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A High Precision Time-frequency Analysis Method

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Abstract

To analysis the signal whose frequency changes as time, this article presents a new time-frequency analysis method based on short-time fractional Fourier transform (FRFT) and integration of midpoints. This method uses window functions to divide the signal to many pieces, then the FRFT is used to calculate the frequency modulation (FM) rate of each piece. A chirp signal can be built based on the FM rate and the intermediate frequency (IF) signal is obtained by mixing the chirp signal and the primary signal. The frequency of the midpoints can be estimated by the chirp-z transform (CZT), therefore the frequency of every piece can be calculated. This method avoids the impact that the quick change of frequency leads to the error of short-time Fourier transform. It also can reduce the error brought by the estimation of chirp rate. Therefore, the method can improve the accuracy of time-frequency analysis. This article also puts forward a method to search the order of fractional Fourier transform, and the computer simulation and measurement verifies the effectiveness of the proposed algorithm.

Keywords: *fractional Fourier transform; integration of midpoints; time-frequency analysis*

1. Introduction

The signal to be processed generally has time parameters and frequency parameters. The frequency of non-stationary signals changes as time and there is a time-frequency distribution of the signal. The time-frequency distribution of the radar signal can be used to check the linearity of linear frequency modulation signal [1], separate the different signals [2] and calculate the Doppler frequency. The time-frequency analysis plays an important role in many fields, such as earthquake [3], sonar [4], oscillating detection [5, 6]. It is significant and necessary to do time-frequency analysis.

In signal processing field, the most used fast Fourier transform (FFT) can calculate the frequency of the signal. However, FFT is a holistic conversion from the time domain to the frequency domain and it does not have time resolution. Because of the defect of FFT, Gabor presented the short-time Fourier transform (STFT) [7] in 1946 and it contains two steps: use window function to choose a subsection and do FFT. Because the time of window function is short enough to regard the segmented signal as stationary signal, we can obtain the time-frequency distribution through moving the window function. There is no crossing-term, but its self-term is not concentrated. When the frequency of segmented signal changes tremendously, the bandwidth of FFT is too wide and STFT cannot calculate the exact frequency of the segmented signal. The common time-frequency analysis methods have Wigner-Ville Distribution (WVD) [8] and wavelet [9]. The WVD is affected by the crossing-term and there are some ways to restrain the crossing-term [10]. Wavelet transformation needs to choose a suitable wavelet base according to the signal types, but sometimes we do not know the type of the signal.

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Fractional Fourier transform (FRFT) can overcome the shortcomings of traditional STFT. Especially for chirp signal, FRFT has a good time-frequency aggregation and does not have crossing-term, therefore FRFT has an advantage in the time-frequency analysis of linear frequency modulation signal. References [1] presents an infinitesimal method to measure the linearity of the VCO, but this method is seriously affected by noise and not very accurate. We need to use a high sampling frequency to sample the transmitting signal of FMCW radar system and the little error can lead to a big mistake. This article presents an innovative method to obtain the time-frequency distribution and achieve a high resolution time-frequency analysis.

The remainder of this article is organized as follows. The methods used to obtain the FM rates and the frequencies of midpoints are introduced in Section II. In Section III we take advantage of computer to simulate and achieve the proposed algorithm. Finally, Section IV concludes the article.

2. Time-Frequency Distribution

2.1 Definition of FRFT

Reference [11] presents a detailed instruction and here give a brief explain. The FRFT is given by

$$S_p(u) = \mathrm{F}^p[\mathrm{s}(t)] = \int_{-\infty}^{+\infty} s(t) K_p(t,u)\,\mathrm{d}t \tag{2.1}$$

p is the older of FRFT and the kernel of FRFT is $K_p(t,u)$, then

$$K_p(t,u) = \begin{cases} \sqrt{\dfrac{1-j\cot\alpha}{2\pi}} \cdot \exp(\mathrm{j}\dfrac{t^2+u^2}{2}\cot\alpha - \mathrm{j}tu\csc\alpha) .. \alpha \neq n\pi \\ \delta(t-u) .... \alpha = 2n\pi \\ \delta(t+u) .... \alpha = (2n \pm 1)\pi \end{cases} \tag{2.2}$$

δ is the impulse function and *n* is inter. α is the angle of rotation and $\alpha = p\pi/2$.

