# Study of a Novel Micro-mechanical Gyroscope using for Rotation Carrier 

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#### Abstract

In this paper, a novel silicon micro-machined gyroscope is introduced, which is driven by the rotating carrier's angular velocity. The principle of structure is analyzed .The mathematic module is also established and the dynamics parameters of the gyroscope is calculated, Design and analysis of signal processing circuit, design and processing of the gyroscope sensing head, at last, the gyroscope is tested, the test results and the theoretical analysis is consistent.


Keywords: Micromechanics, Gyroscope, Angular velocity

## 1. Introduction

With the increasing development of MEMS and inertial guidance technology, all kinds of micro-machined gyros have successfully developed and are gaining increasing popularity for shared use in military and civil applications [1-3]. According to the driving structure, a MEMS gyro can be divided into two types. One is a gyro with a driving structure, and another is gyros without driving structures. The vast majority of reported micro-machined rate gyroscopes utilize a vibratory proof mass suspended by flexible beams above a substrate. The primary working principle is to form a vibratory drive oscillator, coupled to an orthogonal sense accelerometer by the Coriolis force. Using anchor, the proof mass is suspended above the substrate, making the mass free to oscillate in two orthogonal directions-the drive and the sense directions [4-8].

This kind of gyro is difficult to design and manufacture and the cost is high. In order to avoid the difficulties brought about by the driving part, this paper puts forward a novel gyro that uses the rotation of the aircraft itself as a driving part. In this paper, the studied gyro, which has been fabricated on mono-crystalline silicon wafer by means of bulk micromachining manufacturability techniques, belongs to the gyro without driving structure class. Since there is no driving structure, the structure is simple and easy to process $[9,10]$. This new gyro is used on rotating aircraft for flight attitude control. The gyro sensing element is called a silicon pendulum. Aircraft spin provides angular momentum for the silicon pendulum so that it can sense the transverse angular velocity when aircraft appears to turn. Using detection circuits, the angular vibration of the silicon pendulum is transformed into an alternating electric signal. The signal frequency is same as the spin frequency of aircraft, and the signal envelope is proportional to the transverse angular velocity.

## 2. Principle of Structure

Figure 1 shows the structure of the micro-machined silicon gyroscope derived by carrier's angular velocity. In Figure 1, 1 is silicon proof mass (sensing mass, mass), 2 is silicon elasticity torsion girder, 3 is electrode. Four electrode and silicon proof mass form four capacitors. The coordinates $X Y Z$ is fixed on mass of sensor, $\dot{\alpha}$
is angular velocity about mass vibrating around the $O Y$ axes, $\dot{\varphi}$ is carrier's spin angular velocity, $\Omega$ is carrier's yaw or pitching angular velocity. The gyroscope is fixed on the carrier and rotates with the carrier at the speed, yawing or pitching with the speed $\Omega$ at the same time. The mass is affectted by coriolis force changing frequently (the frequent of the coriolis force equals to the frequent of carrier rotating), and then the mass oscillates along the $O Y$ axes. The oscillation causes capacitance variety of four capacitors.
$\left(C_{1}, C_{2}, C_{3}, C_{4}\right)$,which are formed by the mass and four electrode. It is shown in Fig 2, variety of capacitance signal is converted into the variety of voltage signal and then the voltage signal is amplified, so we can obtain the voltage signal in proportion to the angular velocity $\Omega$ that we need detect.


Figure 1. The Principle Structure of Sensor


Figure 2. Principle of Signal Detecting Circuit

## 3. Dynamic Model

### 3.1 Vibration Equation of the Mass

The equation of vibration mass can be described by coordinate transform. In Figure 3, ${ }_{\xi} \xi_{\zeta} \zeta$ is the inertia coordinate (the fixed coordinate) $O X_{1} Y_{1} Z_{1}$ is the yaw or pitching coordinate; $O X_{2} Y_{2} Z_{2}$ is the spin coordinate of the carrier; $O X Y Z$ is the coordinate connected on the mass. In the inertia coordinate $0 \xi \eta \zeta$,


Figure 3. The Coordinate Transform
by the theory of angular momentum for rigid body rotating around fixed point , following formula can be obtained

$$
\frac{d}{d t}\left[\begin{array}{l}
G_{\xi}  \tag{1}\\
G_{\eta} \\
G_{\zeta}
\end{array}\right]=\left[\begin{array}{l}
M_{\xi} \\
M_{\eta} \\
M_{\zeta}
\end{array}\right]
$$

in the formula, $\left[\begin{array}{c}G_{\xi} \\ G_{\eta} \\ G_{\zeta}\end{array}\right]=J\left[\begin{array}{c}\omega_{\xi} \\ \omega_{\eta} \\ \omega_{\zeta}\end{array}\right]$ is the angular momentum of the gyroscope mass in the coordinate $0 \xi \eta \zeta,\left[\begin{array}{l}M_{\xi} \\ M_{\eta} \\ M_{\zeta}\end{array}\right]$ is the moment which operates on the gyroscope mass in the coordinate $05 \eta \zeta$.

