowledgments

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