

Inhomogeneity Image Segmentation with Optimal Spatial Scale

Xiaozhen Xie¹ and Xiaoning Hu^{2*} and Bo Yang³

^{1,2}College of Science, Northwest A&F University, Yangling 712100, PR China

³School of Electronic and Information Engineering, Beihang University, Beijing 100191, PR China

²Xiaoning Hu, hcn1985@163.com

Abstract

A novel local region-based active contour model is proposed to segment medical images with intensity inhomogeneities and various noises. The contribution of the proposed work is twofold. First, the anisotropy of evolution contours is exploited to characterize the local classification information around each pixel. Integrating it with local gray intensity information, the new model stabilizes the active contours in all evolving processes. Second, under the constraint of maximum absolute error of parameter estimation, the optimal spatial scales are automatically selected for the local segmentation models. It is demonstrated from the experiments that our algorithm achieves faster and more robust results than several same-type methods.

Keywords: *Image segmentation, Intensity inhomogeneity, Spatial scale, Active contours, Level set method*

1. Introduction

Intensity inhomogeneity is a low-frequency spatially varying artifact, which occurs often in medical images, such as X-ray radiography, computerized tomography, ultrasound, magnetic resonance images and so on. In addition, complex noise also appears and badly distorts these medical images. They complicate many problems in medical image segmentation due to a strong intensity variation within tissue of the same physical properties.

The active contour models have been proved to be an efficient framework for image segmentation. They basically fall into two categories: edge-based active contour models [1-2], and region-based active contour models [3-4]. The edge-based active contour models mainly use the edge information such as image gradient to stop the contour evolution. Their results depend on the edge detector function which is not only very sensitive to noise but also difficult to detect weak edges. Hence, edge-based models can not obtain the satisfactory segmentation results from images with intensity inhomogeneities. Instead of the gradient information, the region-based models use the image statistical information to basically overcome the above disadvantages. Especially, [5] point out that the local statistical information can appropriately detect the objects in images with intensity inhomogeneities. They use the local intensity variances as the local statistical information to propose the well-known local region-based model, *i.e.*, the LBF model, getting the right segmentation results. Then the local statistic information is widely adopted to segment the inhomogeneity images. Due to the high computational cost of variances, the lower order statistic, *i.e.*, the local intensity mean, is applied by [6] to propose the LIF model. Although the LIF model significantly improves the computational efficiency of LBF model and yields the similar results, it becomes weaker in stabilization

* Corresponding Author

and robustness. Along the line of above local region based models, a series of models are introduced to attempt to achieve better segmentation results [7–11].

As is pointed out by [12], the locality of aforementioned local region based models is generally defined by means of an isotropic spatial kernel. Although many of recent studies notice that the local kernel's scale plays an important role in segmentation models, very few of them investigate how to automatically choose the appropriate spatial scale parameter. In the field of image de-noising, [13] define the scale by the local window size of filters, and use the local polynomial approximation method (LPA) and the intersection of confidence intervals rule (ICI) to find the adaptive window size. Recently, [14] and [15] introduce this approach into the field of the image segmentation. Relying on the mean-square error minimisation of LPA of the observed image conditional on the current segmentation, they apply the ICI algorithm to find the optimal spatial scale parameters. Later on, Boukerroui [12] defines the optimal scale in the sense of the minimum description length principle, introduces an automatic pixel independent scale selection strategy and proposes the optimal scale selection (OSS) model.

To achieve the objective of satisfactory segmentation in the presence of the intensity inhomogeneities and various noises, we focus on the optimal scale selection for a novel local region based segmentation model in this letter. Firstly, we use the anisotropic diffusion filtering to extract the local classification information in the level set function with respect to active contours. Integrating the local classification information with one order statistic of intensity images, we propose a new model which can stabilize the active contours with low computational complexity in all evolving processes. Secondly, by the point estimation theory, the optimal spatial scale for our proposed model is defined under the constraint of maximum absolute error of parameter estimation. By the optimal scale parameters, therefore, the proposed model can obtain better segmentation results than several same-type methods.

