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Damage Location and Extent Identification of Transmission Tower Combining Flexibility Matrix and Signalterm Amplitude Vector

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Abstract

In order to locate damage posotion and quantify damage degree of transmission tower, the method of damage identification of transmission tower based on flexibility matrix and signalterm amplitude vector was proposed, by combining flexibility matrix method of damage identification and time-frequency analysis method of domain vibration signal. Firstly, the coefficient equation about modal parameters and damage parameters was established, according to the linear relationship between flexibility matrix elements and structure parameters. Then, by Wigner-Ville Distribution (WVD) signalterm analytical deduction of free vibration response of the damping structure, the responding function relation between signalterm amplitude and model parameters was founded. Finally, the coefficient equation was solved by substituting mode shape vector with signalterm amplitude vector, avoiding the structural modal parameters identification, the damage location and extent of transmission tower was detected once and for all. The numerical example analysis of single tower model and tower-line system shows that, the proposed method with good anti-noise performance can accurately detect the extent of single or multiple damage of current structure by a single measurement.

Keywords: *Damage location, Damage extent, Damage identification, Transmission tower, Flexibility matrix, Signalterm amplitude vector*

1. Introduction

The traditional idea of damage extent identification based on modal parameters is: Firstly, the condensed stiffness matrix is solved, according to the frequency and corresponding mode shape. Then, the extent of structural damage is detected according to the relationship between stiffness matrix and structural parameters [1-3]. It is extremely difficult to achieve the two steps for the general structure, because the complete stiffness matrix is determined uniquely by all structural modes, while few actual measurement of the high order modes can be achieved [4-5].

However, transmission tower as a kind of typical high slender structure, can be modeled as the concentrate mass cantilever model with equivalent stiffness in theory. Because of clear linear relationship between the elements in flexibility matrix of cantilever structure and structural parameters, the relationship equation between mode parameters and damage parameters can be established directly [6-8], thus the damage identification of transmission tower can be accomplished by solving the equation, instead of the two steps of the traditional damage identification idea.

Structural damage identification method [9-12] based on vibration is actually to identify the change of physical parameters according to the variety of modal parameters or vibration response. Among them, although the physical meaning of modal parameters methods based on vibration mechanics is very clear, the accuracy and application are

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greatly influenced by ambient noise, measurement errors and test incompleteness [13]. In contrast, all kinds of damage identification methods [14-16] based on signal processing directly extracting damage characteristics from vibration signal are effective and feasible, but they mainly depend on various signal processing methods and techniques, the damage identification mechanism linking up with structural system is not much explicit yet [17].

In order to locate damage position and quantify damage degree of transmission tower, the flexibility matrix method of damage identification and time-frequency analysis method of domain vibration signal are combined in this paper. To obtain the initial conditions for solving the physical parameters of the structure, Wigner-Ville Distribution (WVD) signalterm amplitude vector of free vibration response of the damping structure is introduced, the damage location and extent of transmission tower structure are detected by solving the damage coefficient equation. The numerical example analysis of single tower model and tower-line system shows that, the proposed method with good anti-noise performance can accurately detect the extent of single or multiple damage of current structure by a single measurement.

2. Damage Coefficient Equation

Structural local damage generally reduces the stiffness of the structure, but not the quality, accordingly considering that the bending stiffness $(EI)_i$ of i th structural element is reduced to $\alpha_i(EI)_i$, $0 \le \alpha_i \le 1$.With the symmetrical characteristic of flexibility matrix, the flexibility vector $\{f\}$ consisted of upper triangular elements can be expressed as:

$$\{f\} = \{f_{11} \quad f_{12} \quad \cdots \quad f_{1n} \quad f_{22} \quad \cdots \quad f_{2n} \quad \cdots \quad f_{nn}\}^T = [S]\{\alpha\} \tag{1}$$

where, $[S]$ is damage characteristic matrix, $\{\alpha\}$ is damage coefficient vector.

The standard eigenvalue equation of MDOF system is:

$$[B]^{-1}\{x\}_i = \frac{1}{\omega_i^2}\{x\}_i \tag{2}$$

where,

$$[B]^{-1} = [M]^{\frac{1}{2}}[F][M]^{\frac{1}{2}} \tag{3}$$

$$\{x\}_i = [M]^{\frac{1}{2}}\{\phi\}_i \tag{4}$$

where, $[M]$ and $[F]$ is respectively mass matrix and flexibility matrix, ω_i is i th natural frequency, $\{\phi\}_i$ is i th mode shape vector.

