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Convolutive Independent Component Analysis Based on First-order Statistics for Complex-valued Source Extraction

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Abstract

This paper addresses the problem of independent component extraction of complex-valued signals in convolutive mixtures. Most previous research focused on real-valued convolutive ICA, and corresponding solution methods are generally computationally complex and inefficient for real application. In order to solve the problem, we propose a novel method based on first-order statistics, which includes several single-step and iterative separators to satisfy different demands of engineering applications. We also provided the theoretical performance analysis, which is validated by experimental simulations. It is observed from the simulations that various factors (especially the noncircularity) affect the extraction performance of separators; hence we offer some advice on how to choose separators properly. Besides, the proposed iterative separators generally perform better and converge faster compared to two complex FastICA algorithms.

Keywords*: Independent component analysis, convolutive mixture, circular and noncircular signals, first-order statistics*

1. Introduction

Independent component analysis (ICA) for extracting source signals from mixtures has found utility in many applications such as communications [1], face recognition [2], analysis of functional magnetic resonance imaging [3], and radar data [4]. However, in many applications, the mixtures are oversimplified to be instantaneous. The more general case is that the mixtures are convolutive with several different delays, especially in multi-input /multi-output (MIMO) wireless communications where channel models often include multipath propagation.

Currently, the techniques of ICA for convolutive mixtures generally fall into two types: time-domain and frequency-domain methods. Time-domain methods inspired by blind deconvolution methods are the first efforts devoted to the convolutive case [5-6]. One problem with time-domain methods is that they tend to be complex computationally due to the relationship of filter coefficients with each other [7]. This can be overcome by moving to the frequency-domain, where the problem is transformed to multidimensional ICA. But there arise a large problem called permutation, inconsistency of output channels among frequency bins. To correct the permutation disorder, several approaches have been proposed [8-10], which however require a number of samples even larger than time-domain methods [11]. Furthermore, to provide acceptable results, these two types of techniques generally require many iteration steps to converge. This may be unrealistic for some application in timely situation, *e.g.*, telecommunication and wireless communications. Moreover, most researchers pay attention to real-valued convolutive ICA, but neglect the investigation on ICA for convolutive mixtures of complex sources including circular and noncircular

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signals. This fact constrains further the use of abovementioned techniques, especially in wireless communications where sources are complex-valued and most probably noncircular caused by in-phase/quadrature-phase (IQ) imbalance [12].

In this context, it is challenging to solve the complex-valued case of convolutive ICA in an efficient way. Recently, some research has focused on exploiting prior knowledge about the mixing system or the sources themselves, including prior information on the time or frequency indices where the desired source is positive or presents significant power [13-14]. Vicente Zarzoso *et. al.*, proposed an extremely efficient algorithm in real-valued domain which exploits only first-order statistics of the whitened observations, assuming that the sample indices where the source of interest presents positive values are known [15]. Furthermore in [16], we proposed to extract the complex-valued sources from the instantaneous mixtures using the first-order statistics. Nevertheless, it is a pity that these algorithms though simple and efficient can't be directly applied to solve the general complex case of convolutive ICA.

In this paper, we translate the convolutive mixtures into instantaneous ones in time-domain, and then design an effective algorithm for general complex signals using first-order statistics of extended observations. Section 2 introduces some information about complex random variables and develops the problem formulation. Section 3 presents the proposed algorithms including three kinds of single-step algorithms. Generally, prior knowledge of the positive support should be totally required by the method; while in wireless communication, the prior knowledge obtained by training-based communication system usually exhibits inaccuracy. Therefore, an iterative algorithm is also provided to modify the accuracy of the prior knowledge. Furthermore, theoretical performance analysis concerning the above algorithms is accomplished in terms of asymptotic interference-to-signal ratio (ISR) and probability of correct support estimation (PCE) in Section 4. Simulations are carried out in Section 5 to validate the theoretic analysis and investigate the factors that affect the extraction performance of the algorithms, where noncircular property of sources has an extremely important effect on the performance. The performance of two complex FastICA algorithms [17-18] is also used as reference for comparison. Finally, a concise conclusion is given in Section 5. What's more, this paper can be regarded as an important complement for Vicente Zarzoso's method in [15], and a great advancement for our previous work in [16]. In this paper, the main novelties are the following ones:

- ♦ A transformation of the convolutive mixture model into extended instantaneous mixture model with a stacked number of sources which are composed of i.i.d. samples;
- ♦ An efficient method for extracting complex sources including both circular and noncircular signals from the mixtures;
- ♦ Theoretical analysis and empirical rules provided to indicate how to choose the separators for engineering applications.

2. Problem Statement

2.1. Complex Preliminaries

A complex variable z is defined in terms of two real variables z^R and z^I as $z = z^R + jz^I$, where $j = \sqrt{-1}$ is the imaginary unit. The mean of a complex random variable z is given by $E\{z\} = E\{z^R\} + jE\{z^I\}$. Assume that $E\{z\} = 0$, the variance and pseudo-variance of z are defined as $\mathrm{var}(z) = E\{|z|^2\}$ and $\mathrm{p\,var}(z) = E\{z^2\}$ where $|z| = \sqrt{zz^*}$ is the modulus of z [19].

If a complex random variable z has a zero pseudo-variance, it is called second-order circular (or simply as circular), which implies that z^R and z^I are uncorrelated with equal variances. In this paper, noncircular random variable whose real and imaginary parts are uncorrelated with unequal variances is also investigated. A strict but seldom utilized definition of circularity is based on the probability density function of the complex random variable: a complex random variable z is said to be circular if z has the same distribution as $e^{j\theta}z$ for any $\theta \in$ ¡ [20].

2.2. Complex ICA in the Convolutive Mixture Model

Before concentrating on convolutive mixture model, we introduce the instantaneous one first. The instantaneous linear mixture and the extraction of one source can be denoted as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \tag{1}$$

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{2}$$

where $\mathbf{x}(t) = \left[x_1(t), x_2(t), \mathrm{K}, x_m(t)\right]^T$, $\mathbf{s}(t) = \left[s_1(t), s_2(t), \mathrm{K}, s_n(t)\right]^T$, w and A are, respectively, the vector of observations, the original source vector, the separator vector, and the $m \times n$ mixing matrix which is usually assumed to be full rank matrix with $m \geq n$ and invariant over time.
The convolutive mixture and the extraction of one source can be expressed respectively as

$$\mathbf{x}(t) = \sum_{\tau} \mathbf{A}(\tau)\mathbf{s}(t-\tau) \tag{3}$$

$$y(t) = \sum_{\tau} \mathbf{w}(\tau)^H \mathbf{x}(t-\tau) \tag{4}$$

In the above equations, the mixing matrix is replaced by a multi-variate linear time invariant (LTI) system with impulse response $\mathbf{A}(t) \in$ £ $^{m \times n}$, and the separator vector by a LTI vector filter $\mathbf{w}(t) \in$ £ $^{m \times 1}$.