As the OXYZ coordinate is set for inertia principal axis coordinate, rotating inertia matrix $J$ is a constant value matrix.
(1) In inertial coordinate $0 \xi \eta \zeta$, the mass rotates around $O \xi$ axis at the angular rate $\Omega$ and then turns angular $\Omega^{t}$ to the coordinate $O X_{1} Y_{1} Z_{1}$, thus

$$
\left[\begin{array}{l}
G_{\xi} \\
G_{\eta} \\
G_{\zeta}
\end{array}\right]=A^{-1}\left[\begin{array}{l}
G_{X_{1}} \\
G_{Y_{1}} \\
G_{Z_{1}}
\end{array}\right],\left[\begin{array}{l}
M_{\zeta} \\
M_{\eta} \\
M_{\zeta}
\end{array}\right]=A^{-1}\left[\begin{array}{l}
M_{X_{1}} \\
M_{Y_{1}} \\
M_{Z_{1}}
\end{array}\right]
$$

in the formula, $A=\left[\begin{array}{ccc}1 & 0 & 0 \\ 0 & \cos \Omega t & \sin \Omega t \\ 0 & -\sin \Omega t & \cos \Omega t\end{array}\right]$ is transformation matrix, it is the function of time.

Putting the relation formula into the formula (1) and transforming, we can obtain that

$$
\frac{d}{d t}\left[\begin{array}{c}
G_{X_{1}}  \tag{2}\\
G_{Y_{1}} \\
G_{Z_{1}}
\end{array}\right]+\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & -\Omega \\
0 & \Omega & 0
\end{array}\right]\left[\begin{array}{c}
G_{X_{1}} \\
G_{Y_{1}} \\
G_{Z_{1}}
\end{array}\right]=\left[\begin{array}{c}
M_{X_{1}} \\
M_{Y_{1}} \\
M_{Z_{1}}
\end{array}\right]
$$

(2) In the coordinate $O X_{1} Y_{1} Z_{1}$, the mass rotates around $O Z_{1}$ axis at the angular rate $\dot{\varphi}$ and then turns angular $\varphi$ to the coordinate $0 X_{2} Y_{2} Z_{2}$, thus

$$
\left[\begin{array}{l}
G_{X_{1}} \\
G_{Y_{1}} \\
G_{Z_{1}}
\end{array}\right]=B^{-1}\left[\begin{array}{c}
G_{X_{2}} \\
G_{Y_{2}} \\
G_{Z_{2}}
\end{array}\right],\left[\begin{array}{l}
M_{X_{1}} \\
M_{Y_{1}} \\
M_{Z_{1}}
\end{array}\right]=B^{-1}\left[\begin{array}{l}
M_{X_{2}} \\
M_{Y_{2}} \\
M_{Z_{2}}
\end{array}\right]
$$

In the formula, $B=\left[\begin{array}{ccc}\cos \varphi & \sin \varphi & 0 \\ -\sin \varphi & \cos \varphi & 0 \\ 0 & 0 & 1\end{array}\right]$ is transformation matrix, it is the function of time.

Putting the formulation hereinbefore into the formulation (2), we can get that

$$
\left[\begin{array}{ccc}
0 & -\dot{\varphi} & 0  \tag{3}\\
\dot{\varphi} & 0 & 0 \\
0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
G_{X_{2}} \\
G_{Y_{2}} \\
G_{Z_{2}}
\end{array}\right]+\frac{d}{d t}\left[\begin{array}{c}
G_{X_{2}} \\
G_{Y_{2}} \\
G_{Z_{2}}
\end{array}\right]+B\left[\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & -\Omega \\
0 & \Omega & 0
\end{array}\right] B^{-1}\left[\begin{array}{l}
G_{X_{2}} \\
G_{Y_{2}} \\
G_{Z_{2}}
\end{array}\right]=\left[\begin{array}{l}
M_{X_{2}} \\
M_{Y_{2}} \\
M_{Z_{2}}
\end{array}\right]
$$

(3) In the coordinate $O X_{2} Y_{2} Z_{2}$, the mass rotates around $O Y_{2}$ axis at the angular rate $\dot{\alpha}$ and then turns angular $\alpha$ to the coordinate $O X Y Z$, thus

$$
\left[\begin{array}{l}
G_{X_{2}} \\
G_{Y_{2}} \\
G_{Z_{2}}
\end{array}\right]=C^{-1}\left[\begin{array}{c}
G_{X} \\
G_{Y} \\
G_{Z}
\end{array}\right],\left[\begin{array}{l}
M_{X_{2}} \\
M_{Y_{2}} \\
M_{Z_{2}}
\end{array}\right]=C^{-1}\left[\begin{array}{l}
M_{X} \\
M_{Y} \\
M_{Z}
\end{array}\right],
$$

in the formula, $C=\left[\begin{array}{ccc}\cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha\end{array}\right]$ is transformation matrix, it is the function of time.

