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SNR Controlled Anisotropic Diffusion for Rician Noise Reduction in MR Images

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Abstract

To alleviate excessive smoothing of high signal-to-noise ratio (SNR) regions which are correspond to the structural features of MR images by using classical anisotropic diffusion (AD) filter. A novel AD filter called SNR controlled anisotropic diffusion is proposed. In this filter, an adaptive diffusivity function which is based on the correction factor of MR images was constructed. This adaptive diffusivity function of the proposed filter could control smoothing speed in terms of varying of SNR values in space and time scale until smoothing stop. The experimental results verify the effectiveness of novel AD filter.

Keywords: *MR images, SNR, Rician noise, noise reduction*

1. Introduction

Magnetic resonance imaging (MRI) plays an important role in clinical diagnosis. However, MR images usually are affected by noise during its imaging and transmission process. Thus, image denoising becomes an important pre-processing step to improve its quality. But a good trade-off between noise reduction and structural features preservation is dilemma for denoising. Many partial differential equation(PDE) based methods are devoted to solving the dilemma, such as anisotropic diffusion model proposed by Perona and Malik(P-M) [1], and total variation(TV) model proposed by Rudin, Osher and Fatemi [2]. However, these methods could neither preserve structural features sufficiently, nor remove the speckle noise in the background region effectively. The reason is that the gradient range of the edges of interests often overlaps with the gradient range due to noise. In addition, staircase effect [3] is produced by these methods, especially TV model. Thus, fourth-order PDE (FRTH-PDE) methods [4-5] have been attracted attention in recent years. However, they may lead to boundary leakage of images for smoothing high-frequency components of images much faster. So an efficient method is needed to overcome the problems of above methods when used for denoising of MR images. In this paper, inspired by the correction factor which can be used as scale selection of smoothing, SNR controlled anisotropic diffusion model is proposed, and this filter could preserve structural features fairly well and reduce the Rician noise of MR images effectively.

2. Method

2.1. Correction Factor

The magnitude of MR image I is the square root of the sum of the squares of the real and imaginary parts. As each of parts is corrupted by Gussian noise which has the same standard deviation σ_g .Thus, the image is corrupted by Rician noise [6]:

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$$P(I)=\frac{I}{\sigma_g^2}\exp\left(-\frac{I^2+\eta^2}{2\sigma_g^2}\right)I_0\left(\frac{I\eta}{\sigma_g^2}\right) \tag{1}$$

Where η is the ideal intensity of pixel signal, and $I_0(\cdot)$ is the zeroth order Modified Bessel function. There is an analytical relationship between the variance σ_r^2 of pixel signal and Gaussian noise variance σ_g^2 [6]:

$$\sigma_r^2=\xi(\theta)\sigma_g^2 \tag{2}$$

Where $\theta=\eta/\sigma_g$ is the SNR of pixel signal, and $\xi(\theta)$ is the correction factor which is increasing function of $SNR=\theta$ .Thus, the structural features of MR images tend to distribute in the high SNR range. Inversely, the background regions tend to distribute in the low SNR range. $\xi(\theta)$ can be calculated as :

$$\xi(\theta)=2+\theta^2-\frac{\pi}{8}\exp\left(-\frac{\theta^2}{2}\right)\left(\left(2+\theta^2\right)I_0\left(\frac{\theta^2}{4}\right)+\theta^2 I_1\left(\frac{\theta^2}{4}\right)\right)^2 \tag{3}$$

Where $I_1(\cdot)$ is the first order Modified Bessel function. To determine the SNR of pixel signal, we can solve the following fixed point formula by the iterative formula:

$$\theta_{n+1}=\sqrt{\xi(\theta_n)\left(1+\frac{m_r^2}{\sigma_r^2}\right)-2} \tag{4}$$

Where n is the number of iterations, m_r is the mean signal of pixel signal. The standard deviation σ_r of pixel signal can be estimated by the MAD estimator under the Gaussian assumption as follows [7,8]:

$$\sigma_r=1.4826median_{(x,y)}\left[\left|I_{(x,y)}-median_{(x',y')}\left(I_{(x',y')}\right)\right|\right] \tag{5}$$

Where $I_{(x,y)}$ is the intensity of pixel signal (x,y).