The following assumptions on the characteristics of the sources are made

AS1: $\mathbf{s}(t)$ are mutually independent, zero mean, and stochastic processes with unit variance;

AS2: Real and imaginary parts of $\mathbf{s}(t)$ are mutually uncorrelated and respectively symmetric distributed in probability density function;

AS3: $\mathbf{s}(t)$ are composed of i.i.d. samples.

where AS1 and AS3 are basic and original assumptions of ICA [21], and AS2 is satisfied by most of complex signals in communications [19]. Then we translate this linear convolutive mixing model into instantaneous one. We consider the finite impulse response (FIR) mixing system of given length L, thus (3) can be rewritten as

$$\mathbf{x}(t) = \left[\mathbf{A}(0) \quad \mathbf{A}(1) \quad \mathrm{K} \quad \mathbf{A}(L-1)\right] \begin{bmatrix} \mathbf{s}(t) \\ \mathbf{s}(t-1) \\ \mathrm{M} \\ \mathbf{s}(t-L+1) \end{bmatrix}, L \geq 2 \tag{5}$$

On the other hand, the length of FIR separators is denoted by D, and then the extracting filter can be replaced by the following $mD \times 1$ vector which concatenates the vectors of the separator impulse response

$$\underline{\mathbf{w}} @ \left[\mathbf{w}(0)^T \quad \mathbf{w}(1)^T \quad \mathrm{K} \quad \mathbf{w}(D-1)^T \right]^T \tag{6}$$

Similarly, (4) can be rewritten as

$$\begin{aligned} y(t) &= \underline{\mathbf{w}}^H \underline{\mathbf{x}}(t) \\ &= \left[\mathbf{w}(0)^H \quad \mathbf{w}(1)^H \quad \mathrm{K} \quad \mathbf{w}(D-1)^H \right] \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}(t-1) \\ \mathrm{M} \\ \mathbf{x}(t-D+1) \end{bmatrix}, D \geq 2 \end{aligned} \tag{7}$$

where column vector $\underline{\mathbf{x}}(t)$ denotes an extended observation vector that contains the original vector of observations and its delay ones. One can notice from (7) that the first equality is quite similar to (2) which represents the extraction of one source from the instantaneous mixture. Substituting (5) into the extended observation vector, we obtain

$$\begin{aligned} \underline{\mathbf{x}}(t) &= \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{x}(t-1) \\ \mathrm{M} \\ \mathbf{x}(t-D+1) \end{bmatrix} = \begin{bmatrix} \left[\mathbf{A}(0) \quad \mathrm{K} \quad \mathbf{A}(L-1) \right] \begin{bmatrix} \mathbf{s}(t) \\ \mathrm{M} \\ \mathbf{s}(t-L+1) \end{bmatrix} \\ \left[\mathbf{A}(0) \quad \mathrm{K} \quad \mathbf{A}(L-1) \right] \begin{bmatrix} \mathbf{s}(t-1) \\ \mathrm{M} \\ \mathbf{s}(t-L) \end{bmatrix} \\ \mathrm{M} \\ \left[\mathbf{A}(0) \quad \mathrm{K} \quad \mathbf{A}(L-1) \right] \begin{bmatrix} \mathbf{s}(t-D+1) \\ \mathrm{M} \\ \mathbf{s}(t-L-D+2) \end{bmatrix} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}(0) & \mathrm{K} & \mathbf{A}(L-1) & 0 & \mathrm{K} & 0 \\ 0 & \mathbf{A}(0) & \mathrm{K} & \mathbf{A}(L-1) & \mathrm{O} & \mathrm{M} \\ \mathrm{M} & \mathrm{O} & \mathrm{O} & \mathrm{O} & \mathrm{O} & 0 \\ 0 & \mathrm{K} & 0 & \mathbf{A}(0) & \mathrm{K} & \mathbf{A}(L-1) \end{bmatrix} \begin{bmatrix} \mathbf{s}(t) \\ \mathbf{s}(t-1) \\ \mathrm{M} \\ \mathbf{s}(t-L-D+2) \end{bmatrix} \\ &= \underline{\mathbf{A}}\underline{\mathbf{s}}(t) \end{aligned} \tag{8}$$

where $\underline{\mathbf{A}}$ and $\underline{\mathbf{s}}(t)$ are respectively the extended mixing matrix and extended source vector. This leads to the fact that the convolutive model (3) and (4) is transformed into the extended instantaneous model (8) and (7). Noting that $\mathbf{s}(t)$ are composed of i.i.d. samples (AS3), we find $\underline{\mathbf{s}}(t)$ are also mutually independent, zero mean and stochastic processes with unit variance, which means that the problem to be solved is still a classical ICA case. Therefore we assume the extended mixing system has more sensors than sources, *i.e.*, $mD \geq n(L+D-1)$, so that the length of FIR separators is given by

$$D \geq \frac{n(L-1)}{m-n} \tag{9}$$

For $m=n$, the length of mixing filter L is constrained to be 1 and the convolutive model is reduced to instantaneous one. For $m>n$, the condition (9) becomes $D \geq n(L-1)$.

3. Algorithm Development

In the first place, the extended observed vector $\underline{\mathbf{x}}(t)$ is preprocessed to be whitened. In other words, $\underline{\mathbf{x}}(t)$ is linearly transformed into a vector

$$\underline{\mathbf{z}}(t) = \mathbf{U}\underline{\mathbf{x}}(t) = \mathbf{U}\underline{\mathbf{A}}\underline{\mathbf{s}}(t) = \mathbf{Q}\underline{\mathbf{s}}(t) \tag{10}$$

whose covariance matrix equals the identity matrix: $E\left\{\underline{\mathbf{z}}(t)\underline{\mathbf{z}}(t)^H\right\} = \mathbf{I}$, where U is a whitening matrix. We find that the new mixing matrix Q is unitary due to $E\left\{\underline{\mathbf{z}}(t)\underline{\mathbf{z}}(t)^H\right\} = \mathbf{Q}E\left\{\underline{\mathbf{s}}(t)\underline{\mathbf{s}}(t)^H\right\}\mathbf{Q}^H = \mathbf{I}$ and $E\left\{\underline{\mathbf{s}}(t)\underline{\mathbf{s}}(t)^H\right\} = \mathbf{I}$.

Without loss of generality, we will suppose that $s_1(t)$ is the source of interest. The method can be expressed in terms of the conditional mean of the whitened extended observations, as summarized by the following result.

Separator 1:

$$\underline{\mathbf{w}} = \frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| s_1^R > 0\right\} \text{ with } \alpha_1 = E\left\{s_1^R \middle| s_1^R > 0\right\} \tag{11}$$

where s_1^R is the real part of s_1. In practice, (11) means to averaging the observations over samples where s_1^R is positive-valued. Then the source of interest can be estimated as

$$y = \underline{\mathbf{w}}^H \underline{\mathbf{z}} = s_1 \tag{12}$$

The reason for the last equality in (12) is illustrated as follows:

According to (10), we have $\underline{\mathbf{w}} = \frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| s_1^R > 0\right\} = \frac{1}{\alpha_1} E\left\{\mathbf{Q}\underline{\mathbf{s}} \middle| s_1^R > 0\right\} = \mathbf{Qg}$, where

$$\mathbf{g} = \frac{1}{\alpha_1} E\left\{\underline{\mathbf{s}} \middle| s_1^R > 0\right\} \tag{13}$$

Considering AS1 and the uncorrelation assumption in AS2, we have

$$\begin{aligned} g_1 &= \frac{1}{\alpha_1} E\left\{s_1 \middle| s_1^R > 0\right\} = \frac{1}{\alpha_1}\left(E\left\{s_1^R \middle| s_1^R > 0\right\} + jE\left\{s_1^I \middle| s_1^R > 0\right\}\right) = 1 \\ g_k &= \frac{1}{\alpha_1} E\left\{s_k \middle| s_1^R > 0\right\} = 0, k > 2 \end{aligned} \tag{14}$$

Hence, $\mathbf{g} = \mathbf{e}_1 = \begin{bmatrix} 1 & 0 & \mathrm{K} & 0 \end{bmatrix}^T$ and consequently $y = \underline{\mathbf{w}}^H \underline{\mathbf{z}} = \mathbf{g}^H \mathbf{Q}^H \mathbf{Q}\underline{\mathbf{s}} = \mathbf{g}^H \underline{\mathbf{s}} = s_1$.

Similarly, another separator based on the positive support of the imaginary part is given by

Separator 2:

$$\underline{\mathbf{w}} = -\frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| s_1^I > 0\right\} \text{ with } \alpha_2 = E\left\{s_1^I \middle| s_1^I > 0\right\} \tag{15}$$

where α_2 is equal to α_1 only when sources are circular. Furthermore, the third separator based on the positive support of both real and imaginary parts is provided:

Separator 3:

$$\underline{\mathbf{w}} = \frac{\left(\frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| s_1^R > 0\right\} - \frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| s_1^I > 0\right\}\right)}{2} \tag{16}$$

Undoubtedly, Separator 3 requires more prior knowledge concerning the positive support of the sources. All the separators can independently accomplish the source separation task. What should be noticed is that, Separator 3 though utilizing doubled prior knowledge performs worse than the other separators in most conditions. This judgment will be demonstrated by both theoretical and experimental analysis of next sections in detail.