2.2 Calculation of FM Rate

Supposing the primary non-stationary signal is given by

$$s(t) = A \cdot \exp(j2\pi(f_0 + f_1 t + \frac{k_2 t^2}{2} + \frac{k_3 t^3}{3} ....)), \quad t \in [0,T] \tag{2.3}$$

During a very short time, the non-stationary signal can be regarded as stationary signal. When we use a short-time window to cut off the signal to pieces, the time length of widow is T_w and the window moves to right $(2i-1) \cdot T_w / 2$ every piece. The signal of every piece is given by

$$s_i(t) = [A \cdot \exp(j2\pi(f_0 + f_1 t + \frac{k_2 t^2}{2} + \frac{k_3 t^3}{3} ....))] \cdot w(t - (2i-1) \cdot \frac{T_w}{2}) \tag{2.4}$$

If the window function is rectangle, the primary non-stationary signal satisfies $s(t) = \sum_{i=1}^{N} s_i(t)$, $N = T / T_w$.

After weighted by window function, every piece signal can be regard as a chirp signal and the time is from zero to T_w. Because the length of the piece is very short, the power series which is bigger than three of $s_i(t)$ can be ignored and the equation (2.4) can be changed as

$$s_i(t) = [A \cdot \exp(j2\pi(fi_0 + fi \cdot t + \frac{Ki \cdot t^2}{2}))] \cdot w(t - \frac{T_w}{2}), \quad t \in [0, T_w] \tag{2.5}$$

where $2\pi \cdot fi_0$ is the initial phase of $s_i(t)$ and fi is the initial frequency of $s_i(t)$. Ki is the FM rate of $s_i(t)$.

Then let $s_i(t)$ do FRFT:

$$S_i w_\alpha(u) = \mathrm{F}^\alpha[\mathrm{s}_i(t)] = \int_{-\infty}^{+\infty} [A \cdot \exp(j2\pi(fi_0 + fi_1 t + \frac{Ki \cdot t^2}{2}))] \cdot w(t - \frac{T_w}{2}) \cdot \sqrt{\frac{1 - j\cot\alpha_i}{2\pi}} \cdot \exp((\mathrm{j}\frac{t^2 + u^2}{2}\cot\alpha_i - \mathrm{j}tu\csc\alpha_i))\,\mathrm{d}t \tag{2.6}$$

After normalizing discrete and dimensional [12], the fast algorithm [13] can calculate the $S_i w_\alpha(u)$. The peak value of $S_i w_\alpha(u)$ can be found and the corresponding angle α_i also can be obtained. The FM rate is

$$k_i = -\cot\alpha_i \cdot Fs / T_w \tag{2.7}$$

2.3 High Order Moments of FRFT

If we use the method of section 2.2 to calculate the FM rate k_i, it may lead some big errors because the FRFT is sensitive to noise. The high order moments of FRFT can restrain the effect of noise and improve the anti-noise performance [14, 15]. In this article, we use forth order moments to calculate the FM rate. The forth order moments of FRFT is given by

$$P_\alpha = \int_{-\infty}^{+\infty} S_i w_\alpha(\mathrm{u})^4 (\mathrm{u} - \mathrm{m}_\alpha)^4 du \tag{2.8}$$

The m_α is first order moments of FRFT and it can be obtained by

$$m_\alpha = \int_{-\infty}^{+\infty} S_i w_\alpha(\mathrm{u})^2 u du \tag{2.9}$$