The rest of this letter is organized as follows. Section 2 describes the classical local region based segmentation model. The proposed model is presented in Section 3 and its optimal scale selection method is shown in Section 4. Illustrative examples are listed in Section 5. Conclusions are given at the end of the letter.

2. LIF Model

Theoretically, let $\Omega \subset \mathbb{R}^2$ be an open and bounded set, $I: \Omega \rightarrow \mathbb{R}$ be an input greyscale image with intensity inhomogeneities and noises, $\phi: \Omega \rightarrow \mathbb{R}$ be the level set function. Then the evolving contour is the 0-level set, *i.e.*, $\{x \in \Omega | \phi(x) = 0\}$. The Heaviside function $H(\phi)$ denotes the regions inside an evolving contour, and $\delta(\phi)$ is its derivative. Usually, $H(\phi)$ and $\delta(\phi)$ are approximated by their smooth version [3].

In the LIF model [6], the LIF energy, which represents the difference between the original image and the fitted image, is defined by

$$E_{LIF}(\phi) = \frac{1}{2} \int_{\Omega} |I(x) - I^{LFI}(x)|^2 dx,$$

where I^{LFI} is the local fitted image that can be generated as follows

$$I^{LFI}(x) = H(\phi(x))f_1(x) + (1 - H(\phi(x)))f_2(x),$$

with

$$f_1 = \frac{K_{\sigma} * [H(\phi) \cdot I]}{K_{\sigma} * H(\phi)}, \quad f_2 = \frac{K_{\sigma} * [(1 - H(\phi)) \cdot I]}{K_{\sigma} * (1 - H(\phi))}, \quad (1)$$

where $*$ denotes the convolution operation; K_σ is the truncated Gaussian window with the standard deviation σ and of size $4\sigma + 1$ by $4\sigma + 1$. In order to efficiently find the solution, the LIF model utilizes the Gaussian Kernel K_ζ to regularize the level set function. And the algorithm can be summarized as the following alternate iteration scheme

$$\begin{cases} \phi^{t+\frac{1}{2}} = \phi^t - \tau \frac{\partial E_{LIF}(\phi)}{\partial \phi}, \\ \phi^{t+1} = K_\zeta * \phi^{t+\frac{1}{2}}, \end{cases} \quad (2)$$

where t denotes the t -th iteration; τ is the time-step; ζ denotes the standard deviation of K_ζ , which should be larger than the square root of the time-step τ in order to enhance the smoothing capacity. The Gaussian filtering based regularization enables ϕ to be smooth and not to be re-initialized, promoting the efficiency of the LIF model. Nevertheless, the isotropic filtering implemented on ϕ in (2) leads to the strong concussion of active contours during iterations, especially near the 0-level set. Furthermore, it is insufficient to produce accurate segmentation results from images with heavy noises and intensity inhomogeneities.

3. New Local Region Based Model

Inspired by the simple and efficient LIF model, we define a new local region based active contour model to efficiently segment accurate results from the medical images with intensity inhomogeneities and strong noises. The local data energy is defined as follows

$$E_L(\phi) = \int_{\Omega} |I(x) - I^L(x)|^2 dx, \quad (3)$$

where I^L is the robust local fitted images as follows

$$I^L(x) = w_1(x)f_1(x) + w_2(x)f_2(x),$$

with

$$w_1(x) = K_\zeta * H(\phi), \quad w_2(x) = K_\zeta * (1 - H(\phi)), \quad (4)$$

where $\zeta \neq \sigma$; f_1 and f_2 are defined in (1).

Compared with isotropic diffusion filtering in LIF model, the parameters w_1 and w_2 in (4) have three advantages. First, the filtering in w_1 and w_2 can be considered as the anisotropic diffusion filtering implemented on level set function ϕ . And the anisotropy is characterized by the Heaviside function H , which prevents the filtering from crossing the 0-level set and eliminates the undesired concussion of the evolving contour near the real target boundary. Second, $w_1(x)$ and $w_2(x)$ contain the local classification information in the neighborhood centered at pixel x , which ensures the new model is more robust to strong noises. Third, there are two convolution operations in (4), however one convolution is essentially enough to obtain the two parameters due to $w_1(x) + w_2(x) = 1, x \in \Omega$. Hence our local data energy only has one more convolution operation than the LIF energy.