In engineering practice, to capture the prior knowledge about the positive support of sources is difficult and even impossible, leading to the unavailability of the above extraction method. Thus in our previous works, we have also proposed a training-based project for the application in MIMO wireless communications to solve the problem (see [16] for more details). The project is still available in convolution condition. If the MIMO wireless communications follow the rules of the project, the problem of the co-channel interference will be overcome, and thus mutually independent complex sources will be able to transmit simultaneously even in the same frequency band.

However, an inevitable problem in the training-based project is the inaccuracy of prior knowledge. To solve the problem, we add the following iterative separators to replace the single-step ones.

Iterative Separators:

Initiation. Use the single-step separator from the inaccurate prior knowledge to estimate the pilot sequence of the source of interest.

Step1. Recalculate the separator $\underline{\mathbf{w}} = \frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^R > 0\right\}$, $\underline{\mathbf{w}} = -\frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^I > 0\right\}$ or $\underline{\mathbf{w}} = \frac{\left(\frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^R > 0\right\} - \frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^I > 0\right\}\right)}{2}$ where $\tilde{y}$ denotes the estimated pilot.

Step2. Calculate the estimate of pilot, i.e., $\tilde{y} = \underline{\mathbf{w}}^H \underline{\mathbf{z}}$. Repeat these two steps until converges.

We define Iterative Separator 1 that uses $\underline{\mathbf{w}} = \frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^R > 0\right\}$ in Step1, and Iterative Separator 2 with $\underline{\mathbf{w}} = -\frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^I > 0\right\}$ as well as Iterative Separator 3 with $\underline{\mathbf{w}} = \frac{\left(\frac{1}{\alpha_1} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^R > 0\right\} - \frac{j}{\alpha_2} E\left\{\underline{\mathbf{z}} \middle| \tilde{y}^I > 0\right\}\right)}{2}$.

4. Performance Analysis

This section is concerned with the theoretical analysis on source extraction performance. The closed forms of performance on both single-step separators and iterative separators are given in terms of ISR and PCE respectively.

4.1. Theoretical Performance in Terms of ISR

We first assume that the length of observations is T such that the index of samples is expressed as a set $\mathrm{S}=\{0 \quad 1 \quad \mathrm{K} \quad \mathrm{T}-1\}$. The set S can be divided into two exclusive sets S_{r1} and $\overline{\mathrm{S}}_{r1}$ (or S_{i1} and $\overline{\mathrm{S}}_{i1}$) where S_{r1} (or S_{i1}) is the positive support of real parts (or imaginary parts) of the interested source and $\overline{\mathrm{S}}_{r1}$ (or $\overline{\mathrm{S}}_{i1}$) is the complement. The index set estimated as the positive support of real parts (or imaginary parts) of the interested source is denoted by F_r (or F_i), the cardinality of which is $N \approx \frac{\mathrm{T}}{2}$. Here we suppose that the correct support estimation ratios for real and imaginary parts are equal. Therefore, the set F_r (or F_i) is the union of set F_{r1} (or F_{i1}) composed of N_1 indices correctly identified, *i.e.*, for which actually $s_1^R(t)>0$ (or $s_1^I(t)>0$), and its complement $\overline{\mathrm{F}}_{r1}$ (or $\overline{\mathrm{F}}_{i1}$) of N-N_1 indices in F_r (or F_i) where $s_1^R(t)<0$ (or $s_1^I(t)<0$). In short, we can summarize these relations as follows:

$$\mathrm{S}=\mathrm{S}_{r1}\,\mathrm{U}\,\overline{\mathrm{S}}_{r1}, \quad \mathrm{F}_r=\mathrm{F}_{r1}\,\mathrm{U}\,\overline{\mathrm{F}}_{r1}, \quad \mathrm{F}_{r1}=\mathrm{F}_r\,\mathrm{I}\,\mathrm{S}_{r1}, \quad \overline{\mathrm{F}}_{r1}=\mathrm{F}_r\,\mathrm{I}\,\overline{\mathrm{S}}_{r1}$$

and

$$\mathrm{S}=\mathrm{S}_{i1}\,\mathrm{U}\,\overline{\mathrm{S}}_{i1}, \quad \mathrm{F}_i=\mathrm{F}_{i1}\,\mathrm{U}\,\overline{\mathrm{F}}_{i1}, \quad \mathrm{F}_{i1}=\mathrm{F}_i\,\mathrm{I}\,\mathrm{S}_{i1}, \quad \overline{\mathrm{F}}_{i1}=\mathrm{F}_i\,\mathrm{I}\,\overline{\mathrm{S}}_{i1}$$

Next, we show the performance of Separator 1 in (11) explicitly depends on which parameters. In practice, we should first remove the sample mean from every sample of the observations so as to ensure the rationality of the zero-mean assumption in AS1. Therefore the global transformation in (13) can be expressed as

$$\begin{aligned}\tilde{\mathbf{g}}&=\frac{1}{\alpha_1}\left(\frac{1}{N}\sum_{t\in\mathrm{F}_r}\underline{\mathbf{s}}(t)-\frac{1}{\mathrm{T}}\sum_{t\in\mathrm{S}}\underline{\mathbf{s}}(t)\right)=\frac{1}{\alpha_1}\left(\frac{1}{\mathrm{T}}\sum_{t\in\mathrm{F}_r}\underline{\mathbf{s}}(t)-\frac{1}{\mathrm{T}}\sum_{t\in\overline{\mathrm{F}}_r}\underline{\mathbf{s}}(t)\right)\\&=\frac{1}{\alpha_1}\left(\frac{1}{\mathrm{T}}\sum_{t\in\mathrm{F}_r}\underline{\mathbf{s}}^R(t)-\frac{1}{\mathrm{T}}\sum_{t\in\overline{\mathrm{F}}_r}\underline{\mathbf{s}}^R(t)+\frac{j}{\mathrm{T}}\sum_{t\in\mathrm{F}_r}\underline{\mathbf{s}}^I(t)-\frac{j}{\mathrm{T}}\sum_{t\in\overline{\mathrm{F}}_r}\underline{\mathbf{s}}^I(t)\right)\end{aligned} \tag{17}$$

where $\tilde{\mathbf{g}}=[\tilde{g}_1,\tilde{g}_2,\mathrm{K},\tilde{g}_{n'}]^T\in\mathbb{C}^{n(L+D-1)\times 1}$ such that the estimate source of interest can be decomposed as the sum of contributions from the source of interest and the interfering sources:

$$\tilde{y}=\tilde{\mathbf{g}}^H\underline{\mathbf{s}}=\tilde{y}_1+\tilde{b} \text{ with } \tilde{y}_1=\tilde{g}_1 s_1 \text{ and } \tilde{b}=\sum_{k=2}^{n'}\tilde{g}_k s_k \tag{18}$$

To quantify source extraction performance, we refer to [15] and define the average ISR per interfering source as

$$ISR=\frac{E\{\tilde{b}\tilde{b}^*\}}{(n'-1)E\{\tilde{y}_1\tilde{y}_1^*\}}=\frac{\sum_{k=2}^{n'}E\{\tilde{g}_k\tilde{g}_k^*\}}{\left(n(L+D-1)-1\right)E\{\tilde{g}_1\tilde{g}_1^*\}} \tag{19}$$

where $n'=n(L+D-1)$ according to the extended model in Section 2.2. We assume that the non-circularity of sources resulted from the real to imaginary

asymmetry can be defined by $\forall k, \frac{\mathrm{var}\left\{s_k^R\right\}}{\mathrm{var}\left\{s_k^I\right\}} = \eta$, *i.e.*, the ratio of the variances of real and imaginary parts. Hence, $E\left\{\left(s_k^R\right)^2\right\} = \frac{\eta}{\eta+1}$ and $E\left\{\left(s_k^I\right)^2\right\} = \frac{1}{\eta+1}$ according to AS1. For $2 \le k \le n'$, under the uncorrelation assumption of AS2 and AS3, we obtain

$$E\left\{\overset{\circ}{g}_k \overset{\circ}{g}_k^*\right\} = \frac{1}{\alpha_1^2}\left(\frac{1}{\mathrm{T}^2}\sum_{t\in \mathrm{F}_r} E\left\{\left(s_k^R\right)^2\right\} + \frac{1}{\mathrm{T}^2}\sum_{t\in \overline{\mathrm{F}}_r} E\left\{\left(s_k^R\right)^2\right\} + \frac{1}{\mathrm{T}^2}\sum_{t\in \mathrm{F}_r} E\left\{\left(s_k^I\right)^2\right\} + \frac{1}{\mathrm{T}^2}\sum_{t\in \overline{\mathrm{F}}_r} E\left\{\left(s_k^I\right)^2\right\}\right)$$
$$= \frac{1}{\alpha_1^2 \mathrm{T}^2}\left(\frac{\mathrm{T}}{2}\frac{\eta}{\eta+1} + \frac{\mathrm{T}}{2}\frac{\eta}{\eta+1} + \frac{\mathrm{T}}{2}\frac{1}{\eta+1} + \frac{\mathrm{T}}{2}\frac{1}{\eta+1}\right) = \frac{1}{\alpha_1^2 \mathrm{T}}$$
(20)