When the SNR (signal and noise rate) is about -20 dB, we use the above two equations to calculate the forth order moments with α increasing from zero to π gradually, and the forth order moments spectrum can be obtained

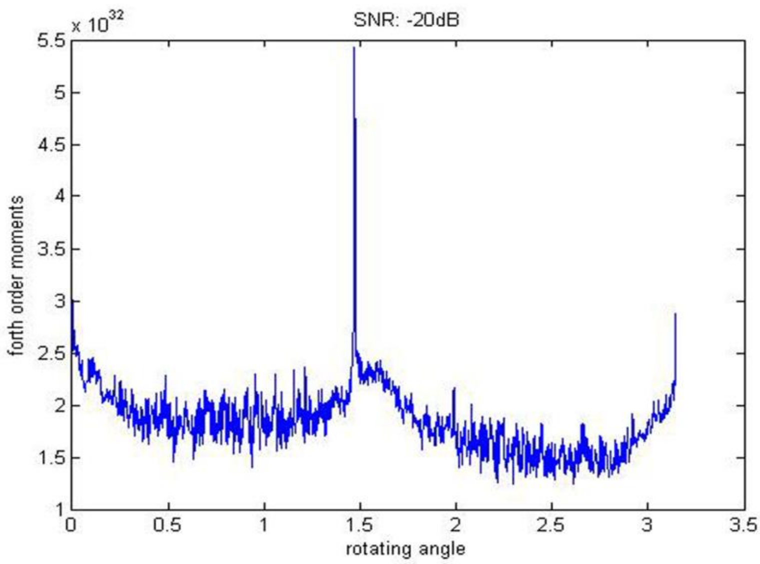


Figure 1. The Forth Order Moments Spectrum of FRFT

In the Figure 1, we can find the peak valve of forth order moments and the corresponding rotating angle also can be obtained from the abscissa. The FM rate can be calculated by the equation (2.7). If the SNR is lower, we can use higher order moments to calculate the FM rate at the cost of more calculation complexity.

2.4 Obtain Initial Frequency

In section 2.2 and 2.3, the FM rate k_i is obtained and we can use it to construct a LFM signal $s_i^{'}(\mathrm{t})$ whose initial frequency is zero.

$$s_i^{'}(t) = e^{j2\pi(\frac{k_i}{2}t^2)}, t \in [0, T_w] \tag{2.10}$$

Let $s_i^{'}(\mathrm{t})$ mix with $s_i(t)$, and the IF signal $x(t)$ can be obtained

$$x(t) = s_i^{'}(t) \cdot s_i^{*}(t) = A \cdot e^{j2\pi(-fi\cdot t-(\frac{Ki-k_i}{2})t^2)}, \quad t \in [0, T_w] \tag{2.11}$$

From the above equation, the time-frequency representation of IF signal is $f(t) = fi + (Ki - k_i)t$.

In order to calculate the frequency of IF signal, ZFFT (Zoom-FFT) [16] and CZT (Chirp Z-transform) [17] can be used. When the FM rate k_i in section 2.2 and 2.3 is an unbiased estimation, that is to say, k_i is equal to K_i, then the frequency of IF signal $x(t)$ is equal to the initial frequency of $s_i(t)$ and $f(t) = fi$. When the FM rate k_i is not an unbiased estimation, the calculated frequency F_0 of IF signal $x(t)$ can be expressed as

$$F_0 = fi + \frac{(Ki - k_i)T_w}{2} \tag{2.12}$$

The estimating time-frequency representation $\hat{F}_i(\mathrm{t})$ of $s_i(t)$ during the short time T_w is obtained and is given by

$$\hat{F}_i(t) = F_0 + k_i t = fi + \frac{(Ki - k_i)T_w}{2} + k_i t \tag{2.13}$$

2.5 Integration of Midpoints

If the power series which is bigger than three of $s_i(t)$ is ignored, the time-frequency function of $s_i(t)$ is $F_i(t) = fi + Ki \cdot t$.