With the symmetrical characteristic of $[B]^{-1}$, the vector $\{b\}$ consisted of upper triangular elements can be expressed as:

$$\{b\} = \begin{Bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1n} \\ b_{22} \\ \vdots \\ b_{2n} \\ \vdots \\ b_{nn} \end{Bmatrix} = \begin{bmatrix} \sqrt{m_1 m_1} & & & & & & & & \\ & \sqrt{m_1 m_2} & & & & & & & \\ & & \ddots & & & & & & \\ & & & \sqrt{m_1 m_n} & & & & & \\ & & & & \sqrt{m_2 m_2} & & & & \\ & & & & & \ddots & & & \\ & & & & & & \sqrt{m_2 m_n} & & \\ & & & & & & & \ddots & \\ & & & & & & & & \sqrt{m_n m_n} \end{bmatrix} \begin{Bmatrix} f_{11} \\ f_{12} \\ \vdots \\ f_{1n} \\ f_{22} \\ \vdots \\ f_{2n} \\ \vdots \\ f_{nn} \end{Bmatrix} = [M^*]\{f\} \tag{5}$$

Then, the equation (2) is deformed as:

$$\begin{bmatrix} x_{1i} & x_{2i} & x_{3i} & \cdots & x_{ni} & \vdots & 0 & \cdots & \cdots & 0 & \vdots & 0 & \cdots & 0 & \vdots & \cdots & 0 \\ & x_{1i} & 0 & \cdots & 0 & \vdots & x_{2i} & x_{3i} & \cdots & x_{ni} & \vdots & 0 & \cdots & 0 & \vdots & \cdots & 0 \\ & & x_{1i} & \cdots & 0 & \vdots & 0 & x_{2i} & \cdots & 0 & \vdots & x_{3i} & \cdots & x_{ni} & \vdots & \cdots & 0 \\ & & & \ddots & \vdots & \vdots & & & \ddots & \vdots & \vdots & & \ddots & & \vdots & \cdots & 0 \\ & & & & x_{1i} & \vdots & & & & x_{2i} & \vdots & & & x_{3i} & \vdots & \cdots & x_{ni} \end{bmatrix} \cdot \{b\}$$

$$= [X_i]\{b\} = \frac{1}{\omega_i^2}\{x\}_i \tag{6}$$

From equation (1), (4), (5) and (6), the damage coefficient equation is established:

$$\{\alpha\} = \frac{1}{\omega_i^2}([X_i][M^*][S])^{-1}[M]^{\frac{1}{2}}\{\phi\}_i \tag{7}$$

Solving the structural damage coefficient from equation (7) has significant advantage, because the number of unknowns is equal with that of independent equations, only single order mode parameter is required to detect the location and extent of structural damage.Especially for the transmission tower with so many degrees of freedom, the high order mode is hardly obtained, the damage identification application of damage coefficient equation is particularly convenient, the assessment of damage status of current structure can be done by one measurement, the contrast test before and after damage is not needed.

3. Signalterm Amplitude Vector

Assuming $s(t)$ is a real signal, $z(t)$ is the analytic signal corresponding to $s(t)$ by Hilbert transform, the Wigner-Ville Distribution (WVD) is defined as:

$$W_z(t, f) = \int_{-\infty}^{\infty} z(t+\frac{\tau}{2}) \cdot z^*(t-\frac{\tau}{2}) \cdot e^{-\mathrm{i}\cdot 2\pi\tau f}\,\mathrm{d}\tau \tag{8}$$

Where, $z^*(t)$ is the complex conjugate of $z(t)$.

Under initial displacement condition by suddenly unloading, the free vibration expression at location k of a damping structural system with n degrees of freedom is:

$$y_k(t) = \sum_{i=1}^{n} \phi_{ki} \cdot \phi_i^T M y_0 \cdot e^{-\xi_i \omega_i t} \cdot \cos(\omega_{di} t + \varphi_i) \tag{9}$$

Where, ϕ_i is i th mode shape, M is mass matrix, y_0 is initial displacement condition, ξ_i is i th damping ratio, ω_i is i th natural frequency, ω_{di} is i th damped frequency, and φ_i is i th initial phase.