For k=1, given the symmetry assumption of distribution in AS2, we have

$$E\{s_1^R(t)\} = \begin{cases} \alpha_1 & t \in \mathrm{S}_{r1} \\ -\alpha_1 & t \in \overline{\mathrm{S}}_{r1} \end{cases}$$
$$\mathrm{var}\{s_1^R(t)\} = \frac{\eta}{\eta+1} - \alpha_1^2, \quad t \in \mathrm{S}_{r1}\, or \quad t \in \overline{\mathrm{S}}_{r1} \tag{21}$$

Now, N_1 indices of F_r belong to S_{r1} and the remaining N-N_1 to $\overline{\mathrm{S}}_{r1}$; by symmetry, set $\overline{\mathrm{F}}_r$ contains N-N_1 indices in S_{r1} and N_1 in $\overline{\mathrm{S}}_{r1}$. Hence, we obtain

$$E\{\overset{\circ}{g}_1\} = (2r-1)$$
$$\mathrm{var}\{\overset{\circ}{g}_1\} = \frac{1}{\alpha_1^2 \mathrm{T}}\left(1-\alpha_1^2\right) \tag{22}$$

where r=N_1/N represents the correct support estimation ratio. For sufficient sample size, $\mathrm{var}\{\overset{\circ}{g}_1\}$ can be neglected and then $E\{\overset{\circ}{g}_1 \overset{\circ}{g}_1^*\} \approx E\{\overset{\circ}{g}_1\}E\{\overset{\circ}{g}_1\}^*$, as a result, $E\{\overset{\circ}{g}_1 \overset{\circ}{g}_1^*\} = (2r-1)^2$. Substituting this equation and (20) into (19), the closed form of ISR is given by

$$ISR_1 = \frac{1}{\mathrm{T}(2r-1)^2 \alpha_1^2} \tag{23}$$

Similarly, referring to the derivation of performance of Separator 1, the ISR performance of Separator 2 in (15) can be achieved easily:

$$ISR_2 = \frac{1}{\mathrm{T}(2r-1)^2 \alpha_2^2} \tag{24}$$

To deduce the closed form of ISR for Separator 3, we note that another global transformation in (16) can be derived:

$$
\begin{aligned}
\tilde{\mathbf{g}} &= \frac{1}{\alpha_1}\left(\frac{1}{\mathrm{T}}\sum_{t\in \mathrm{F}_r}\underline{\mathbf{s}}(t)-\frac{1}{2\mathrm{T}}\sum_{t\in \mathrm{S}}\underline{\mathbf{s}}(t)\right)-\frac{j}{\alpha_2}\left(\frac{1}{\mathrm{T}}\sum_{t\in \mathrm{F}_i}\underline{\mathbf{s}}(t)-\frac{1}{2\mathrm{T}}\sum_{t\in \mathrm{S}}\underline{\mathbf{s}}(t)\right)\\
&= \frac{1}{\alpha_1}\left(\frac{1}{2\mathrm{T}}\sum_{t\in \mathrm{F}_r}\underline{\mathbf{s}}(t)-\frac{1}{2\mathrm{T}}\sum_{t\in \overline{\mathrm{F}}_r}\underline{\mathbf{s}}(t)\right)-\frac{j}{\alpha_2}\left(\frac{1}{2\mathrm{T}}\sum_{t\in \mathrm{F}_i}\underline{\mathbf{s}}(t)-\frac{1}{2\mathrm{T}}\sum_{t\in \overline{\mathrm{F}}_i}\underline{\mathbf{s}}(t)\right)\\
&= \frac{1}{2\mathrm{T}\alpha_1}\left(\sum_{t\in \mathrm{F}_r}\underline{\mathbf{s}}^R(t)-\sum_{t\in \overline{\mathrm{F}}_r}\underline{\mathbf{s}}^R(t)+j\sum_{t\in \mathrm{F}_r}\underline{\mathbf{s}}^I(t)-j\sum_{t\in \overline{\mathrm{F}}_r}\underline{\mathbf{s}}^I(t)\right)\\
&-\frac{j}{2\mathrm{T}\alpha_2}\left(\sum_{t\in \mathrm{F}_i}\underline{\mathbf{s}}^R(t)-\sum_{t\in \overline{\mathrm{F}}_i}\underline{\mathbf{s}}^R(t)+j\sum_{t\in \mathrm{F}_i}\underline{\mathbf{s}}^I(t)-j\sum_{t\in \overline{\mathrm{F}}_i}\underline{\mathbf{s}}^I(t)\right)
\end{aligned}
\tag{25}
$$

Under the assumption AS1-AS3, for $2 \le k \le n'$ we have

$$
\begin{aligned}
E\left\{\overset{\circ}{g}_k\,\overset{\circ}{g}_k^*\right\} &= \frac{1}{4\mathrm{T}^2\alpha_1^2}\left(\sum_{t\in \mathrm{F}_r}E\left\{\left(s_k^R\right)^2\right\}+\sum_{t\in \overline{\mathrm{F}}_r}E\left\{\left(s_k^R\right)^2\right\}+\sum_{t\in \mathrm{F}_r}E\left\{\left(s_k^I\right)^2\right\}+\sum_{t\in \overline{\mathrm{F}}_r}E\left\{\left(s_k^I\right)^2\right\}\right)\\
&+\frac{1}{4\mathrm{T}^2\alpha_2^2}\left(\sum_{t\in \mathrm{F}_i}E\left\{\left(s_k^R\right)^2\right\}+\sum_{t\in \overline{\mathrm{F}}_i}E\left\{\left(s_k^R\right)^2\right\}+\sum_{t\in \mathrm{F}_i}E\left\{\left(s_k^I\right)^2\right\}+\sum_{t\in \overline{\mathrm{F}}_i}E\left\{\left(s_k^I\right)^2\right\}\right)\\
&= \frac{1}{4\mathrm{T}^2\alpha_1^2}\left(\frac{\mathrm{T}}{2}\frac{\eta}{\eta+1}+\frac{\mathrm{T}}{2}\frac{\eta}{\eta+1}+\frac{\mathrm{T}}{2}\frac{1}{\eta+1}+\frac{\mathrm{T}}{2}\frac{1}{\eta+1}\right)\\
&+\frac{1}{4\mathrm{T}^2\alpha_2^2}\left(\frac{\mathrm{T}}{2}\frac{\eta}{\eta+1}+\frac{\mathrm{T}}{2}\frac{\eta}{\eta+1}+\frac{\mathrm{T}}{2}\frac{1}{\eta+1}+\frac{\mathrm{T}}{2}\frac{1}{\eta+1}\right)=\frac{1}{4\mathrm{T}}\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right)
\end{aligned}
\tag{26}
$$