2.2. Proposed Model

It is clear that the correction factor $\xi(\theta)$ could not only reflects the structural features effectively, but also measures the noise level of MR images. The correction factor is normalized as follows:

$$\bar{\xi}(\theta)=\frac{\xi(\theta)-\xi(\theta)_{\min}}{\xi(\theta)_{\max}-\xi(\theta)_{\min}} \tag{6}$$

Where $\xi(\theta)_{\max}$ and $\xi(\theta)_{\min}$ denotes the maximum and the minimum correction factor value across a given image

Based on the above analysis, the new diffusivity function is proposed as follows to effectively reflect the structural features of MR images:

$$c(\theta,\|\nabla I\|)=\left(1-c_0\bar{\xi}(\theta)\right)g\left(\|\nabla I\|\right) \tag{7}$$

Where ∇ is the gradient operator, c_0 is the tunable parameter which is slightly less than one and the structural feature regions in the high SNR range can be smoothed slightly along with smoothing process by reasonably setting this parameter. The $g(\|\nabla I\|)$ can be selected as the diffusivity function of P-M model:

$$g(\|\nabla I\|)=\frac{1}{1+\left(\|\nabla I\|/K\right)^2} \tag{8}$$

Where K is the conductance parameter. Thus, the SNR controlled anisotropic diffusion model is:

$$\frac{\partial I}{\partial t} = div\left[c\left(\theta, \|\nabla I\|\right)\nabla I\right] \quad (9)$$

Where div is the divergence operator.

The discrete form of model (9) is:

$$I_{(x,y)}(t+1) = I_{(x,y)}(t) + \tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right]\nabla I_{(i,j)}(t) \quad (10)$$

Where $I_{(x,y)}(t)$ and $\theta_{(x,y)}(t)$ is the intensity and SNR of pixel (x,y) at iteration t respectively, $\eta(x,y)$ are four neighborhood pixel sets of pixel (x,y).

As $c\left(\theta, \|\nabla I\|\right) \in [0,1]$, and if we choose the time step size $\tau \in [0, 0.25]$ .Thus, we have:

$$\begin{aligned} &I_{(x,y)}(t+1) = \\ &I_{(x,y)}(t) + \tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right]\nabla I_{(i,j)}(t) = \\ &I_{(x,y)}(t) + \tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right]\left[I_{(i,j)}(t) - I_{(x,y)}(t)\right] = \\ &\left[1 - \tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right]\right] I_{(x,y)}(t) + \\ &\tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right] I_{(i,j)}(t) \le \\ &\left[1 - \tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right]\right] I_{\max} + \\ &\tau \sum_{(i,i)\in\eta(x,y)} c\left[\theta_{(x,y)}(t), \left\|\nabla I_{(i,j)}(t)\right\|\right] I_{\max} = I_{\max} \end{aligned} \quad (11)$$

Similarly, we also have:

$$I_{(x,y)}(t+1) \ge I_{\min} \quad (12)$$

Thus, we have:

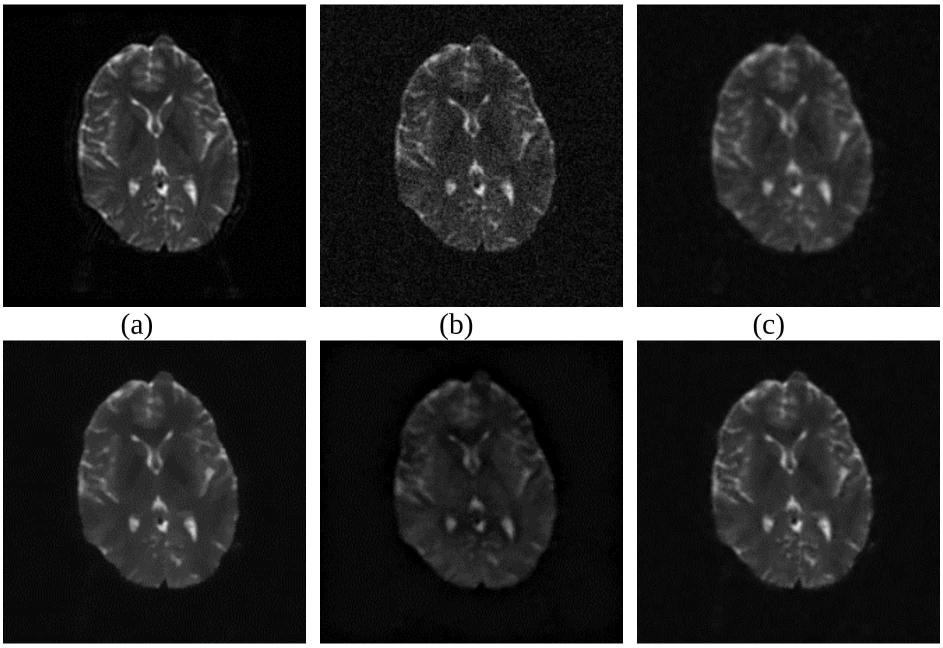
$$I_{\min} \le I_{(x,y)}(t+1) \le I_{\max} \quad (13)$$

Where $I_{\max}$ and $I_{\min}$ denotes the maximum and the minimum of intensities across a given image. Thus, the formula (13) guarantees the stability of the proposed model.