Putting the formulation hereinbefore into the formulation (3), we can obtain that $\left[\begin{array}{ccc}0 & \dot{\varphi} & 0 \\ \dot{\varphi} & 0 & 0 \\ 0 & 0 & 0\end{array}\right] C^{-1}\left[\begin{array}{c}G_{X} \\ G_{Y} \\ G_{Z}\end{array}\right]+\frac{d}{d t}\left(C^{-1}\left[\begin{array}{c}G_{X} \\ G_{Y} \\ G_{Z}\end{array}\right]\right)+B\left[\begin{array}{ccc}0 & 0 & 0 \\ 0 & 0 & -\Omega \\ 0 & \Omega & 0\end{array}\right] B^{-1} C^{-1}\left[\begin{array}{l}G_{X} \\ G_{Y} \\ G_{Z}\end{array}\right]=C^{-1}\left[\begin{array}{c}M_{X} \\ M_{Y} \\ M_{Z}\end{array}\right]$
After the formulation hereinbefore has been predigested, we can get that

$$
\left[\begin{array}{ccc}
0 & -\Omega \cos \varphi \sin \alpha-\dot{\varphi} \cos \alpha & -\Omega \sin \varphi+\dot{\alpha}  \tag{4}\\
\Omega \sin \alpha \cos \varphi+\dot{\varphi} \cos \alpha & 0 & \dot{\varphi} \sin \alpha-\Omega \cos \varphi \cos \alpha \\
\Omega \sin \varphi-\dot{\alpha} & \Omega \cos \varphi \cos \alpha-\dot{\varphi} \sin \alpha & 0
\end{array}\right]\left[\begin{array}{l}
G_{X} \\
G_{Y} \\
G_{Z}
\end{array}\right]+\frac{d}{d t}\left[\begin{array}{l}
G_{X} \\
G_{Y} \\
G_{Z}
\end{array}\right]=\left[\begin{array}{c}
M_{X} \\
M_{Y} \\
M_{Z}
\end{array}\right]
$$

In the coordinate $O X Y Z$, angular momentum of the gyroscope mass is

$$
\left[\begin{array}{c}
G_{X} \\
G_{Y} \\
G_{Z}
\end{array}\right]=J\left[\begin{array}{l}
\psi_{X} \\
\psi_{Y} \\
\psi_{Z}
\end{array}\right]=\left[\begin{array}{l}
J_{X} \psi_{X} \\
J_{Y} \psi_{Y} \\
J_{Z} \psi_{Z}
\end{array}\right]
$$

in the formulation, $J_{X}, J_{Y}, J_{Z}$ are the moments of inertia for the gyroscope mass in $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ axises and $\psi_{X}, \psi_{Y}, \psi_{Z}$ are projection component of the angular velocity vector in coordinate $O_{X Y Z}$.

$$
\left[\begin{array}{l}
\psi_{X} \\
\psi_{Y} \\
\psi_{Z}
\end{array}\right]=\left[\begin{array}{c}
\Omega \cos \varphi \cos \alpha-\dot{\varphi} \sin \alpha \\
-\Omega \sin \varphi+\dot{\alpha} \\
-\Omega \cos \varphi \sin \alpha+\dot{\varphi} \cos \alpha
\end{array}\right]
$$

Thus,

$$
\left[\begin{array}{l}
G_{X} \\
G_{Y} \\
G_{Z}
\end{array}\right]=J\left[\begin{array}{l}
\psi_{X} \\
\psi_{Y} \\
\psi_{Z}
\end{array}\right]=\left[\begin{array}{c}
J_{X} \psi_{X} \\
J_{Y} \psi_{Y} \\
J_{Z} \psi_{Z}
\end{array}\right]=\left[\begin{array}{c}
J_{X}(\Omega \cos \varphi \cos \alpha-\dot{\varphi} \sin \alpha) \\
J_{Y}(-\Omega \sin \varphi+\dot{\alpha}) \\
J_{Z}(-\Omega \cos \varphi \sin \alpha+\dot{\varphi} \cos \alpha)
\end{array}\right]
$$