Moreover, the local intensity means f_1 and f_2 in our data energy (3) are the lower order statistics than the ones used by [5-12] and other researchers. This leads our model to

capture the characters of the target region in images with lower computational complexity.

As a typical level set method, we need to regularize the zero level set by penalizing its length to derive a smooth contour during the evolution. Therefore, the proposed local region based model is

$$E(\phi) = E_L(\phi) + \mu \int_{\Omega} |\nabla H(\phi)| dx,$$

where μ is the positive regularization parameter.

4. Scale Selection

In aforementioned local-region based models, locality is generally defined by an isotropic Gaussian kernel which is rewritten as

$$K_{\sigma}(x) = \exp\left(-\frac{\|x\|_2^2}{2\sigma^2}\right), x \in (-2\sigma, 2\sigma) \times (-2\sigma, 2\sigma).$$

The scale parameter σ is crucial to the segmentation model. As we have mentioned earlier in Section 1, some works appear on pixel dependent scale selection methods [14–17] and the pixel independent scale selection method [12]. But the choice of the appropriate scale is still a difficult problem. Because bigger σ means the better estimation accuracy of the local statistics, while bringing in the higher computational cost. In the following under a weak assumption, we aim to balance the above conflict and introduce a new adaptive pixel dependent scale selection method.

Unlike the method proposed by [12], we assume that the given image intensities I are stochastic variables obeying unknown distributions. The local intensity means, *i.e.*, the expectations of stochastic variable $I(x)$ are estimated by $f_1(x)$ and $f_2(x)$ in (1). By use of the Central Limit Theorem, the maximum absolute errors of these estimators can be represented as

$$\Delta(x) = U_{\alpha/2} \frac{S(x)}{\sqrt{n}},$$

where U_{α} is the α -th percentile of Gaussian distribution and usually $\alpha=0.01$; n is the sample size and is equal to $(4\sigma+1)^2$ in this paper; $S^2(x)$ are the unbiased estimators of the local variances of stochastic variables $I(x)$ [18], which can be defined by

$$S^2 = K_{\sigma} * I^2 - (K_{\sigma} * I)^2.$$

Consequentially for all $I(x)$, $x \in \Omega$, the minimization of the maximum absolute errors is required. And the simplified penalty term according to σ is defined by

$$E_{\sigma}(\sigma) = \gamma \int_{\Omega} \frac{S(x)}{\sigma} dx,$$

where γ is a positive parameter.

Ultimately, the proposed local region-based segmentation model is

$$E(\phi, \sigma) = E_L(\phi) + E_{\sigma}(\sigma) + \mu \int_{\Omega} |\nabla H(\phi)| dx. \quad (5)$$

And the minimization of the energy functional (5) can be easily performed using gradient decent techniques. On one hand, the gradient with respects to ϕ is

$$\frac{\partial E}{\partial \phi} = [I - (w_1 f_1 + w_2 f_2)](f_2 - f_1) [K_\zeta * \delta(\phi)] - \mu \delta(\phi) \operatorname{div} \left(\frac{\nabla \phi}{\|\nabla \phi\|} \right).$$

On the other hand, the gradient with respects to σ is

$$\frac{\partial E}{\partial \sigma} = \frac{\partial E_L}{\partial \sigma} + \frac{\partial E_\sigma}{\partial \sigma},$$

where

$$\frac{\partial E_L}{\partial \sigma} = - \int_{\Omega} [I - (w_1 f_1 + w_2 f_2)] \left(w_1 \frac{df_1}{d\sigma} + w_2 \frac{df_2}{d\sigma} \right) dx,$$

$$\frac{\partial E_\sigma}{\partial \sigma} = \gamma \int_{\Omega} \left[\frac{K'_\sigma * I^2 - 2(K_\sigma * I) \cdot (K'_\sigma * I)}{2\sigma S} - \frac{S}{\sigma^2} \right] dx,$$