After $y_k(t)$ is transformed to analytic signal $z_k(t)$ by Hilbert transform, the WVD of $y_k(t)$ is deducted as:

$$W_k(t, f) = \int_{-\infty}^{\infty} (\sum_{i=1}^{n} \phi_{ki}\phi_i^T M x_0 \cdot e^{-\xi_i \omega_i t} \cdot e^{j(\omega_{di} t + \varphi_i)} \cdot e^{-\xi_i \omega_i \frac{\tau}{2} + j\omega_{di}\frac{\tau}{2}}) \cdot$$

$$(\sum_{l=1}^{n} \phi_{kl}\phi_l^T M x_0 \cdot e^{-\xi_l \omega_l t} \cdot e^{-j(\omega_{dl} t + \varphi_l)} \cdot e^{\xi_l \omega_l \frac{\tau}{2} + j\omega_{dl}\frac{\tau}{2}}) \cdot e^{-\mathrm{j}\cdot 2\pi f\tau}\,\mathrm{d}\tau \tag{10}$$

With the reason that signalterms derive from signal components themselves, that is $i = l$, utilizing the δ fuction integrals result of $\int_{-\infty}^{\infty} e^{-\mathrm{j}\cdot 2\pi(f-f')\tau} \mathrm{d}\tau = \delta(f - f')$, the signalterms in WVD of free vibration response is deducted as:

$$W_k^{auto}(t, f) = \sum_{i=1}^{n} (\phi_{ki} \cdot \phi_i^T M y_0 \cdot e^{-\xi_i \omega_i t})^2 \cdot \delta(f - \frac{\omega_{di}}{2\pi}) \tag{11}$$

Can be seen from analytic expression, the signalterms in WVD of free vibration response are as follows: n impulse functions with their amplitude decaying along time axis, located at frequency $f_i = \omega_{di} / 2\pi$ in time-frequency plane.

If frequency parameter f_i of an signalterm is determined, while the i th damped frequency ω_{di} is easy to be acquired by frequency spectrum analysis or time-frequency, then the time-decaying amplitude $A_k(t) = (\phi_{ki} \cdot \phi_i^T M y_0 \cdot e^{-\xi_i \omega_i t})^2 = W_k^{auto}(t, f_i)$ can be determined, that is the response function of i th mode shape at test point k also can be acquired, by analogy with $k = 1,2,\cdots n$, the response function of i th mode shape at one point ($t = t_0$) can be obtained at any test point, finally the i th mode shape can be acquired. Thus, the i th mode shape can be got from signal time-decaying amplitude vector A of WVD signalterms.

$$\begin{aligned} A &= [\sqrt{A_1(t)}, \cdots, \sqrt{A_k(t)}, \cdots, \sqrt{A_n(t)}]^T \\ &= (|\phi_{1i}|, \cdots, |\phi_{ki}|, \cdots, |\phi_{ni}|)^T \cdot \left| \phi_i^T M y_0 e^{-\xi_i \omega_i t} \right| \end{aligned} \tag{12}$$

The positive and negative relations between mode shape components can be determined by the phase relationship of cross-power spectrum at test points, with same direction as positive and different direction as negative. The above derivation is based on the displacement response of the structure system, but acceleration response which is the most common parameter in dynamic testing of structures, it can be obtained by the second order derivative of the time in the formula (9):

$$a_k(t) = \ddot{y}_k(t) = \sum_{i=1}^{n} \phi_{ki} \phi_i^T M x_0 \cdot e^{-\xi_i \omega_i t} \cdot \cos(\omega_{di} t + \theta_i) \tag{13}$$

where, θ_i is i th initial phase of acceleration response. With the same cosine function expression as the displacement response, thus the above deduction also can be applied to the acceleration response signal.

With signalterm amplitude vector A in place of mode shape vector, the damage identification by solving damage coefficient equation (7) do not need the mode parameter messages, and can avoid the measurement error and test incompleteness.