Putting the formulation hereinbefore into the formulation (4), we can obtain three dynamic equations, the dynamic equation in $O Y$ axis is $\left(J_{X}+J_{Z}\right) \Omega^{2} \cos ^{2} \varphi \sin \alpha+\left(J_{Z}-J_{X}\right) \dot{\varphi}^{2} \sin \alpha \cos \alpha+J_{X} \Omega \dot{\varphi} \cos \varphi \cos 2 \alpha-J_{Z} \Omega \dot{\varphi} \cos \varphi+$
$J_{Y} \ddot{\alpha}-J_{Y} \Omega \dot{\varphi} \cos \varphi-J_{Y} \frac{d \Omega}{d t} \sin \varphi=M_{Y}$
The summation for moment of external force in $O Y$ axis is

$$
M_{Y}=-K_{T} \alpha-D \dot{\alpha}
$$

Considering $\Omega \ll \dot{\varphi}$, item $\Omega^{2}$ can be neglected. As $\alpha=0$, we can get $\sin \alpha \approx \alpha, \cos \alpha=\cos 2 \alpha=1$, Let

$$
\frac{d \Omega}{d t}=0
$$

get following equation:

$$
\begin{equation*}
J_{Y} \ddot{\alpha}+D \dot{\alpha}+\left[\left(J_{Z}-J_{X}\right) \dot{\varphi}^{2}+K_{T}\right] \alpha=\left(J_{Z}+J_{Y}-J_{X}\right) \Omega \dot{\varphi} \cos (\dot{\varphi} t) \tag{6}
\end{equation*}
$$

in the formulation hereinbefore, $J_{X}, J_{Y}, J_{Z}$ are the moments of inertia for the gyroscope mass in $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ axises, $K_{T}$ is the coefficient of torsion rigidity, $D$ is damping coefficient.

### 3.2 Solution of Angular Vibration Vibrating Equation

Reducing the formulation (6), we can obtain that

$$
\begin{equation*}
\ddot{\alpha}+2 \xi \omega_{0} \dot{\alpha}+\omega_{0}^{2} \alpha=f_{0} \cos (\dot{\varphi} t) \tag{8}
\end{equation*}
$$

In the formulation,

$$
\begin{gather*}
\omega_{0}^{2}=\frac{1}{J_{Y}}\left[\left(J_{Z}-J_{X}\right) \dot{\varphi}^{2}+K_{T}\right]  \tag{9}\\
\xi=\frac{D}{2 \omega_{0} J_{Y}}=\frac{D}{2 \sqrt{\left[\left(J_{Z}-J_{X}\right) \dot{\varphi}^{2}+K_{T}\right] J_{Y}}}  \tag{10}\\
f_{0}=\frac{1}{J_{Y}}\left(J_{Z}+J_{Y}-J_{X}\right) \Omega \dot{\varphi}
\end{gather*}
$$

Its solution is

$$
\begin{aligned}
& \alpha=A e^{-n t} \cos \left(\sqrt{\omega_{0}^{2}-n^{2}} t+\delta\right)+B \cos (\dot{\varphi} t-\beta) \\
& \operatorname{tg} \beta=\frac{2 n \dot{\varphi}}{\omega_{0}^{2}-\dot{\varphi}^{2}}
\end{aligned}
$$

$A$ and $\delta$ are integration constant and can be defined by movement initial condition, $B$ is amplitude for stable vibration, $\beta$ is phase difference, phase vibration drops behind excitation force phase angular $\beta, n=\xi \omega_{0}$, it is damping factor.
The first part of the formulation hereinbefore is attenuated soon as the vibration time rising. The second part is determined by excitation force and its frequent equals to excitation force frequent (namely spin angular velocity of carrier), vibration amplitude is affected by not only excitation force but also excitation frequent and parameters of vibration system $J_{X}, J_{Y}, J_{Z}, K_{T}, D$. Stable solution for the equation is

$$
a=B \cos (\dot{\varphi} t-\beta)=\frac{f_{0}}{\sqrt{\left(\omega^{2}{ }_{0}-\dot{\varphi}^{2}\right)+4 n^{2} \varphi^{2}}} \cos (\dot{\varphi} t-\beta)
$$

Substituting parameters and reducing, we can get

$$
\alpha=\frac{\left(J_{Z}+J_{Y}-J_{X}\right) \Omega \dot{\varphi}}{\sqrt{\left[\left(J_{Z}-J_{X}-J_{Y}\right) \dot{\varphi}^{2}+K_{T}\right]^{2}+(D \dot{\varphi})^{2}}} \cos (\dot{\varphi} t-\beta)
$$

Amplitude of angular vibration is

$$
\begin{equation*}
\alpha_{m}=\frac{\left(J_{Z}+J_{Y}-J_{X}\right) \dot{\varphi}}{\sqrt{\left[\left(J_{Z}-J_{X}-J_{Y}\right) \dot{\varphi}^{2}+K_{T}\right]^{2}+(D \dot{\varphi})^{2}}} \Omega \tag{11}
\end{equation*}
$$

## 4. Dynamic Parameter Analysis and Calculation

### 4.1 Torsion Rigidity Coefficient of Elastic Girder

The structure of elastic girder is shown in Figure 5, length, width and thickness of girder are $\mathrm{L}, \mathrm{W}$, and t . In order to be convenient for get the torsion rigidity coefficient of elastic girder, supposing:
(1) Turning angular is direct proportion to girder length;
(2) Warp of all brace girder ' $s$ cross-section are equal;
(3) Twist moments of girder's ends are equative, and their orientations are contrary.