$$\frac{df_1}{d\sigma} = \frac{K'_\sigma * [H(\phi) \cdot I]}{K_\sigma * H(\phi)} - \frac{K'_\sigma * H(\phi)}{K_\sigma * H(\phi)} f_1,$$

$$\frac{df_2}{d\sigma} = \frac{K'_\sigma * [(1-H(\phi)) \cdot I]}{K_\sigma * (1-H(\phi))} - \frac{K'_\sigma * (1-H(\phi))}{K_\sigma * (1-H(\phi))} f_2,$$

$$K'_\sigma(x) = \frac{\|x\|_2^2}{\sigma^3} \exp \left(-\frac{\|x\|_2^2}{2\sigma^2} \right).$$

Then minimizing (5) can be substituted by the following alternate iteration scheme

$$\begin{cases} \phi^{t+1} = \phi^t - \tau \frac{\partial E}{\partial \phi}, \\ \sigma^{t+1} = \sigma^t - \tau \frac{\partial E}{\partial \sigma}. \end{cases}$$

5. Experimental Results

We have tested the performance of the proposed method on a set of challenging images with intensity inhomogeneities and various noises.

In this Section, we present some typical results to demonstrate its effectiveness compared with the ones of LBF model [5], LIF model [6], LCK model [11] and OSS model [12]. All algorithms are performed on a workstation of Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz with 8G memory. In all experiments for convenience, we initiate the important parameters as follows: $\mu = 0.1 \times 255^2$, $\gamma = 1$, $\zeta = 4\tau$. And one can adjust the parameters for better segmentation results.

As shown in Figure 1, the first experiment is carried out on a T-shaped image with intensity inhomogeneity and different noises. From the left column to the right one, there are the original images and images with Poisson noise, 0.1 and 0.2 level Gaussian noise.

The first row lists the initial curves. From the 2nd row to the last one, the yellow curves are the segmentation results of the LIF model, LBF model, LCK model, OSS model and our model respectively. From four initial values of the scale parameter, *i.e.*, $\sigma = 10, 20, 30, 40$, the same results can be segmented from the images with different noises by the proposed model. The evolution of scale parameters from different initializations are presented for images with different noises in Figure 2. During a few iterations, the scale fast converges to a stable constant, and the higher level noise in the image will lead to bigger scale parameter. Compared with the results of other models, the ones of the proposed model are more accurate and robust to noise, avoiding the concussion of the final curves near the real boundaries. Furthermore, the quantitative evaluation, *i.e.*, Dice similarity Coefficient (DSC) in Table 1, shows the advantage of the proposed model.