It is can be seen from the model (9) that, for high SNR regions which are correspond to the structural features of MR images, the value of $1 - c_0\bar{\xi}(\theta)$ tends to zero approximately, then such pixels are preserved, and for low SNR regions which are correspond to the background regions, the value of $1 - c_0\bar{\xi}(\theta)$ tends to one, then such pixels are smoothed out. In addition, according to SNR $\theta(x,y,t) = \eta(x,y,t)/\sigma_g(t)$,it can be seen that the model(9) is how to end the smooth process to prevent the structural features from excessive smoothing in terms of noise level distributed in the image. This is because that $\lim\limits_{t\to+\infty}\sigma_g(t)\to 0$ will lead to $\lim\limits_{t\to+\infty}\theta(x,y,t)\to+\infty$ and $\lim\limits_{\theta\to+\infty}\bar{\xi}(\theta)\to 1$, further leads to $\lim\limits_{\theta\to+\infty}\left(1 - c_0\bar{\xi}(\theta)\right)\to 0$ approximately.

3. Experimental results

In order to verify the validity of the proposed model. We set $K = 7$, $c_0 = 0.98$, $\tau = 0.25$. A layer of MR brain image added Rician noise(The noise level is $\sigma_g^2 = 50,100,150$ respectively) is selected as test image, then used to compare the proposed model with the P-M model [1], TV model[2] and FRTH-PDE model[4].The results as shown in Figure 1(Results when noise level is $\sigma_g^2 = 100$ are showed) and Table 1.



(a) (b) (c) (d) (e) (f)

Figure1. Experimental results. (a) Noise-free MR brain image. (b) Noise MR brain image. (c) Denoised by P-M model. (d) Denoised by TV model. (e) Denoised by FRTH-PDE model. (f) Denoised by proposed model

From the denoised image by P-M model, it can be seen that there are not only some speckles in the background region, but also some fine structural features in the high SNR region are smoothed out or blurred; Although noises are thoroughly smoothed out by TV model, many staircases are formed in the region of interests (ROI) and some structural features have become distortion. There are no staircases distributed in the denoised image by FRTH-PDE model, but its ability of fine structural features preservation is similar to the P-M model; It is clear that the proposed model performs better than the other three models in preserving fine structural features, and achieves a better compromise results between structural features preservation and noise reduction.

Table 1. The Maximum PSNR (dB) and MSSIM Values

Evaluation Index	Noise level	P-M	TV	FRTH-PDE	Proposed
PSNR(dB)|MSSIM	50	31.43|0.94	31.74|0.94	31.16|0.93	32.68|0.95
	100	29.49|0.87	29.65|0.88	29.19|0.87	30.52|0.89
	150	25.96|0.78	26.14|0.79	25.62|0.78	26.88|0.81

Where the larger values of PSNR and MSSIM [8] indicate a better effect in noise reduction and structural features preservation. As can be seen from Table 1, the propose model provides the best performance on both PSNR and MSSIM value among the comparison models.

In order to verify that the proposed model could stop smoothing process automatically in terms of noise level distributed in the image. Another layer of MR brain image added Rician noise (The noise level is $\sigma_g^2 = 150$) is selected as test image for iterative experiment and we record their MSSIM values of each iteration. The result is shown in Figure 2 and Figure 3 (Results of the proposed model when the number of iterations are t=10, 18, 26, 34, 42, 50 are showed).

