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Images Denoising and Enhancement Based on Dyadic Wavelet Domain Hidden Markov Models and Interpolation

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Abstract

For image denoising and enhancement, we combine dyadic wavelet transform and hidden Markov tree (HMT) model, and propose an image edge enhancement method based on dyadic wavelet domain hidden Markov models and interpolation algorithm. Making use of scale correlation between HMT model and binary wavelet coefficients, we establish different interpolation classification for different pixel distribution, realize enhanced image through transforming low resolution picture into high resolution picture, and simplify the complexity of computing at the same time. The simulation results show that compared with dyadic wavelet transform, both visual effects and quantitative analysis has significantly improved by wavelet domain HMT model method.

Keywords: *Dyadic wavelet transform, Hidden Markov tree model, Interpolation algorithm, Image enhancement*

1. Introduction

Image will always be mixed with noise during acquire and transmission, which not only affect the image quality, but also further affect the image analysis, understanding and value. As a result, image needs to be preprocessed. Nowadays, there are several denoising methods in the field of image processing, and most of these methods are developed according to features of the actual image and statistical characteristics of noise. For example, spatial domain method, such as mean filtering, median filtering and Wiener filtering, makes use of various filtering methods to remove noise. Frequency domain method reduces noise by retaining low-frequency component or low-scale components. The basic idea of these methods is based on the fact that noise energy is generally concentrated in high-frequency, while images are mainly distributed in low frequency. These methods can eliminate part of the noise to some extent, but at the same time, edge information of image will inevitably be lost, especially for image with rich edge. Image interpolation is of concern since it can resolve images from low resolution to high resolution. [1] proposed an interpolation algorithm: by weighted interpolation of gradient control, part of image information can be retained during image enlargement, and more importantly, some details of image edge can be preserved. Analysis results show that interpolation algorithm can not only handle image amplification, but also deal with image denoising and enhancement by increasing resolution. But it should be noticed that improving image resolution will bring image edges blur or jaggy at the same time. In recent years, a lot of work has already been done for interpolating the images using discrete wavelet transform. It was targeted at edges and texture enhancements such that after interpolation the smoothness of the visual images was maintained for photographic

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and printing purposes [2, 3]. Comparatively, less emphasis was given on interpolation for the images of required size. Most of the algorithms were computationally heavy [4, 5]. However, we could not come across any interpolation technique using discrete wavelet transform (DWT) which can predict the required image resizing factor in terms of DWT level, as proposed in this work. However, how to fully access the image information, only the interpolation algorithm of wavelet and multi-scale geometric analysis method is still difficult to solve the multi-morphological characteristics of the image[6], respectively.

Taking into account the hidden Markov tree (HMT) model, it can successfully be applied to a different class so soon, and include image denoising and related applications. HMT model can be used to find the most likely state of the estimated coefficients, estimated by using randomly generated partition coefficient, the lower resolution of the status information state transition.

2. Dyadic Wavelet Transform and Denoising Image

In two-dimensional signal analysis, multiscale tensor space $\{V_j^2 : V_j \otimes V_j\}_{j\in Z}$ makes up the multiresolution analysis of $L^2(R^2)$. Set wavelet space, $W_i : W_i = V_{i+1}/V_i \quad V_{j+1} = V_j \oplus W_j$, then space $L^2(R^2)$ is expressed as $L^2(R) = \underset{i\in Z}{\oplus} W_i$ from subspace W_i.

Setting $\psi(x) \in \{V_j\}$ as one-dimensional scaling function, $\varphi(x) \in \{W_j\}$ as the corresponding dyadic wavelet, thus an orthonormal basis of $L^2(R^2)$ is defined as follow:

$$\psi(x,y) = \psi(x)\psi(y) \qquad (x,y) \in R^2$$

Three two-dimensional dyadic wavelet are available as follows,

$$\begin{aligned} &\varphi^{(1)}(x,y) = \psi(x)\varphi(x) \\ &\varphi^{(2)}(x,y) = \psi(y)\varphi(x) \\ &\varphi^{(3)}(x,y) = \varphi(x)\varphi(y) \end{aligned} \tag{1}$$

(1) can be expressed as

$$\varphi^{(i)}_{2^j,m,n}(x,y) = \frac{1}{\sqrt{2^j}}\varphi^{(i)}\left(\frac{x-m}{2^j}, \frac{y-n}{2^j}\right), \quad j,n,m \in Z,\ j \geq 0,\ i = 1, 2, 3$$

According to two-dimensional multiresolution analysis, low frequency components of two-dimensional image signal $f(x,y)$ under the scale 2^j can be showed as

$$a_j[m,n] = (f(x,y), \psi_{2^j,m,n}(x,y)) = \frac{1}{\sqrt{2^j}}\iint_{R^2} f(x,y)\psi\left(\frac{x-m}{2^j}, \frac{y-n}{2^j}\right)dxdy$$

The high frequency components are showed follows

$$c_j^3[m,n] = (f(x,y), \varphi_{32^j,m,n}(x,y))$$

The image signal is decomposed into a low frequency component and three high frequent components, a_j is the low frequency component of scale 2^j, c_j^1, c_j^2, c_j^3 are the vertical, horizontal, diagonal high frequency components of scale 2^j respectively. Low frequency component a_j can still be decomposed according to above way.