According these assumptions, from elasticity mechanics we can get:

$$
\begin{equation*}
K=\frac{512 G a^{3} b}{\pi^{4} L} \sum_{n=1,3,5, \ldots}^{\infty} \frac{1}{n^{4}}\left(1-\frac{2 a}{n \pi b} \tanh \frac{n \pi b}{2 a}\right) \tag{12}
\end{equation*}
$$

In the formulation, $a$ and $b$ are width and length of rectangle cross-section, G is shear modulus of material for girder.

From formulation (12) we get total rigidity of two girders.

$$
\begin{equation*}
K_{T}=0.657 \times \frac{G t^{3} w}{L} \sum_{n=1,3,5}^{\infty} \frac{1}{n^{4}}\left(1-\frac{2 t}{n \pi w} \tanh \frac{n \pi w}{2 t}\right) \approx \frac{2}{3} \cdot \frac{G t^{3} w}{L} \tag{13}
\end{equation*}
$$



Figure 5. Structure of Supporting Girder
Put $w=0.8 \mathrm{~mm}, \mathrm{~L}=0.8 \mathrm{~mm}, \mathrm{t}=0.025 \mathrm{~mm}, \mathrm{G}=5.1 \times 10^{10}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$ into the formulation (13), get

$$
K_{T}=5.313 \times 10^{-4} \mathrm{~N} \cdot \mathrm{~m} .
$$

### 4.2 Angular Vibration Damping Coefficient of Vibration Devices

When a rectangle plane with length A and width B moves towards underside whose gap breadth is H , the press membrane damping coefficient is

$$
\begin{equation*}
f=\frac{F_{\text {danp }}}{d h / d t}=\frac{A B^{3} \mu}{h^{3}}\left[1-\frac{192 B}{A \pi^{5}} \sum_{n=1.35 . \ldots} \frac{1}{n^{5}} \tanh \frac{n \pi A}{2 B}\right] \tag{14}
\end{equation*}
$$

In the formulation, $\mu$ is gas adhesion coefficient, infinite series convergence is swiftness, so only get the first item, namely

$$
\begin{equation*}
f \approx \frac{A B^{3} \mu}{h^{3}}\left[1-\frac{192 B}{A \pi^{5}} \tanh \frac{\pi A}{2 B}\right] \approx \frac{96 \times \mu}{h^{3} \cdot \pi^{4}} B^{3} A\left[1-\frac{2}{\pi} \cdot \frac{B}{A} \tanh \left(\frac{\pi}{2} \cdot \frac{A}{B}\right)\right] \tag{15}
\end{equation*}
$$



Figure 6. Damping Partition
As the structure of the vibrating mass is complex, calculating the damping coefficient is very difficult. For reducing the calculation difficulty, the gyroscope mass is divided into three areas with distinct colours, it is shown in Figure 6, then add damping of three district as the gyroscope vibration global damping approximately. Consult Figure 3, get angular vibration damping factor of three distinct are

$$
\begin{align*}
& D_{1}(d, \alpha)=\frac{4 \times 96 \mu}{\pi^{4}}\left[\frac{1}{\left(d+r_{1} \alpha\right)^{3}}+\frac{1}{\left(d-r_{1} \alpha\right)^{3}}\right]\left(\frac{b_{3}-b_{2}}{2}\right)^{3} \times \frac{a_{3}-a_{1}}{2}\left[1-\frac{2 b_{3}-b_{2}}{\pi \alpha_{3}-\alpha_{1}} \tanh \left(\frac{\pi \alpha_{3}-a_{1}}{2 b_{3}-b_{2}}\right)\right] r_{1}^{2} \\
& D_{2}(d, \alpha)=\frac{2 \times 96 \mu}{\pi^{4}}\left[\frac{1}{\left(d+r_{2} \alpha\right)^{3}}+\frac{1}{\left(d-r_{2} \alpha\right)^{3}}\right]\left(\frac{a_{3}-a_{2}}{2}\right)^{3} \times b_{2}\left[1-\frac{1}{\pi} \frac{a_{3}-a_{2}}{b_{2}} \tanh \left(\pi \frac{b_{2}}{a_{3}-a_{2}}\right)\right] r_{2}^{2} \\
& D_{3}(d, \alpha)=\frac{4 \times 96 \mu}{\pi^{4}}\left[\frac{1}{\left(d+r_{3} \alpha\right)^{3}}+\frac{1}{\left(d-r_{3} \alpha\right)^{3}}\right]\left(\frac{a_{1}}{2}\right)^{3} \times \frac{b_{1}-b_{2}}{2}\left[1-\frac{2}{\pi} \frac{a_{1}}{b_{1}-b_{2}} \tanh \left(\frac{\pi}{2} \frac{b_{1}-b_{2}}{a_{1}}\right)\right] r_{3} \tag{16}
\end{align*}
$$