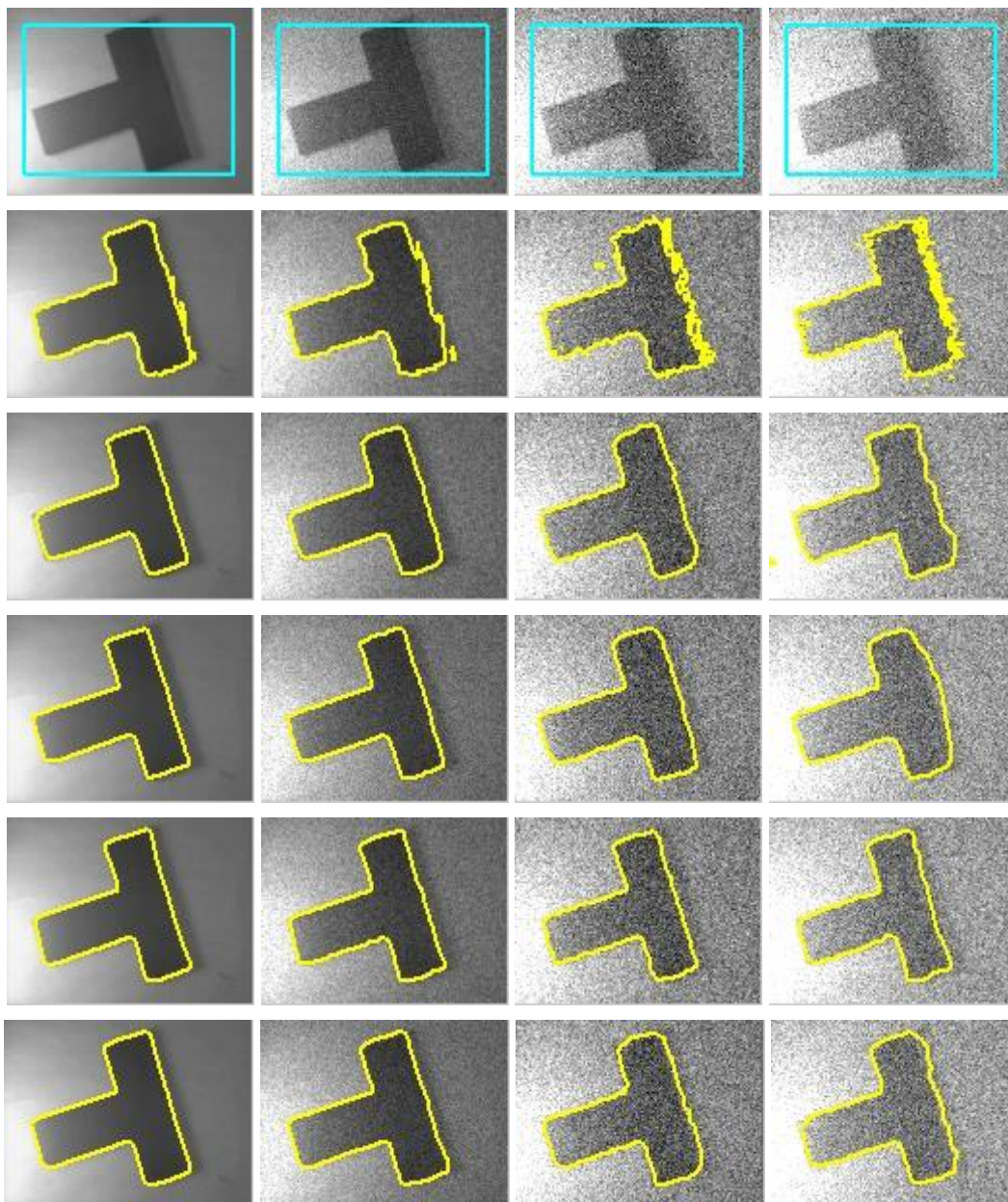


Figure 1. Segmentation Results on Synthetic Image with Different Noises.

Figure 3, shows the visual results on the ultrasound image. To conduct a fair comparison, we strictly fix the same conditions of the proposed model as the ones of OSS model. For example, the cyan curves in Figure 3, are the two initializations. The initial value of the scale parameter is 200. The left image shows the result of OSS model and the left one lists the result of the proposed model. From Figure 3, it is observed that the proposed model achieves the smoother and more stable result due to the usage of the local classification information and the low order statistic. Figure 4, lists some segmentations from real medical images with intensity inhomogeneities and noises.

Table 1. DSC of Segmentation Results

DSC	Clean image	Poisson noise	Gaussian noise 0.1	Gaussian noise 0.2
LIF model	0.9385	0.9344	0.9076	0.9013
LBF model	0.9572	0.9513	0.9403	0.9348
LCK model	0.9724	0.9654	0.9467	0.9465
OSS model	0.9716	0.9649	0.9494	0.9251
Our model	0.9792	0.9658	0.9590	0.9589