For scale space V_0, the two-dimensional $f(x,y) \in V_0 \otimes V_0 \in L_2(R^2)$ any pixel (x,y) is corresponded to gray value of image signal $f(x,y)$. In the scale 2^j $(j = 1,2....)$, the decomposition can be written as

$$f(x,y) = \sum_{j=1}^{J}\sum_{m,n}[a_j\psi(x,y) + c_j^1\varphi^{(1)}(x,y) + c_j^2\varphi^{(2)}(x,y) + c_j^3\varphi^{(3)}(x,y)] \tag{2}$$

3. Hidden Markov Tree Model

According to thorough analysis of the relationship between the quality of image processing and wavelet coefficients, wavelet domain HMT has the characteristic that large (or small) coefficients between scales of wavelet decomposition pass progressively along wavelet quadtrees. Two-dimensional discrete wavelet transform is able to obtain the multiresolution characteristics of three subbands singularities at different scales of image, and it is also able to model edge probability density function $f(C_i)$ of each wavelet coefficient C_i into mixed Gaussian distribution with hidden state $S_i \in \{S, L\}$. S_i is decided by component of hybrid model corresponding to C_i. State S represents small wavelet coefficient with zero mean and small variance Gaussian distribution, while status L represents large wavelet coefficient with zero mean and large variance Gaussian distribution, and they can better capture the passing properties between wavelet coefficients [7].

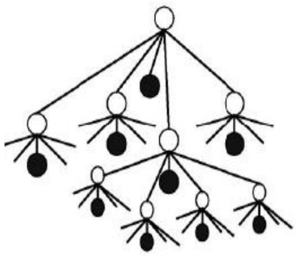


Figure 1. HMT Models

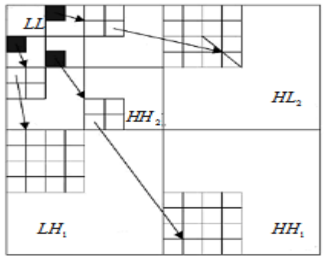


Figure 2. 2-D Dyadic Wavelet Decomposition

Figure 1 and Figure 2 shows the black node represents a wavelet coefficient, the white nodes represent the implicit state relates to wavelet coefficients, and the connection shows the relationship among the states. Each channel uses a quadtrees to describe the mutual independency among them. Each parent node join four children nodes, and distribution of each child node i is determined by the distribution of its parent node $p(i)$ and the transition probability between parent child node. The scaling relationship of dyadic wavelet coefficients is represented by simple quadtrees structure of HMT model, and there is a sub coefficient of dyadic wavelet coefficient $C_{i,k}^{b}$ in the i th layer, $b \in \{LH, HL, HH\}$.

The large and small dyadic wavelet coefficients can pass between adjacent scales, and wavelet amplitude is closely related to the amplitude of its parent node. As a result, if the state of coefficient C_i is given as S_i, then C_i has nothing to do with all the other random variables. Describing the dependence between state variables of coefficient and of its children nodes by their connecting probability tree, HMT can obtain quadtrees structure diagram, just like wavelet coefficient. It can be seen that three subbands of the coefficients are independent with each other, and in the same subband, one wavelet coefficient under j ($2 \le j \le J$) scale correspond to four wavelet coefficients under $j-1$ scale.

For the two-dimensional image of wavelet coefficients, the HMT model is built along three subbands directions of horizontal, vertical and diagonal. The model parameters corresponding to channel HH, HL , LH are respectively recorded as θ^{LH}, θ^{HL} and θ^{HH}, the probabilities of root node state variables are noted as p_0^L, $\varepsilon_{i,p(i)}^{mn}$ ($m,n \in \{S, L\}$),expressing the probability of child node status n while the parent node m, and the mean value and variance of the mixed Gaussian distribution are namely $\mu_{i,m}$ $\mu_{i,m}$ and $\sigma_{i,m}^2$. Each dyadic wavelet coefficient C_i and S represented by six parameters, thus there is the vector of model parameter