In the formulation,

$$
r_{1}=\frac{a_{3}+a_{1}}{4} \quad r_{2}=\frac{a_{3}+a_{2}}{4}, r_{3}=\frac{a_{1}}{4}
$$

Global damping factor of angular vibration is

$$
\mathrm{D}(d, \alpha)=\mathrm{D}_{1}(d, \alpha)+\mathrm{D}_{2}(d, \alpha)+\mathrm{D}_{3}(d, \alpha)
$$

Three relationship curves about damping coefficient and vibration angular are shown in Fig 7.
When $d=0.017 \mathrm{~mm}, d=0.020 \mathrm{~mm}, d=0.023 \mathrm{~mm}$. From Fig 7, we get
$\mathrm{D}\left(2 \times 10^{-5}, 0\right)=1.231 \times 10^{-5}(\mathrm{~N} \cdot m \cdot s)$,
When $d=0.020 \mathrm{~mm}, a=0$.


Figure 7. Relationship of Damping Coefficient and Swing Angle

## 5. Signal Detection

The signal detection circuit of micro-machined gyroscope is shown in Figure 3. When silicon pendulum spin with angular rate $\dot{\varphi}^{\dot{\varphi}}$, deflection angular $\alpha$ variation leads to four capacitors $\mathrm{C} 1, \mathrm{C} 2, \mathrm{C} 3, \mathrm{C} 4$ changing which are formed by silicon pendulum and electrode plane. The capacitance variety signal changes into voltage signal and the signal is amplified, getting the signal whose amplitude correspond to detected angular velocity.

Capacitance variety of micro-machined gyroscope is very small and be affected by distribution capacitance easily. So signal processing use alternating current bridge as transformation circuit of interface and capacitance sense device is used as operation arms of the bridge. The bridge supply is equivalency amplitude high frequent alternating voltage. When operation capacitors are changed, obtain the amplitude modulation wave signal modulated by operation capacitors variety at the output port of the bridge and signal is amplified and demodulated, then get low frequent output signal.

### 5.1 Signal Process Circuit

The signal process circuit is shown in Figure 8. Supply voltage stabilizer gives stable voltage to pulse generator in order to make pulse generator generate stable pulse signal. When inputted voltage of trigger reaches certain value, pulse generator begins turning from stillness stated to operation state, and as the input voltage fall to certain value, generator turns from operation state to stillness state.


Figure 8. Sketch Map of Signal Process Circuit

The output signal through a feedback resister and a charging- discharging capacitor, then period rectangle wave $U_{g e n}$ can be obtained. At last, in the fig9, $U_{g e n}$ is voltage of signal picking up capacitor.
The capacitance-to-voltage conversion circuit is shown in Fig 10(a), namely capacitance-to-voltage conversion bridged, it consists of signal picking up capacitors $C_{S_{1}}$ and $C_{S_{2}}$ with variety clearance,


Figure 9. Impulse Builder and Voltage of Sense Capacitance
charging-discharging diodes D1, D2 and resisters R1, R2. The voltage $U_{g e n}$ generated by pulse generator is put on voltage input tip of bridge and chargingdischarging diodes adjust the bridge performance. It is shown in Figure 10(b), the difference amplifier AD620 is an instrument amplifier consisted of three operation amplifiers, it's amplify multiple is determined by $R_{5}$.

(a) Capacitance-to-voltage conversion circuit

(b) Difference amplify circuit

Figure 10. Signal Process Schematic

### 5.2 Circuit Analysis

The pulse generator generates rectangle pulse voltage $U_{g e n}$, whose high voltage is +5 V and low voltage is 0 V . The voltage is put on bridge, and then current through $R_{1}, ~ R_{2}$ charging capacitors $C_{S_{1}}$ and $C_{S_{2}}$ The charging current also through reverse diodes, but this part of charging current is very small and can be neglected. When $\mathrm{t}=0 \sim T_{1}\left(U_{\text {gen }}=5 \mathrm{~V}\right)$, capacitor's picking up voltage $U_{\text {gen }}$ is decided by value of capacitor. When $\mathrm{t}=T_{1} \sim T\left(U_{g e n}=0 \mathrm{~V}\right)$, the voltage is decided by diode character and it approximates zero.
(1) As charging, namely

$$
k T \leq t \leq k T+T_{1} \quad k=0, \pm 1, \pm 2 \cdots
$$

the following expressions can be obtained.