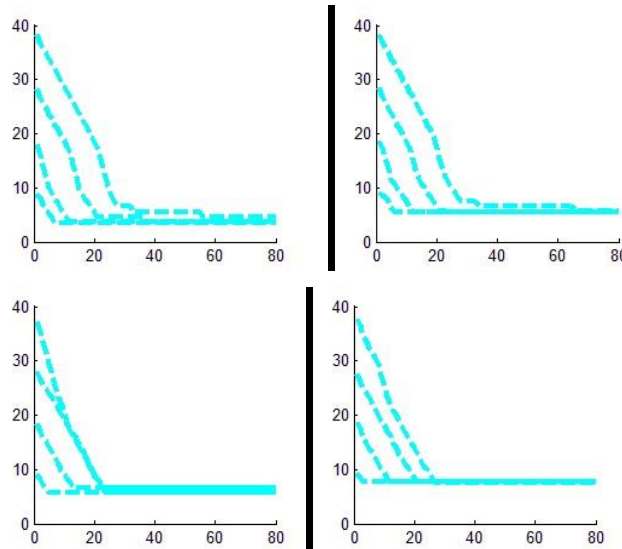


Figure 2. Scale Evolution with 4 Different Initializations Over Iterations

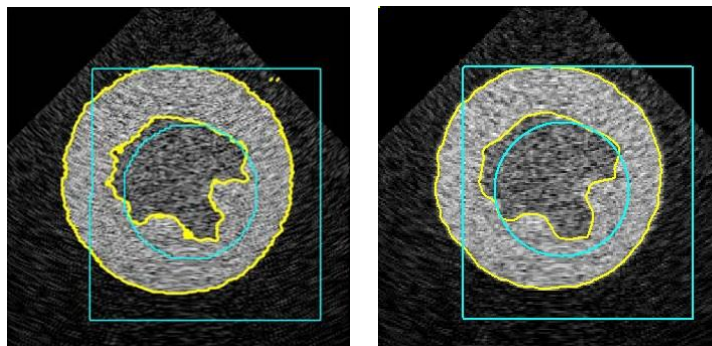


Figure 3. Segmentation Results on Ultrasound Image

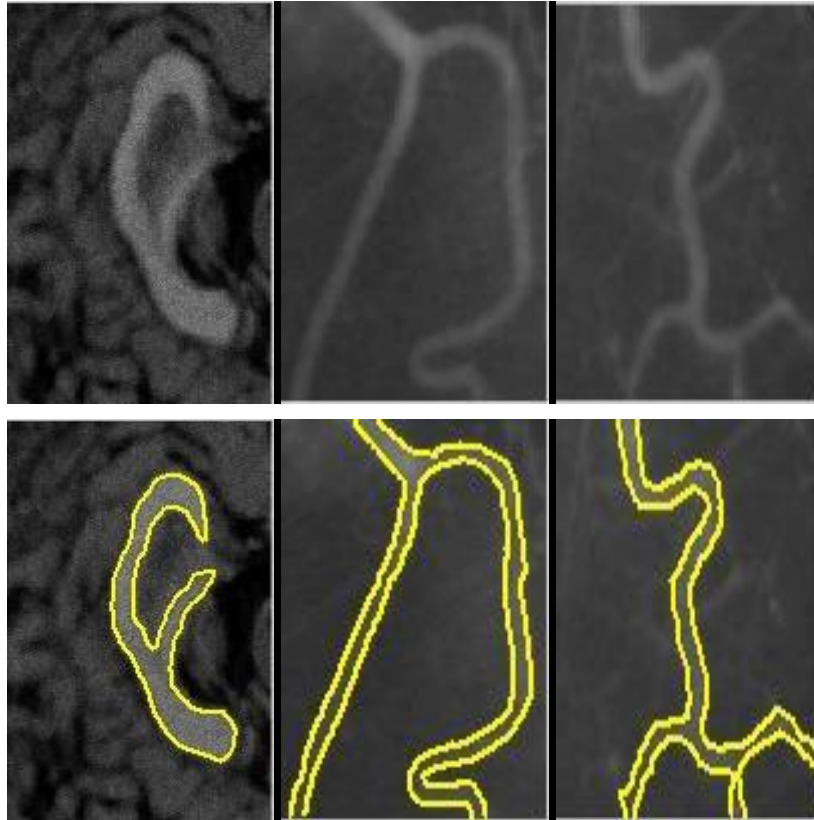


Figure 4. Segmentation Results on Real Medical Images.

6. Conclusion

This letter presents a new local region-based active contour model to segment images with intensity inhomogeneities and various strong noises. The anisotropic information of the active contours can ensure the stabilization and robustness of the new model with low computational complexity. Meanwhile, by the theory of estimation error, we investigate the automatic approach of the optimal scale selection for the local region-based segmentation model. Relying on the optimal scale, the proposed model can achieve better results than the related local region-based models.