When the time t is equal to $T_w/2$, the frequency valve of $\hat{F}_i(T_w/2)$ is equal to the frequency valve of $F_i(T_w/2)$. Therefore, we can get the conclusion: Whether or not the FM rate k_i is an unbiased estimation, the midpoints frequency of $\hat{F}_i(\mathrm{t})$ is equal to the midpoints frequency of $F_i(t)$ and these points are unbiased estimations $\hat{F}_i(T_w/2) = F_i(T_w/2)$. Every short window has an unbiased estimation frequency and we can exploit linear interpolation method to calculate the frequency of other points. In this article, cubic interpolation and cubic spline interpolation are used and the error of the two interpolations will be compared in the following text.

3. Simulation and Measurement

3.1 Simulation

The simulation uses MATLAB to achieve the algorithm of this article. In this simulation, the frequency change of the signal is a sine function and the center frequency is 600Hz and the max frequency variation is 500 Hz/s.

$$s(t) = \cos(2\pi(600t - 500 \cdot \frac{T}{2\pi} \cdot \cos(2\pi \frac{t}{T}))) + A_n \cdot N(t) \quad (3.1)$$

In the equation (3.1), T is the length of time, $N(t)$ is white Gaussian noise, A_n is the coefficient of white Gaussian noise and the SNR can be changed by changing A_n. The time-frequency representation of the signal is

$$F(t) = 600 + 500\sin(2\pi \frac{t}{T}) \quad (3.2)$$

The sampling rate Fs is equal to 20480 Hz/s and the length of window is $T_w = T/50$. So the signal is sectioned to 50 pieces. When the SNR is about 5dB, the frequency estimation picture and error picture of the proposed algorithm are shown in Figure 2.

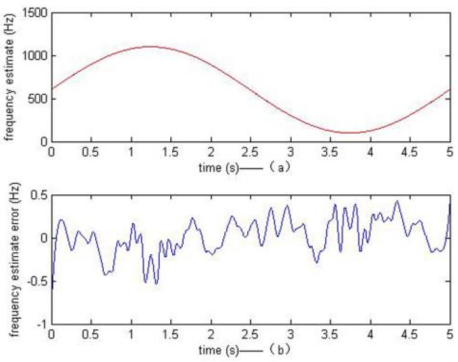


Figure 2. The Time-frequency Picture

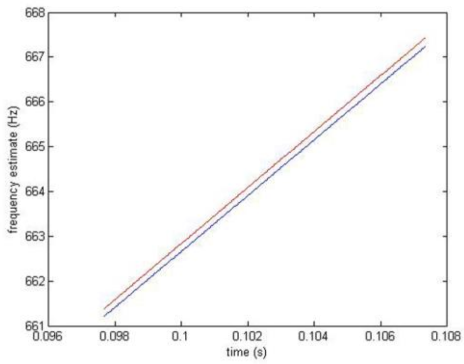


Figure 3. Local Zoom Image of Time-frequency

In the Figure 2, picture (a) is the frequency estimation picture of the proposed algorithm and picture (b) is the error picture of frequency estimation. The error is below 0.5Hz when the SNR is about 5dB. Figure 3 provides to us the local zoom image for certain position in Figure 2.

Moreover, the simulation also compares the error of the three algorithms: the proposed algorithm, WVD [8] and infinitesimal algorithm [1].