$$
\begin{gather*}
U_{a}=U_{m}\left(1-e^{-\frac{1}{R_{1} C_{S_{\mathrm{s}}}}}\right)  \tag{17}\\
U_{b}=U_{m}\left(1-e^{-\frac{1}{R_{2} C_{s_{2}}}}\right) \tag{18}
\end{gather*}
$$

In the formulation, $U_{m}$ is amplitude of inspiriting signal $U_{g e n}$.
(1) As discharging, namely

$$
k T+T \leq t \leq(k+1) T \quad k=0, \pm 1, \pm 2 \ldots
$$

Because forward resistance of diode is very small, charge of capacitor $C_{S_{1}}$ is discharged instantly through diode, then $U_{a}=0$. During charging and discharging $(\mathrm{t}=0 \sim T)$, the DC component of output voltage for a port is

$$
\begin{equation*}
\bar{U}_{a}=\frac{1}{T} \int_{0}^{T_{1}} U_{\alpha} d t=\frac{U_{m}}{T}\left[T_{1}-R_{1} C_{s 1}\left(1-e^{\frac{-T_{1}}{R_{1} C_{S 1}}}\right)\right] \tag{19}
\end{equation*}
$$

Considering the structure of sensing device, let $C_{S_{1}}=C_{0}-\Delta C, C_{S_{2}}=C_{0}+\Delta C, C_{0}$ is inherence capacitor for electrode plane clearance with no Coriolis force, $\Delta C$ is capacitance variety with force. As $\Delta C \ll C_{0}, C_{S_{1}}=C_{0}-\Delta C \approx C_{0}$ thus

$$
\begin{equation*}
\bar{U}_{a} \approx \frac{U_{m}}{T}\left[T_{1}-R_{1}\left(C_{0}-\Delta C\right)\left(1-e^{\frac{-T_{1}}{R_{1} C_{0}}}\right)\right] \tag{20}
\end{equation*}
$$

Samely

$$
\begin{equation*}
\bar{U}_{a} \approx \frac{U_{m}}{T}\left[T_{1}-R_{2}\left(C_{0}+\Delta C\right)\left(1-e^{\frac{-T_{1}}{R_{2} C_{0}}}\right)\right] \tag{21}
\end{equation*}
$$

Setting $R_{1}=R_{2}$ thus

$$
\begin{equation*}
\bar{U}_{a b} \approx \bar{U}_{\alpha}-\bar{U}_{b}=\frac{2 U_{m} R_{1}}{T}\left(1-e^{\frac{-T_{1}}{R_{1} C_{0}}}\right) \cdot \Delta C \tag{22}
\end{equation*}
$$

The amplifier AD620 amplifies the output voltage of bridge

$$
\begin{equation*}
U_{o u t}=G \bar{U}_{a b} \tag{23}
\end{equation*}
$$

In the formulation, G is the amplifier's amplification factor, $\mathrm{G}=49.4 \mathrm{k} \Omega / R_{5}+1$.
The formulation (24) can be expressed

$$
\begin{equation*}
U_{\text {out }}=K \Delta C \tag{24}
\end{equation*}
$$

In the formulation,

$$
\begin{equation*}
K=\frac{2 U_{m} R_{1}}{T}\left(1-e^{\frac{-T_{1}}{R_{1} C_{0}}}\right) \cdot G \tag{25}
\end{equation*}
$$

As $R_{1}, R_{2}$ and $R_{5}$ are selected values, $G$ is constant value, $T$ and $T_{1}$ are constant value decided by pulse generator and connected resisters and capacitors, inspiriting signal amplitude $U_{g e n}$ is decided by REF-02 and $K$ is also constant value. The output signal has direct ratio with capacitance variety.

## 6. Gyroscope Sensor Structure and Processing Technology

## 6.1"Sandwiches" Sensor

Gyro sensor is formed by the electrode on the top and the bottom and the vibration silicon modules in the middle, they form the "sandwiches" structure. As is shown in figure 11 ,the temperature expansion coefficient of silicon is $2.6 \times 10^{-6}$ / ${ }^{\circ} \mathrm{C}$. To ensure the stability of the "sandwich" structure, the temperature coefficient of expansion of the electrode has been closed with the temperature coefficient of expansion of the silicon. We choose the No. 75 ceramic Substrate.

Vibration silicon modules is shown in fig 12, the silicon quality is in the middle, outside are the frames, two flexible beams put the quality and the frames together. So that the quality can rotate around the axis consisted of two beams, the thickness of the frames are 30 um thicker than the thickness of the quality, so the two facets of the frame are 15 um higher than the two facets of the quality. The thickness of the beam is 48 um . There is prescribe hole in the center of mass of silicon, and 7 grooves outside, they are used to reduce the damping, the whole vibration modules is one structure through the micro-machined processing used the silicon as the material Figure 13 is the chart of the plates, blotted out regional in the map is copper electrode, electrode substrate are made by ceramics, the electrode on the top and the bottom and the vibration silicon modules in the middle formed the "sandwiches" sensor.