4. Numerical Example Analysis

A transmission tower with square plane shape is cup type self-supporting composite steel tube tower, which is modeled to the concentrate mass segmentation model with 12 degrees of freedom with static condensation method, shown in Figure 1, ensuring the first three modes basically same between theoretical model and the real tower, the two models of single tower and tower-line system are established. Through one vibration response test of damage status, the current damage can be detected by solving the damage coefficient equation (7) with signalterm amplitude vector.

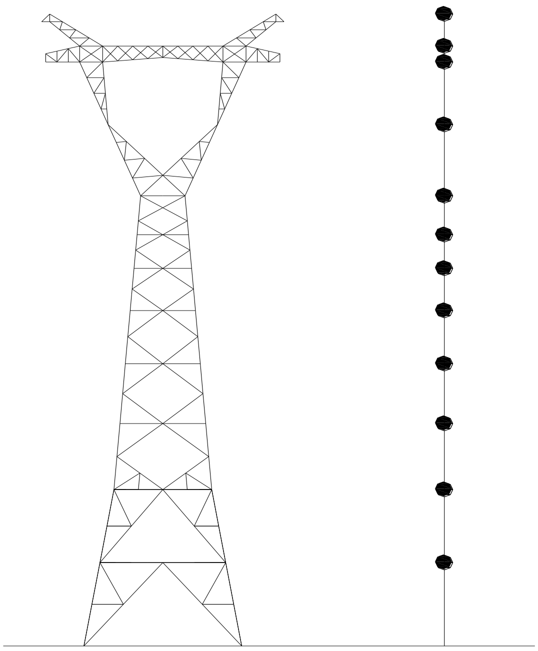
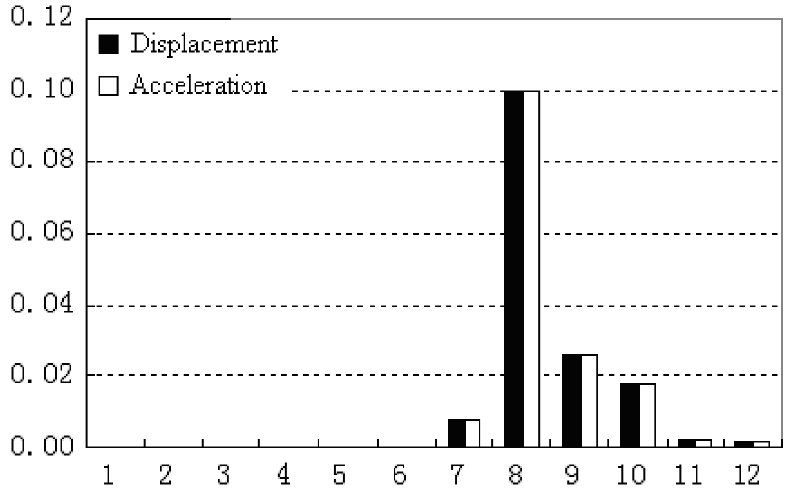


Figure 1. Concentrate Mass Model of Transmission Tower

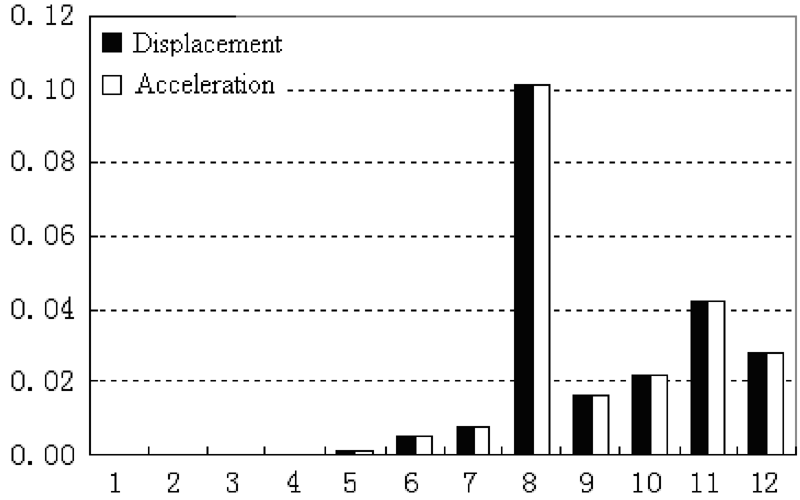
The simulated damage position and extent are set up: (1) Single damage is at 8th segment of tower, with bending stiffness reduced by 10%, (2) Two damage are at 6th and 10th segment of tower, with bending stiffness reduced by 20% and 10% respectively, (3) Three damage are at 7th, 9th and 11th, with bending stiffness reduced by 30%, 20% and 10% respectively.