$$\theta = \{p_0^L, \varepsilon_{i,p(i)}^{mn}, \mu_{i,m}, \sigma_{i,m}^2\}$$

4. Interpolation Calculation

The nearest neighbor interpolation, bilinear interpolation and bi-cubic interpolation are the commonly used image interpolation algorithms. Bilinear interpolation method is a modified nearest neighbor method. The bilinear interpolation method uses the pixel values of four adjacent points to do linear interpolation in both directions (horizontal axis direction, the vertical axis direction), gets the new pixel value of the awaiting sampling point, and calculates the pixel values of sampling point according to the corresponding weights of the distance between the awaiting sampling points and its adjacent points. Bilinear interpolation method is more accurate than the nearest neighbor interpolation method, but jaggy or blur phenomena still happen [8]. The existing research experiments show that the wavelet transform can be ratio combined with bilinear interpolation, and better reflect the advantage of improving image quality.

For images $f(x,y)$, x, y are the horizontal and vertical coordinates of the original pixels, respectively, and by wavelet transform they are decomposed into four components, HH, HL, LH and LL, where LL is the major low frequency components of the input image, HL is high frequency components in vertical direction and low frequency components in horizontal direction, LH is low frequency components in vertical direction and high frequency components in horizontal direction, while HH is the high frequency components in vertical and horizontal directions. In this work, we propose the interpolation calculation based on wavelet transform: get a high resolution subgraph from low resolution subgraph, and further improve the resolution of image by implementing inverse transform, thus achieving image enhancement. Denoting binary wavelet transform by T, bilinear interpolation formula is expressed as follows:

$$f(x,y) = [f(1,0) - f(0,0)]x + [f(0,1) - f(0,0)]y + [f(1,1) + f(0,0) - f(0,1) - f(1,0)]xy + f(0,0) \qquad (3)$$

where $f(x,y)$ is an $M \times N$ image, $Y_{i,j}$ $(i = 1,\cdots M, j = 1,\cdots N)$ is the grey value of the i th line and the j^{th} column pixel in $f(x,y)$, and $f(x,y) = Y_{i,j}$.

Suppose low resolution image is $L_{i,j}$, and the corresponding high resolution image is $H_{i,j}$, by interpolation amplification, we can get

$$H_{2i,2j} = L_{i,j} \quad i = 0,1,2,....M, \quad j = 0,1,2,.....N$$

By interpolation we can get the values of pixel points $H_{2i,2j+1}$, $H_{2i+1,2j}$ and $H_{2i+1,2j+1}$ in high resolution image. According to the algorithm proposed in [9], the wavelet domain is divided into flat areas and edge areas, and in order to improve the image pixel, different interpolation algorithms are implemented to pixels in different regions. The main steps are as follows. Minimum pixel difference expresses the highest level of similarity between pixel points. By setting threshold, the difference between pixel pairs is calculated along the horizontal, vertical and two diagonal directions, respectively

$$\Delta H_n = \left| H_{2i,2j} - H_{2i+2p,2j+2q} \right| \qquad (p,q) \in \{(0,1),(1,0)\} \quad n = 1.2.3$$

If the difference between pixel pair is less than the setting threshold, the pixel point is regarded as a pixel in flat areas, which is implemented by bilinear interpolation (3), otherwise, the pixel point is recorded as an edge pixel point.

5. Images Denoising and Enhancement based on Wavelet Domain HMT Models

Bilinear interpolation plays a certain role in improving the image resolution. However, since it is difficult to completely acquire the high frequency signals, a gap is easily produced between the interpolated image and the original image, thus the mosaic phenomenon is emerged in the edge of image. In order to reflect the local characteristics of image from different resolution spaces, binary wavelet transform is applied to effectively extract information from the signal [10]. By dilation and translation operations, it can do multiscale refinement analysis to functions or signals, ultimately achieve time subdivision at high frequency and frequency subdivision at low frequency, and automatically adapt to time frequency signal analysis, thus can focus on any detail of the image. From the previous analysis, the main steps of image denoising and enhancement, which is based on the interpolation of dyadic wavelet domain HMT model, are as follows:

Step 1: For image $f(x,y)$, do two-dimensional dyadic wavelet transform $WT_{f(x,y)}$. According to àtrous algorithm and two-dimensional wavelet transform, the $M \times N$ amplitude image $f(x,y)$, (2) is divided into four subbands images HH, HL, LH and LL with different scale levels and details, which is denoted by $Tf(x,y)=(HH,HL,LH,LL)$

Step 2: Establish HMT model on wavelet transform coefficients, and classify high frequency coefficients based on the fact that details and noise are more concentrated in high frequency subbands. By

$$\sigma_n^2 = \frac{median(|w_{ij}|)}{0.6745}, \quad \text{where} \quad w_i \in HH_1$$

layered threshold is constructed, and the image denoised by layered threshold is denoted as
$f_T(x,y)$.