Figure 11. "Sandwiches"Sens or


Figure 12,
Vibration Silicon Modules


Figure 13. Chart of the Plates

### 6.2 Processing Technology of the Silicon Modules

Used for the 4 -inch silicon, type N, double Polishing, (100) crystal face. In the experiment, we spent $30 \%$ of the concentration of KOH solution for corrosion, temperature is $104{ }^{\circ} \mathrm{C}$, corrosion rate is approximately 4.3. Process is shown in figure 14.
a. $2000 \AA$ A silicon dioxide layer double growth;
b. Double lithography, get rid of the oxide layer, get rid of the plastic;
c. 15 um silicon surfaces of the deep corrosion each plane;
d. .5um silicon dioxide layer double growth;
e. Second double-lithography, 24um silicon surfaces of the deep corrosion each plane;
f. Third double-lithography, 80um silicon surfaces of the deep corrosion each plane;
g. Forth double-lithography, 64um silicon surfaces of the deep corrosion each plane, Link up.


Figure 14. Processing Technology of the Silicon Modules
After seven steps above, we got the vibration silicon modules, But then the silicon beam vibration unit has not been processed out, next, the process is make the elastic beam of the silicon modules vibration. We put on a single silicon module for lithography, silicon corrosion, then, we got the vibration silicon modules, as is shown in Figure 12.

## 7. The Test of Gyroscope

The test was held on the test platform, shown in Figure 15. It was held on dynamic stand controlled by personal computer. The sensor rotation rate was set in the range $5^{\sim} 25 \mathrm{~Hz}$ by a rotation simulator. The tested sensors proved the total efficiency on all the conditions of the carried tests..
Dependence of the sensor output signal on the measured angular rate at different rotation frequencies of simulator is shown in Table 1 and Figure 16. Scale factor stability at changes of the measured angular rate is shown in Table 2.


Figure 15. Measuring Platform
Table 1. Dependence of the Sensor Output Signal on the Measured Angular Rate at Different Rotation Frequencies of Simulator

| \%/s | 50 |  | 100 |  | 150 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Hz | CW. | CCW | CW | CCW | CW | CCW |
| 12 | 1.942 | 1.96 | 3.92 | 3.905 | 6 | 6 |
| 17 | 2.14 | 2.138 | 4.315 | 4.315 | 6.6 | 6.6 |
| 22 | 2.258 | 2.26 | 4.556 | 4.56 | 6.9 | 6.9 |


| Stability \% | $\pm 7.38$ | $\pm 7$ | $\pm 7.37$ | $\pm 7.6$ | $\pm 6.82$ | $\pm 6.82$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Table 2. Scale Factor Stability at Changes of the Measured Angular Rate

|  | 50 |  | 100 |  | 150 |  | $\begin{array}{\|l} \text { Stability } \\ \% \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | CW. | CCW | CW | CCW | CW | CCW |  |
| 12 | 38.84 | 39.2 | 39.2 | 39.05 | 40 | 40 | 2.94 |
| 17 | 42.8 | 42.76 | 43.15 | 43.15 | 44 | 44 | 2.86 |
| 22 | 45.16 | 45.2 | 45.56 | 45.6 | 46 | 46 | 1.84 |



Figure 16. Dependence Gyroscope Output Signal from Input Rate (V)

## 8. Discussion and Conclusion

The influence of the rotate speed $\dot{\varphi}$ of the rotating substrate on proportional coefficient and output signals of the gyroscope is significant (up to $4.1 \%$ ), because output signals of silicon micro-machined gyroscope is proportional to the rotate speed of the aircraft under low damp situation.to avoid the disadvantage which the rotate speed error caused by electron circuit makes output signals out of stability, the microprocessor in gyroscope modifies the rotating frequency achieved by measuring different angular rate, then the influence is decreased.

The instability of outputting damping coefficient D is mainly caused by varying of aerodynamic viscosity coefficient ${ }^{\mu}$ of gas in the case of the gyroscope according to temperature. For nitrogen, $\Delta \mu=29.3 \% ~\left(100^{\circ} \mathrm{C}\right)$, The variety of damping factor D
because of varying temperature is $\pm 17 \%$. For low damp gyroscope, the main factor is the angular velocity of the rotating carrier itself ${ }^{\dot{\varphi}}$, not damping coefficient.

Using carrier's rotating angular velocity as driving force, and then forming micro-machined gyroscope without driving circuit and driving girder, this principle is correct.

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