Figure 2, is the identification result of simulated damage (1), it is found that the position and extent of single damage of current structure is correctly detected by one test. The identification errors mainly concentrate in the vicinity of damage position, which indicates that the proposed method is sensitive to the change of physical parameters caused by local damage. The damage identification result based on single tower model is better than that computed by tower-line model, the reason is that the dynamic characteristic error between tower-line model and theoretical model is larger than single tower model, because of the influence from transmission lines. Meanwhile, the identification results based on displacement and acceleration response are equivalent, which brings great convenience to the actual damage identification test.

In order to measure the anti-noise ability of the method of damage identification combining flexibility matrix and signalterm amplitude vector, the different level of the Gaussian white noise is added to vibration response signal for contrast analysis. Figure 3, and Figure 4, is respectively the identification result of simulated damage (2) and (3), it is shown that the introduction of measurement noise can not submerge the damage identification message at small damage location, neither can change the damage identification result at major damage location. Although the noise just causes a small increase in identification error with maximum 6.2%, the location and extent of multiple damages of transmission tower is accurately detected, which demonstrates that the damage identification method proposed in this paper has well noise immunity.

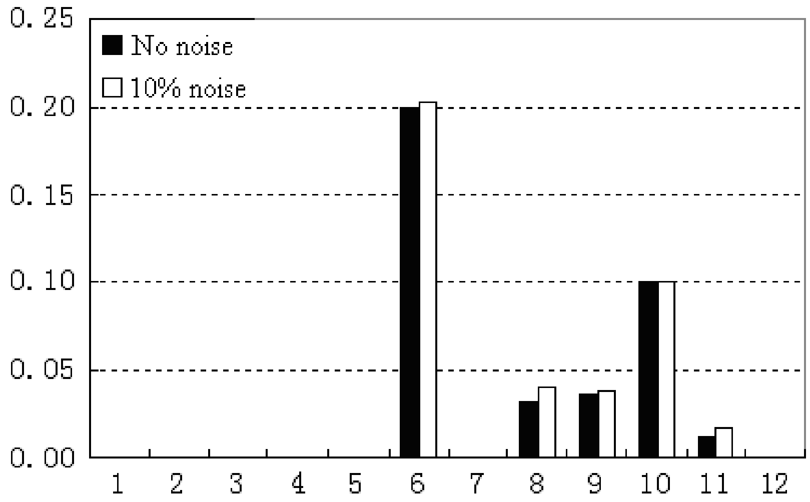


(a) Single Tower Model

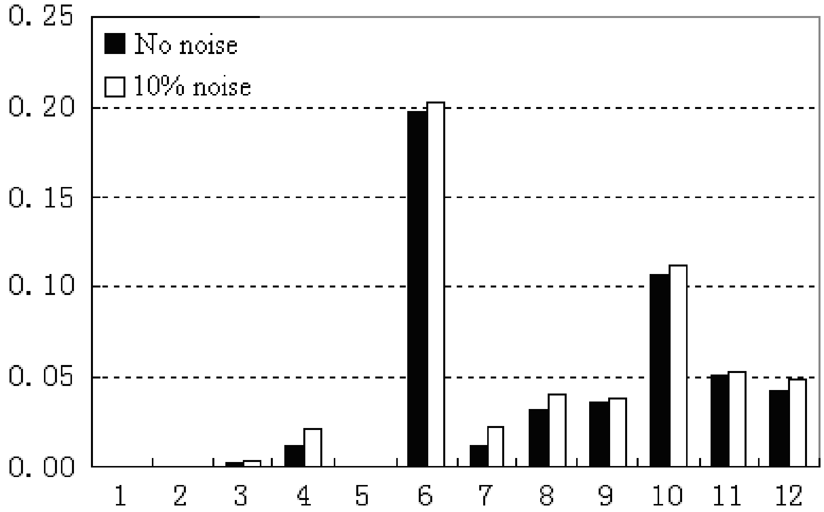


(b) Tower-Line Model

Figure 2. The Identification Result of Damage (1)