For k=1, we obtain

$$
\begin{aligned}
&E\{s_1^R(t)\}=\begin{cases}\alpha_1 & t\in \mathrm{S}_{r1}\\ -\alpha_1 & t\in \overline{\mathrm{S}}_{r1}\end{cases} \qquad E\{s_1^I(t)\}=\begin{cases}\alpha_2 & t\in \mathrm{S}_{i1}\\ -\alpha_2 & t\in \overline{\mathrm{S}}_{i1}\end{cases}\\
&\mathrm{var}\{s_1^R(t)\}=\frac{\eta}{\eta+1}-\alpha_1^2,\quad t\in \mathrm{S}_{r1}\ or\quad t\in \overline{\mathrm{S}}_{r1}\\
&\mathrm{var}\{s_1^I(t)\}=\frac{1}{\eta+1}-\alpha_2^2,\quad t\in \mathrm{S}_{i1}\ or\quad t\in \overline{\mathrm{S}}_{i1}
\end{aligned}
\tag{27}
$$

Thereby, N_1 indices of F_r belong to S_{r1} and the remaining N-N_1 to $\overline{\mathrm{S}}_{r1}$; by symmetry, set $\overline{\mathrm{F}}_r$ contains N-N_1 indices in S_{r1} and N_1 in $\overline{\mathrm{S}}_{r1}$. So it is the same with sets F_i and $\overline{\mathrm{F}}_i$ Hence, we obtain

$$
\begin{aligned}
&E\{\overset{\circ}{g}_1\}=(2r-1)\\
&\mathrm{var}\{\overset{\circ}{g}_1\}=\frac{1}{4\alpha_1^2\mathrm{T}}\left(1-\alpha_1^2\right)+\frac{1}{4\alpha_2^2\mathrm{T}}\left(1-\alpha_2^2\right)
\end{aligned}
\tag{28}
$$

For sufficient sample size, $E\{\overset{\circ}{g}_1\overset{\circ}{g}_1^*\}=(2r-1)^2$ so that consequently

$$
ISR_3=\frac{1}{\mathrm{T}(2r-1)^2\alpha_{eff}^2}\ \text{ with }\ \alpha_{eff}^2=\frac{4\alpha_1^2\alpha_2^2}{\alpha_1^2+\alpha_2^2}
\tag{29}
$$

The equations (23), (24) and (29) indicate that ISR is in inverse proportion to sample size T, correct support estimation ratio r and the conditional means α_1,α_2, α_{eff}. When r tends to 0.5, *i.e.*, very little prior knowledge of positive support is available, ISR will extremely increase. The fitness of these approximate equations will be assessed by the experiments in Section 5. The constants α_1 and α_2 are derived and summarized in Table 1 for some common sources.

To explain the outcome of Table 1, we take the uniform source as an example to derive its conditional mean α_1 and α_2. Assume that the real part of normalized source is uniformly distributed during the interval [-λ, λ], *i.e.*,

$$p_{s^R} = \begin{cases} \dfrac{1}{2\lambda}, & s^R \in [-\lambda, \lambda] \\ 0, & otherwise. \end{cases}$$

Therefore, the variance of the real part and conditional mean α_1 are given respectively:

$$\mathrm{var}\left(s^R\right) = E\left\{\left(s^R\right)^2\right\} = \int_{-\lambda}^{\lambda} \left(s^R\right)^2 p_{s^R} ds^R = \frac{1}{3}\lambda^2 ;$$

$$E\left\{s^R \middle| s^R > 0\right\} = 2\int_0^{\lambda} s^R p_{s^R} ds^R = \frac{\lambda}{2}.$$

The sources are normalized with unit variance such that $E\left\{\left(s^R\right)^2\right\} = \dfrac{\eta}{\eta+1}$, which has been just proved during the derivation of ISR performance. Hence we obtain $\lambda = \sqrt{\dfrac{3\eta}{\eta+1}}$ and the conditional mean $\alpha_1 = \dfrac{1}{2}\sqrt{\dfrac{3\eta}{\eta+1}}$. Analogically, we can derive another conditional mean $\alpha_2 = \dfrac{1}{2}\sqrt{\dfrac{3}{\eta+1}}$ since $E\left\{\left(s^I\right)^2\right\} = \dfrac{1}{\eta+1}$.

Table 1. Conditional Mean α_1 and α_2 for Some Normalized Sources

Source	1.	2.	3.	4.	5.
Distribution	Bernoulli	Sinusoid	Uniform	Laplacian	Gaussian
$\alpha_1 = E\left\{s^R \middle| s^R > 0\right\}$	$\sqrt{\frac{\eta}{\eta+1}}$	$\frac{2}{\pi}\sqrt{\frac{2\eta}{\eta+1}}$	$\frac{1}{2}\sqrt{\frac{3\eta}{\eta+1}}$	$\frac{\sqrt{2}}{2}\sqrt{\frac{\eta}{\eta+1}}$	$\sqrt{\frac{2\eta}{\pi(\eta+1)}}$
$\alpha_2 = E\left\{s^I \middle| s^I > 0\right\}$	$\sqrt{\frac{1}{\eta+1}}$	$\frac{2}{\pi}\sqrt{\frac{2}{\eta+1}}$	$\frac{1}{2}\sqrt{\frac{3}{\eta+1}}$	$\frac{\sqrt{2}}{2}\sqrt{\frac{1}{\eta+1}}$	$\sqrt{\frac{2}{\pi(\eta+1)}}$

4.2. Theoretical Performance Concerning Iterative Separators

To evaluate the performance of iterative separators, we define PCE as $r' = P\left(s_1^R > 0 \middle| \tilde{y}^R > 0\right)$ for Iterative Separator 1, $r' = P\left(s_1^I > 0 \middle| \tilde{y}^I > 0\right)$ for Iterative Separator 2, and $r' = \frac{1}{2}\left(P\left(s_1^R > 0 \middle| \tilde{y}^R > 0\right) + P\left(s_1^I > 0 \middle| \tilde{y}^I > 0\right)\right)$ for Iterative Separator 3. Assume that the real and imaginary parts of the estimate are composed of source component and interfering components, *i.e.*, $\tilde{y}^R = \tilde{y}_1^R + \tilde{b}_r$ and $\tilde{y}^I = \tilde{y}_1^I + \tilde{b}_i$.

To deduce the closed form of $r' = P\left(s_1^R > 0 \middle| \tilde{y}_1^R > 0\right)$, we note that it can be approximately reduced to $P\left(\tilde{y}_1^R > 0 \middle| s_1^R > 0\right)$ according the Bayesian Theorem. Therefore, the probability density function of $\tilde{y}_1^R$ given $s_1^R > 0$ which is denoted by $p_{\tilde{y}_1^R|s_1^R>0}(u)$ would be required to calculate $P\left(\tilde{y}_1^R > 0 \middle| s_1^R > 0\right)$. Otherwise, for sufficient samples using (18) and (22), we find that $\tilde{y}_1 \approx E\{\tilde{g}_1\}s_1 = (2r-1)s_1$ and that $\tilde{y}_1^R = (2r-1)s_1^R$. Thereby, we obtain

$$\begin{aligned} p_{\tilde{y}_1^R|s_1^R>0}(u) &= \int_{-\infty}^{+\infty} p_{(2r-1)s_1^R|s_1^R>0}(\upsilon)\, p_{\tilde{b}_r}(u-\upsilon)d\upsilon \\ &= 2\int_0^{+\infty} p_{(2r-1)s_1^R}(\upsilon)\, p_{\tilde{b}_r}(u-\upsilon)d\upsilon \end{aligned} \tag{30}$$

where the second equality follows from the symmetry assumption of distribution in AS2. According to the Central Limit Theorem, the interference term $\tilde{b}_r$ performs as a zero-mean Gaussian random variable with variance $\sigma^2 = E\{b_r^2\} = \dfrac{\eta(n'-1)}{(\eta+1)\alpha_1^2 \mathrm{T}}$, where we consider that $E\{\tilde{b}\tilde{b}^*\} = \sum_{k=2}^{n'} E\{\tilde{g}_k \tilde{g}_k^*\} = (n'-1)/\alpha_1^2 \mathrm{T}$ and $E\{\tilde{b}\tilde{b}^*\} = E\{\tilde{b}_r^2\} + E\{\tilde{b}_r^2\} = (1+\dfrac{1}{\eta})E\{\tilde{b}_r^2\}$. Hence, PCE is given by integration of $p_{\tilde{y}_1^R|s_1^R>0}(u)$ over $[0,+\infty)$:

$$\begin{aligned} r' &= P\left(\tilde{y}_1^R > 0 \middle| s_1^R > 0\right) = \int_0^{+\infty} p_{\tilde{y}_1^R|s_1^R>0}(u)du \\ &= 2\int_0^{+\infty}\int_0^{+\infty} p_{(2r-1)s_1^R}(\upsilon)\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(u-\upsilon)^2}{2\sigma^2}\right)d\upsilon du \\ &= \frac{1}{2} + \int_0^{+\infty} p_{(2r-1)s_1^R}(\upsilon) erf\left(\sqrt{\frac{(\eta+1)\mathrm{T}\alpha_1^2}{2\eta(n'-1)}}\upsilon\right)d\upsilon \end{aligned} \tag{31}$$

where the last equality uses the error function $erf(x) = \dfrac{2}{\sqrt{\pi}}\int_0^x e^{-v^2}dv$ and follows from the change of variable $v = (u-\upsilon)/(\sqrt{2}\sigma)$.