Step 3: Execute the above interpolation algorithm, and interpolate HH, HL, LH, which are recorded as HH^*, HL^*, LH^*, respectively, and the low frequency part of the original image is denoted by $f(x,y)=LL^*$.

Step 4: Inverse wavelet transform to HH^*, HL^*, LH^* and LL^*, and get the image $f^*(x,y)=T^{-1}(HH^*,HL^*,LH^*,LL^*)$ denoised and enhanced by wavelet transform and interpolation algorithm.

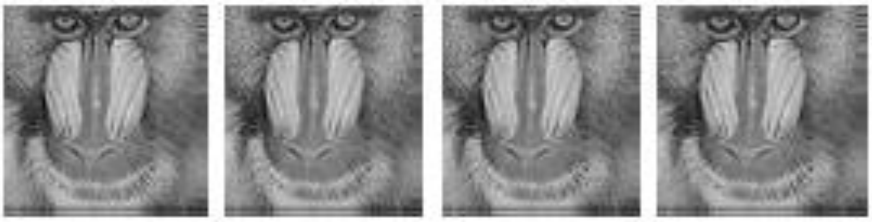
6. Experiment

In order to verify the effectiveness of algorithm for image processing, comparison simulation experiments are constructed in the Lena and Baboon images with the size of 256×256: noise are Gaussian white noise with variance being 15, and mean being zero; in the interpolation algorithm, the initial interpolation threshold is 30, and after interpolation, the image is enlarged to 512×512. Observed from the testing image, the regional and detail feature of pixel value even distributed has been enhanced significantly. Through experimental analysis, the experiment uses wavelet db 4 to decompose the image into three-layers, denoise the image by different threshold function, and compares the denoising effect and the method in the paper. The Peak Signal to Noise Ratio (PSNR) is

$$PSNR = 10\log[\max_{i,j}\{f^2(i,j)\}/MSE]$$

where M and N are the numbers of ranks of the image respectively, MSE is mean square error, $\hat{f}(i,j)$ and $f(i,j)$ $(1 \le i \le M,\ 1 \le j \le N)$ present the gray value of the original image and the denoised image at the point (i,j) respectively.

The denoising and enhancement effects are evaluated in the two aspects: the peak signal to noise ratio and subjective visual effects. The figures as follows are: image of Baboon and Lena (use the white Gaussian noise whose standard deviation is 20, mean value is 0). Three methods are compared: the two-dimensional bidirectional linear interpolation (denoted by "BLI"), two-dimensional dyadic wavelet soft threshold (denoted by "DWST"), and the proposed HMT and interpolation in dyadic wavelet domain scheme in the simulations.



(a) Noisy image (b) BLI (c) DWST (d) Proposed

Figure 3. Comparing the Performance of the Various Methods on Baboon

(a) Noisy image (b) BLI (c) DWST (d) Proposed

Figure 4. Comparing the Performance of the Various Methods on Lena

It shows from the figures that compared with wavelet denoising, denoising by HMT and interpolation in dyadic wavelet domain keeps the edge portions of the image comparing better, especially the edge portions of the image can be clearly seen, so the visual effects are greatly improved. The proposed method in this paper, which can be seen from the figure, retains the high frequency details and textures of the image and improves the visual effects of the image. Table 1 shows the MSE and PSNR values (dB) of the simulation results for the reconstructed images.

Table 1: Comparison of experimental results of image denoising and enhancement using different algorithms where BSNR, DSNR, and PSNR are Signal to Noise Ratio (SNR) on BLI, DWST and proposed, respectively. BPSNR, DPSNR and PPSNR are Peak Signal to Noise Ratio (PSNR) on BLI, DWST and proposed, respectively.

Table 1. The Three Kind Values using Different Algorithms

Image	BSNR	DSNR	PSNR	BPSNR	DPSNR	PPSNR
Baboon	10.063182	9.94719	9.02714	17.75102	19.5028	22.0151
Lena	1.24149	1.07106	0.75401	12.7103	13.5082	14.2971

7. Conclusions

Based on wavelet domain HMT model, the images denoising and enhancement methods is proposed in dyadic wavelet domain HMT image interpolation. We have chosen different thresholds for different scales of wavelet coefficients for the high frequency signal distribution of the image noise and image details. HMT model generates random coefficient estimates, and generates different pixel distribution to take a different classification of interpolation. The state of images is converted from low resolution to high resolution, as well as textures details of the image are enhanced. The overall feature profile visual effect. The simulation results show that this method is effective and feasible.

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