Acknowledgments

This work has been supported by the National Natural Science Foundation of China under Grant no. 61401368 and Chinese Universities Scientific Fund under Grant no. 2452015098.

References

- [1] V. Caselles, R. Kimmel and G. Sapiro, "Geodesic active contours", *International Journal Comput. Vis.*, vol. 22, no. 1, (1997), pp. 61-79.
- [2] W. Kass, A. Witkin and D. Terzopoulos, "Snakes: active contour models", *International Journal Comput. Vis.*, vol. 1, no. 1, (1998), pp. 312-331.
- [3] T. Chan and L. Vese, "Active contours without edges", *IEEE Transactions on Image Processing*, vol. 10, no. 2, (2001), pp. 266-277.
- [4] D. Mumford and J. Shah, "Optimal approximation by piecewise smooth functions and associated variational problems", *Commun. Pure Appl. Math.*, vol. 42, no. 5, (1989), pp. 577-685.
- [5] C. M. Li, C. Y. Kao, J. C. Gore and Z. Ding, "Minimization of region scalable fitting energy for image segmentation", *IEEE Transactions on Image Processing*, vol. 17, no. 10, (2008), pp. 1940-1949.
- [6] K. Zhang, H. Song and L. Zhang, "Active contours driven by local image fitting energy", *Pattern recognition*, vol. 43, no. 4, (2010), pp. 1199-1206.

- [7] S. Lankton, S. Member and A. Tannenbaum, "Localizing region-based active contours", *IEEE Trans Image Process*, vol. 17, no. 11, (2008), pp. 2029-2039.
- [8] T. Brox and D. Cremers, "On local region models and a statistical interpretation of the piecewise smooth Mumford-Shah functional", *Int. J. Comput. Vis.*, vol. 84, no. 184, (2009), pp. 184-193.
- [9] J. Liu and H. Zhang, "Image segmentation using a local GMM in a variational framework", *Journal of Math. Imaging & Vision*, vol. 46, no. 2, (2013), pp. 161-176.
- [10] Z. Ji, Y. Xia, Q. Sun, G. Cao and Q. Chen, "Active contours driven by local likelihood image fitting energy for image segmentation", *Information Sciences*, vol. 301, (2015), pp. 285-304.
- [11] L. Wang and C. Pan, "Robust level set image segmentation via a local coreentropy-based K-means clustering", *Pattern Recognition*, vol. 47, no. 5, (2014), pp. 1917-1925.
- [12] D. Boukerroui, "Optimal spatial scale for local region-based active contours", *Proceedings of The International Conference on Image Processing (ICIP)*, Paris, France, (2014) October 27-30.
- [13] V. Katkovnik, K. Egiazarian and J. Astola, "Adaptive window size image de-noising based on intersection of confidence intervals (ICI) rule", *Journal of Mathematical Imaging and Vision*, vol. 16, no. 3, (2002), pp. 223-235.
- [14] Q. Yang and D. Boukerroui, "Optimal spatial adaptation for local region based active contours: An intersection of confidence intervals approach", *Proceedings of the International Conference on Imaging Theory and Applications and International Conference on Information Visualization Theory and Applications*, Vilamoura, Algarve, Portugal, (2011) March 5-7.
- [15] Q. Yang and D. Boukerroui, "Ultrasound image segmentation using local statistics with an adaptive scale selection", *Proceedings of 9th IEEE International Symposium on Biomedical Imaging (ISBI)*, (2012) May 2-5.
- [16] D. Boukerroui, "A local Rayleigh model with spatial scale selection for ultrasound image segmentation", *Proceedings of British Mach. Vis. Conf.*, Surrey, United Kingdom, (2012) September 3-7.
- [17] J. Piovanò and T. Papadopoulos, "Local statistic based region segmentation with automatic scale selection", *Proceedings of European Conf. Comput. Vis.*, Marseille, France, (2008) October 12-18.
- [18] A. Papoulis and S. Pillai, Editor, "Probability, Random Variables and Stochastic Processes, 4th ed.", McGraw-Hill, New York, NY, (2002).