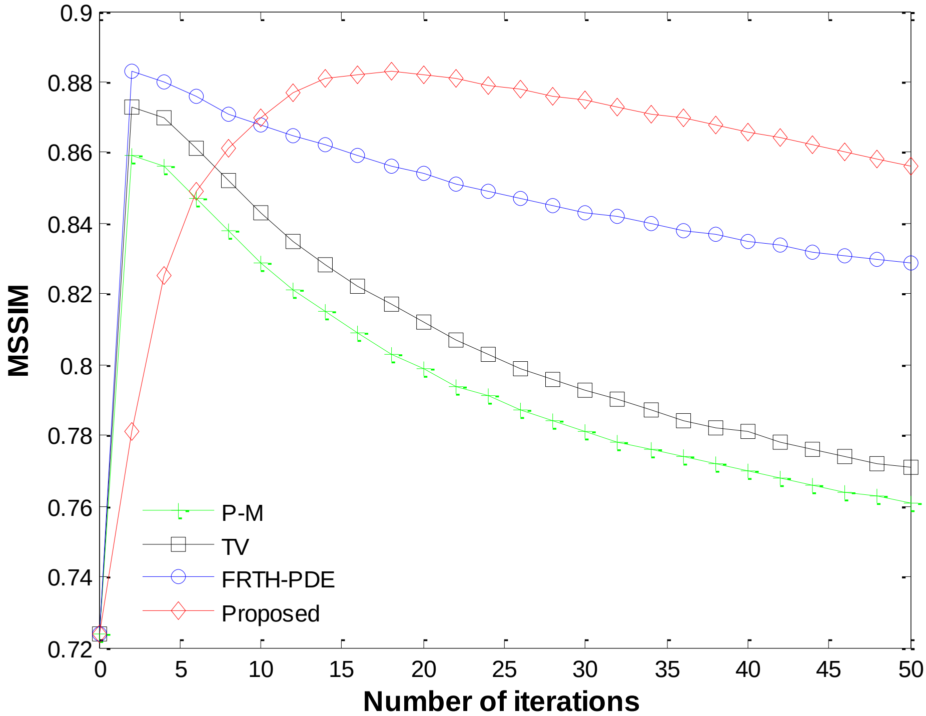
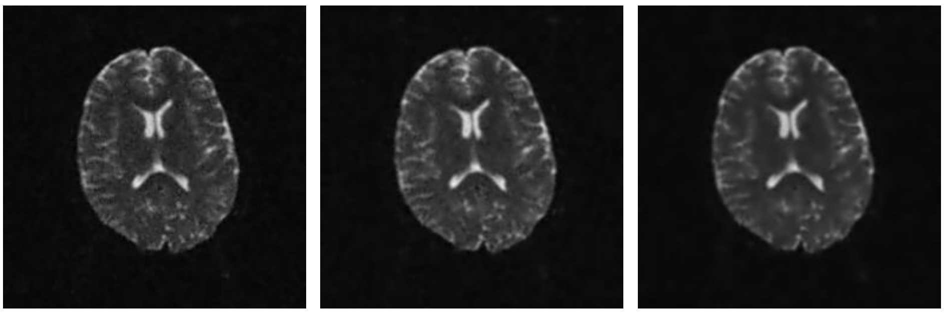
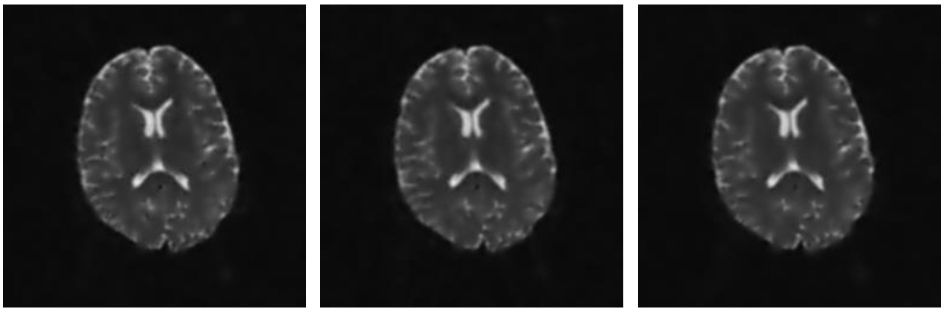


Figure 2. Experimental Results of MSSIM



(a) (b) (c)



(d) (e) (f)

Figure 3. Experimental Results; (a) t=10. (b) t=18. (c) t=26. (d) t=34. (e) t=42. (f) t=50

It can be seen from the Figure 2 that the MSSIM values tend to relative steaty state after several iterations when reached the maximum value of MSSIM. This phenomenon indicates that the proposed model will approximately stop smoothing to preserve the image structural features when the noise are smooth out thoroughly along with the noise reduction process. In addition, experimental results in Figure 3 further confirmed this phenomenon from the visual effects.

4. Conclusion

SNR controlled based anisotropic diffusion model is proposed for MR images denoising. This model uses the mapping relationship between the correction factor of MR images and image structural features, and it constructs a new adaptive diffusivity function based on the correction factor to adaptively control smoothing speed in terms of varying of SNR values in space and time scale. Experimental results show the effectiveness of proposed model by comparison with some classical models.

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