Similarly, referring to the derivation of performance of Iterative Separator 1, the performance of Iterative Separator 2 in terms of PCE can be achieved easily:

$$\begin{aligned} r' &= P\left(s_1^I > 0 \middle| \tilde{y}_1^I > 0\right) \\ &= \frac{1}{2} + \int_0^{+\infty} p_{(2r-1)s_1^I}(\upsilon) erf\left(\sqrt{\frac{(\eta+1)\mathrm{T}\alpha_2^2}{2(n'-1)}}\upsilon\right)d\upsilon \end{aligned} \tag{32}$$

To evaluate the closed form of PCE for Iterative Separator 3, we note that $P\left(s_1^R > 0 \middle| \tilde{y}_1^R > 0\right) \approx P\left(s_1^I > 0 \middle| \tilde{y}_1^I > 0\right)$ and the probability can be also reduced to

$r' = P\left(s_1^R > 0 \middle| \tilde{y}^R > 0\right)$, whereas the variance of the interference term $\tilde{b}_r$ becomes $\sigma^2 = E\{b_r^2\} = \frac{\eta(n'-1)}{4(\eta+1)\mathrm{T}}\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right)$ where we consider that $E\{\tilde{b}\tilde{b}^*\} = \sum_{k=2}^{n'} E\{\tilde{g}_k \tilde{g}_k^*\} = \frac{n'-1}{4\mathrm{T}}\left(\frac{1}{\alpha_1^2}+\frac{1}{\alpha_2^2}\right)$.

Therefore, the closed form of PCE is given by

$$\begin{aligned} r' &= P\left(\tilde{y}^R > 0 \middle| s_1^R > 0\right) \\ &= \frac{1}{2} + \int_0^{+\infty} p_{(2r-1)s_1^R}(\upsilon) erf\left(\sqrt{\frac{(\eta+1)\mathrm{T}\alpha_{eff}^2}{2\eta(n'-1)}}\upsilon\right) d\upsilon \end{aligned} \tag{33}$$

where $\alpha_{eff}^2 = \frac{4\alpha_1^2\alpha_2^2}{\alpha_1^2+\alpha_2^2}$. As an example, let us consider the case of 4QAM source with symmetric Bernoulli distribution for both real and imaginary parts, in which $p_{(2r-1)s_1^R}(\upsilon) = \frac{1}{2}\delta\left(\upsilon-(2r-1)\right)$ and $p_{(2r-1)s_1^I}(\upsilon) = \frac{1}{2}\delta\left(\upsilon-(2r-1)\right)$ for $\upsilon > 0$. Substituting these terms into (31), (32) and (33) leads to the fixed-point like update

$$2r'-1 = erf\left(\sqrt{\frac{\mathrm{T}\alpha_1^2}{(n'-1)}}(2r-1)\right) \tag{34}$$

$$2r'-1 = erf\left(\sqrt{\frac{\mathrm{T}\alpha_2^2}{(n'-1)}}(2r-1)\right) \tag{35}$$

$$2r'-1 = erf\left(\sqrt{\frac{\mathrm{T}\alpha_{eff}^2}{(n'-1)}}(2r-1)\right) \tag{36}$$

where the equations also follow from the assumption that 4QAM sources are circularly symmetric, *i.e.*, $\eta = 1$.

Finally in this section, we checked the complexity of iterative separators for extraction of one interested source. One can see that the complexity of the algorithms mainly depends on step2, which means that the algorithms need $o(n(L+D-1)\mathrm{T})$ products per iteration. Hence the proposed algorithms have the same quantity level of complexity as convolutive FastICA [22], an extended algorithm of FastICA in instantaneous case. In contrast, another second-order technique called joint block Toeplitzation and block-inner diagonalization (JBTBID) [23], which converges slower than convolutive FastICA, requires $o(n^3(L+D-1)^3\mathrm{T})$ products.

5. Simulation Results and Discussion

Here, several sets of simulation results are provided to demonstrate the performance of the proposed algorithm. Generally speaking, experiments on both single-step and iterative algorithms have been carried out.

5.1. Performance of Single-step Algorithms

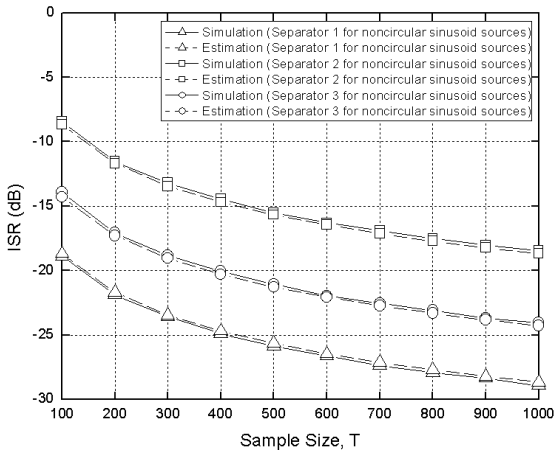
In the simulation, complex Gaussian and sinusoid sources with the number of sources n=10 have been generated respectively. They have been mixed by mixing

filters with the length L=10 whose coefficients are randomly driven. The number of sensors is set to m=11, and the separating FIR separators have been searched with the length $D = n(L-1)/(m-n) = 90$, *i.e.*, the number of extended observations equals the number of the extended sources according to (9), an exactly determined model. The extended observations are prewhitened before the source extraction in all experiments.

In the first experiment, the positive support of the source of interest is assumed to be perfectly known. Figure 1 illustrates the ISR obtained by different single-step separators (11), (15) and (16) for different complex sources with the noncircularity η=10. Results are averaged over 100 Monte Carlo runs. The fitness of theoretical approximation is very precise in all cases. The Separator 1 that is real-based algorithm exhibits better than other separators, and Separator 2 behaves worst among them, due to the internal fact that the power of real parts is larger than that of imaginary parts. Besides, Separator 3 though using the doubled prior knowledge performs poorer than Separator 1, which indicates that doubled prior knowledge from noncircular sources may probably lead to the degeneration of performance.

Next, we study the performance for different values of the correct support estimation ratio r when T=1000 and η=10. Figure 2 shows that the simulation results are also well approximated by the estimation in theory.

Moreover, Figure 3 provides the ISR performance with respect to the asymmetry η when sample size T=1000 and the prior knowledge of positive support is totally known. As seen in the figure, the performance of Separator 3 is about 3dB better than other two separators when η=1, *i.e.*, sources are circular symmetric distributed. However, as η increases, the performance of Separator 1 is improved and the other two degrade. If η is set between 0 and 1, *i.e.*, the power of imaginary parts is larger than that of real parts, it can be expected obviously that Separator 2 will be modified just as Separator 1 does when η>1. Therefore, Separator 3 not only uses doubled prior knowledge, but also performs worse than the other separators in most conditions, except when sources are close to circular (1/3<η<3). Again, these results demonstrate the conformity of the theoretical approximations to the experiments.