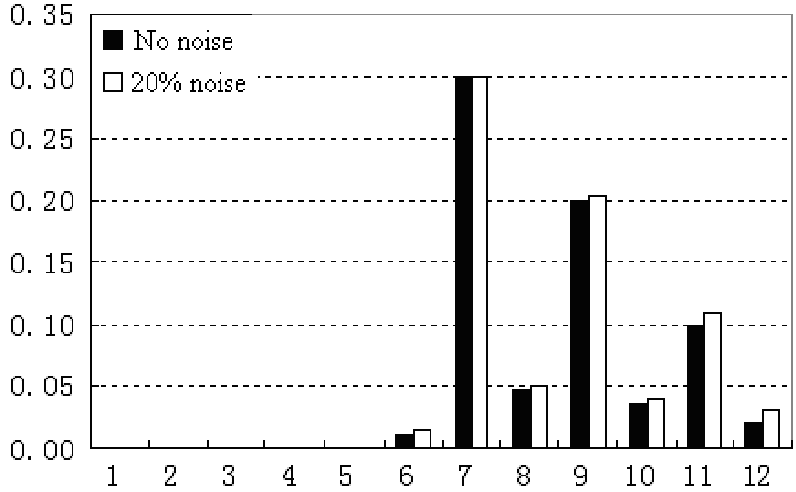


(a) Single Tower Model

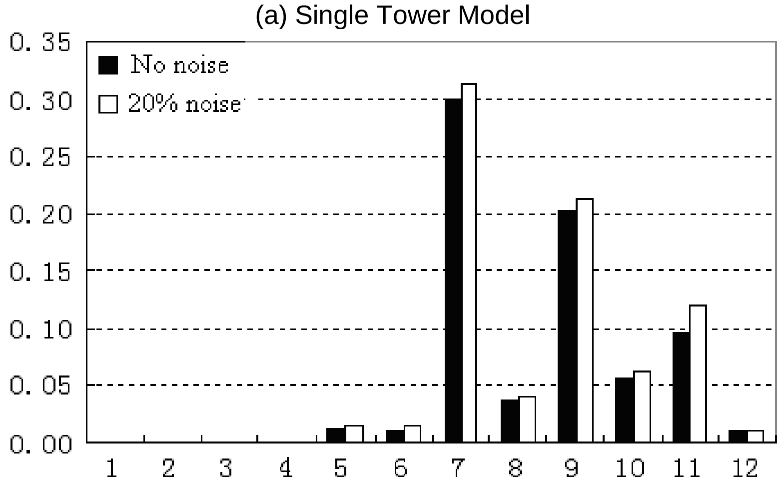


(b) Tower-Line Model

Figure 3. The Identification Result of Damage (2)



(a) Single Tower Model



(b) Tower-Line Model

Figure 4. The Identification Result of Damage (3)

Summarizing all damage identification results, the stiffness changing points on tower head, corssarm of structure itself and error between theoretical model and actual structure cause some interference to the damage identification, there are some small value of damage coefficient at non-damage position, but generally does not affect the damage location and extent identification.

5. Conclusions

The method of damage location and extent identification of transmission tower combining flexibility matrix and signalterm amplitude vector is proposed, which has clear physical meaning and does not require model shape parameters identification. Firstly, the coefficient equation about modal parameters and damage parameters is established, according to the linear relationship between flexibility matrix elements and structure parameters.Then, by Wigner-Ville Distribution (WVD) signalterm analytical deduction of free vibration response of the damping structure, the responding function relation between signalerm amplitude and model parameters is founded. Finally, the coefficient equation is solved by substituting mode shape vector with autoterm amplitude vector, the damage location and extent of transmission tower was detected once and for all.

A numerical example analysis of transmission tower demonstrates that the proposed method with well noise immunity can accurately detect the location and extent of current single or multiple damages by one test, the comparison test before and after damage status is not needed. In addition, the damage identification results based on displacement and acceleration response are equivalent, which brings great convenience to the actual damage identification work.

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