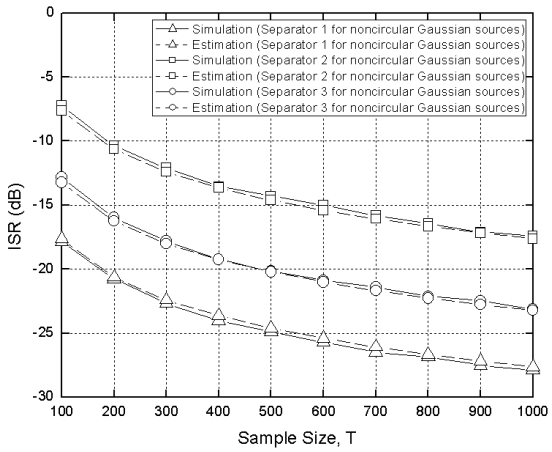


Figure 1. ISR Versus Sample Size for Three Separators when *n*=10, *η*=10, and *r*=1

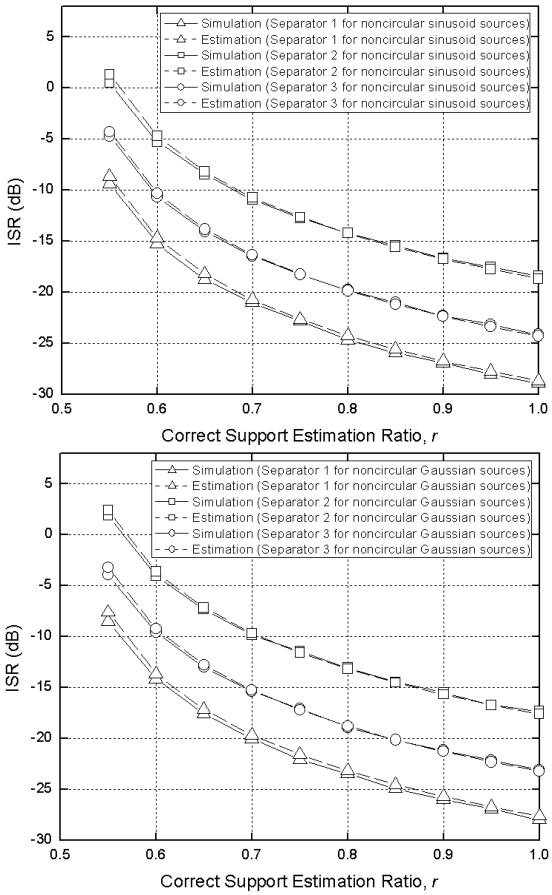


Figure 2. ISR Versus Correct Support Estimation Ratio for Three Separators when *n*=10, *η*=10, and T=1000

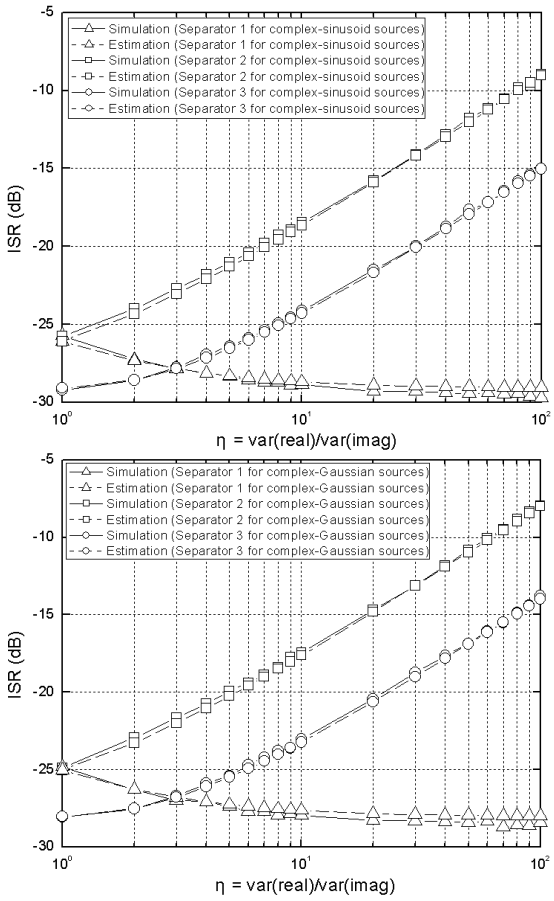


Figure 3. ISR Versus Asymmetry of the Real to Imaginary Parts for Three Separators when *n*=10, T=1000, and *r*=1

5.2. Performance of Iterative Algorithms

The iterative implementation of Section 4 is applied on the extraction of circular and noncircular 4QAM sources from convolutive mixtures. The convergence speed of iterative separators is investigated when n=2, m=3, L=3, D=4 and T=1000. Likewise, results are obtained over 100 Monte Carlo runs.

Figure 4 shows the average values of r and ISR obtained by three kinds of iterative separators for circular 4QAM sources, when different initial values of the correct support estimation ratio r_1 are adopted. Also shown are the theoretical approximations given by (34-36) and the corresponding ISR defined by (23-24) and (29). One can observe that, after several iterations the correct support estimation ratio r converges around 1.0. With the same initial r_1, Iterative Separator 3 (Figure 4, right) converges faster than other separators (Figure 4, left), and the predicted r and ISR of Iterative Separator 3 are more accurate.

Next, we study the influence from the noncircularity of sources on the performance of iterative separators. Considering that Iterative Separator 2 is unsuitable for extracting the sources when the asymmetry η>1, we adopted Iterative Separator 1 and 3 here. Figure 5 and Figure 6 show the average performance for 4QAM sources with the asymmetry η=2 and 3 respectively. The theoretical approximations given by (31-33) and the corresponding ISR are also provided. What should be noticed is that, as the asymmetry value increase, the performance of Iterative Separator 1 (Figure 5, left and Figure 6 left) is improved while Iterative Separator 3 (Figure 5, right and Figure 6 right) behaves worse. Additionally, the predicted r and ISR of Iterative Separator 1

become more accurate. The comparison between these two separators indicates that Iterative Separator 3 which requires doubled prior knowledge may be cost-effective only for the extraction of nearly circular sources (η<2), whereas Iterative Separator 1 using less prior knowledge is generally more effective when the sources are noncircular ($\eta \geq 2$). This judgment can be easily extended to the reverse case that 0<η<1, in which Iterative Separator 2 behaves more effective for $0 < \eta \leq 1/2$ while Iterative Separator 3 is proper only when 1/2<η<1.

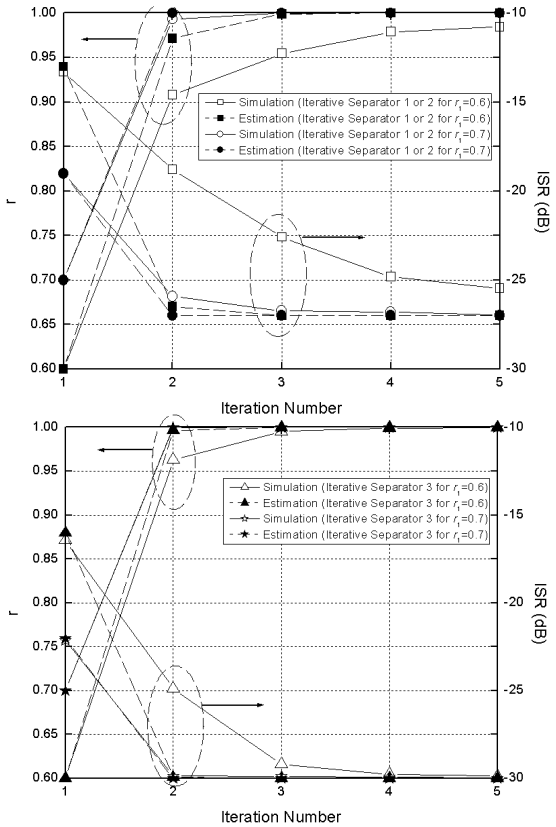
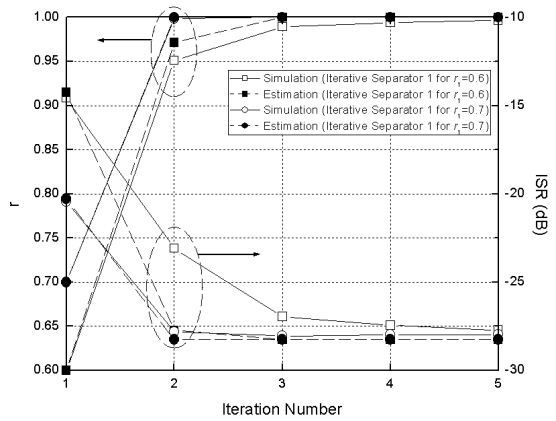


Figure 4. *r* and ISR of Iterative Separators for Different Initial Correct Support Estimation Ratios when Circular 4QAM Sources with *η*=1 are Adopted



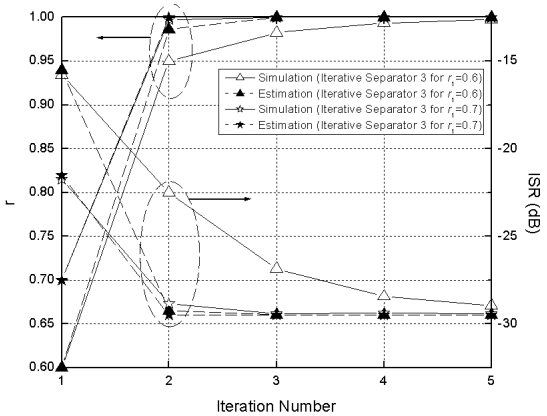


Figure 5. *r* and ISR of Iterative Separators for Different Initial Correct Support Estimation Ratios when Noncircular 4QAM Sources with *η*=2 are Adopted

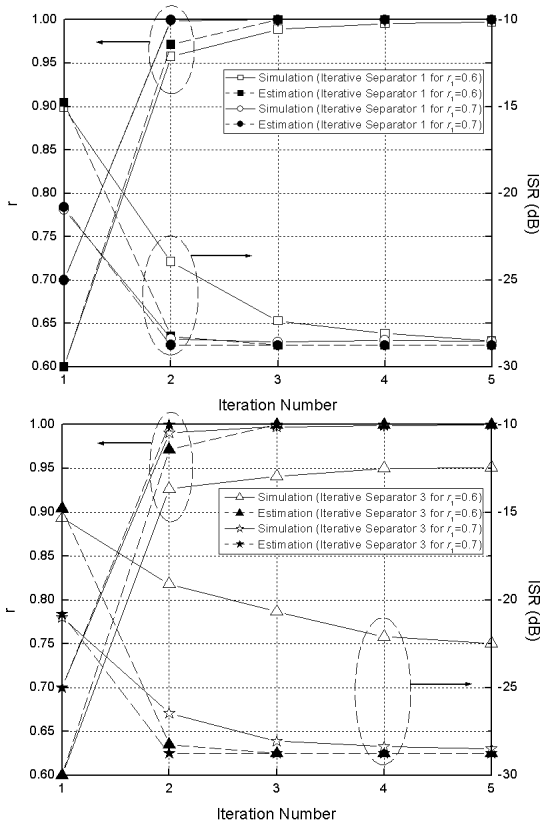


Figure 6. *r* and ISR of Iterative Separators for Different Initial Correct Support Estimation Ratios when Noncircular 4QAM Sources with *η*=3 are Adopted

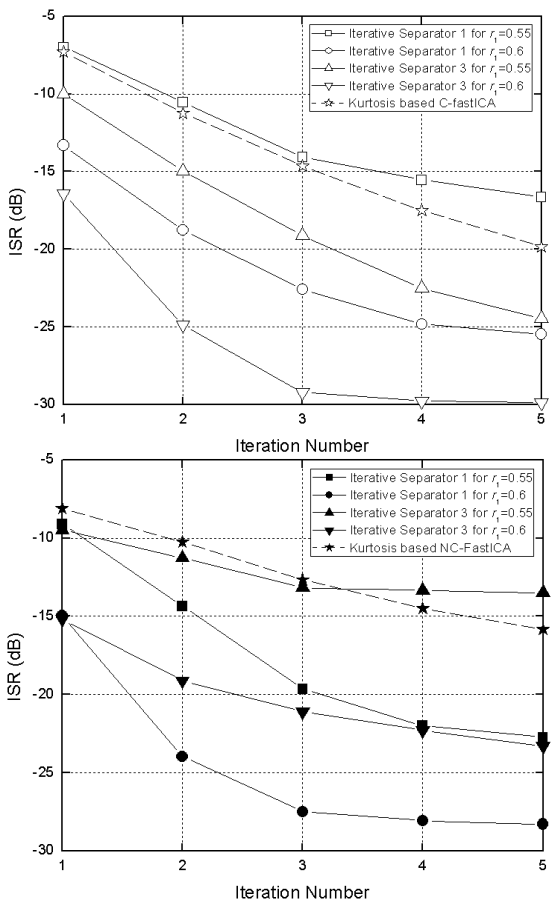


Figure 7. Comparison of Performance with Kurtosis based C-FastICA and NC-FastICA in terms of ISR. (Left) Performance for Extracting Circular 4QAM sources. (Right) Performance for Extracting Noncircular 4QAM Sources with the Asymmetry η=3

Finally, the performance of iterative separators is compared with two complex FastICA algorithms, *i.e.*, Complex FastICA (C-FastICA) [17] and Noncircular FastICA (NC-FastICA) [18] in terms of ISR. Although these two algorithms were used to deal with the instantaneous case for circular and noncircular sources respectively, they are now available for the convolutive case since we have transformed the convolutive mixture into instantaneous one in Section 2. While recovering both circular sources (Figure 7, left) and noncircular sources (Figure 7, right), iterative separators generally converge faster and perform better than two complex FastICA algorithms, except when improper separators are chosen also with low value of r_1. Meanwhile, considering the complexity with equally $o(n(L+D-1)\mathrm{T})$ products per iteration, the proposed algorithms are more efficient than the complex FastICA algorithms.

The above-mentioned analysis can be summed up by a few empirical rules to indicate how separators should be chosen:

♦ If the prior knowledge of positive support is accurate enough (r>0.9), using single-step separators given in (11), (15) and (16) is a satisfactory choice to achieve extraction efficiently. But iterative separators should be preferable to improve the performance when the accuracy of prior knowledge is poor (0.5<r<0.9).

♦ For single-step separators, Separator 3 in (16) requiring doubled corresponding prior knowledge really costs and behaves cost-effective only when sources are close to

circular ($1/3 < \eta < 3$), whereas Separator 1 in (11) and Separator 2 in (15) that is more economical should be chosen in most cases, especially when the sources are noncircular with the asymmetry $\eta \geq 3$ and $0 < \eta \leq 1/3$ respectively.

♦ For iterative separators, Iterative Separator 3 converges faster and performs better only when sources are nearly circular ($1/2 < \eta < 2$), whereas Separator 1 in (11) and Separator 2 in (15) converge equally fast or even faster than Separator 3 in most cases, especially when the sources are noncircular with the asymmetry $\eta \geq 2$ and $0 < \eta \leq 1/2$ respectively.

6. Conclusion

In this paper, we propose a novel method to solve the problem of extracting complex source signals from convolutive mixtures in a simple yet effective way. The single-step and iterative algorithms are derived for both complex circular and noncircular sources. In order to show effects of different factors including sample size, correct support estimation ratio and noncircularity on the performance, ISR and PCE is provided theoretically and experimentally. Theoretical analysis approximates the simulation results precisely, and indicates that special attention should be paid to the noncircularity while choosing separators to recover the source of interest. The judgment concerning the choice of separators is made after deliberation specifically. What's more, compared with two complex FastICA algorithms, the proposed iterative algorithms have equally low complexity and converge faster in most cases, except when improper separators are chosen also with very low initial value of correct support estimation ratio. This work is an important advancement of our previous work for the application in wireless communication system, but future research is still needed to evaluate its performance in real-world applications.

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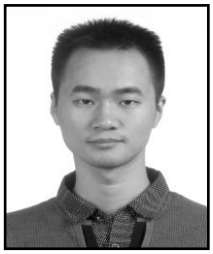

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