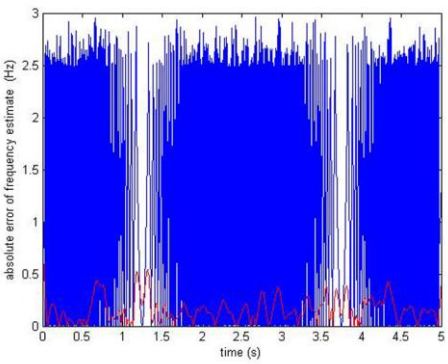


Figure 4. Errors Comparing with WVD

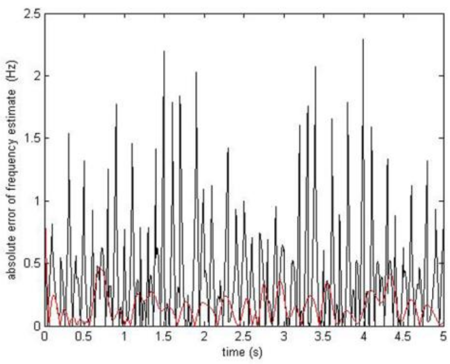


Figure 5. Errors Comparing with Infinitesimal

The Figure 4 is the absolute error picture of WVD and the proposed algorithm. The blue line represents the error of the WVD and the red line represents the error of the proposed algorithm.

The Figure 5 is the absolute error picture of infinitesimal algorithm and the proposed algorithm. The black line represents the error of the infinitesimal algorithm and the red line represents the error of the proposed algorithm. From the Figure 4 and Figure 5, the time-frequency analysis precision of the proposed algorithm is highest.

The following Table 1 shows the MSE (mean square errors) at different SNR:

Table 1. The MSE of the Three Algorithms

SNR(dB)	WVD	infinitesimal algorithm	proposed algorithm with cubic interpolation	proposed algorithm with spline interpolation
30	1.4273	0.4344	0.1142	0.0860
25	1.4276	0.4365	0.1246	0.0979
20	1.4283	0.4649	0.1239	0.1004
15	1.4286	0.4571	0.1322	0.1120
10	1.4305	0.5412	0.1668	0.1455
5	1.4351	0.5329	0.1222	0.1114
0	1.4422	0.5851	0.1825	0.1751
-5	1.4585	0.8546	0.2506	0.2756
-10	36.6634	1.0520	0.3324	0.3484
-15	1185.5680	1.3676	0.4504	0.4490
-20	3410.7154	1.8505	0.5402	0.5475
-25	3788.5040	2.4912	0.5267	0.5648
-30	3806.4867	2302.2219	1985.1809	2273.2592

From the Table 1, the proposed algorithm has a better precision than the WVD and the infinitesimal algorithm when the SNR is from -30dB to 30dB.

The two linear interpolation methods have different error. When the SNR is higher than 0 dB, the cubic spline interpolation is better. When the SNR is lower than 0 dB, the cubic interpolation is better. Therefore, different interpolation methods have different effects on the time-frequency analysis of reconstructed signal. It is necessary to choose a suitable interpolation method.

3.2 Measurement

In order to test this algorithm, a signal source system was designed. This system can produce the signal which is expected. The system chart is shown in Figure 6.

Figure 6. The System Chart of Signal Source

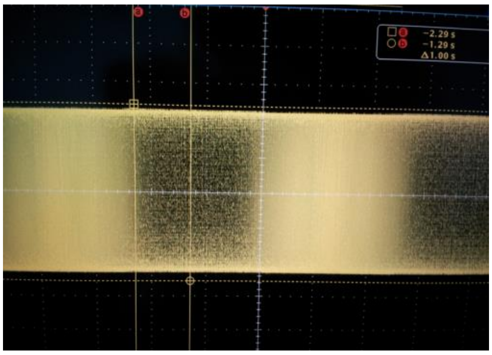


Figure 7. The Time Domain Signal

Let the signal source system produce the signal and the representation of this signal is

$$s(t) = \cos(2\pi(600t - 500 \cdot \frac{T}{2\pi} \cdot \cos(2\pi \frac{t}{T}))) \quad (3.3)$$

The period T is 5s and the sampling rate Fs is 100 KHz. The oscilloscope is used to show the time domain signal as the Figure 7.

The oscilloscope samples the signal and inputs the signal into computer. The MTLAB is used to achieve the three algorithms and the time-frequency distribution images and the local zoom images are shown in Figure 8.

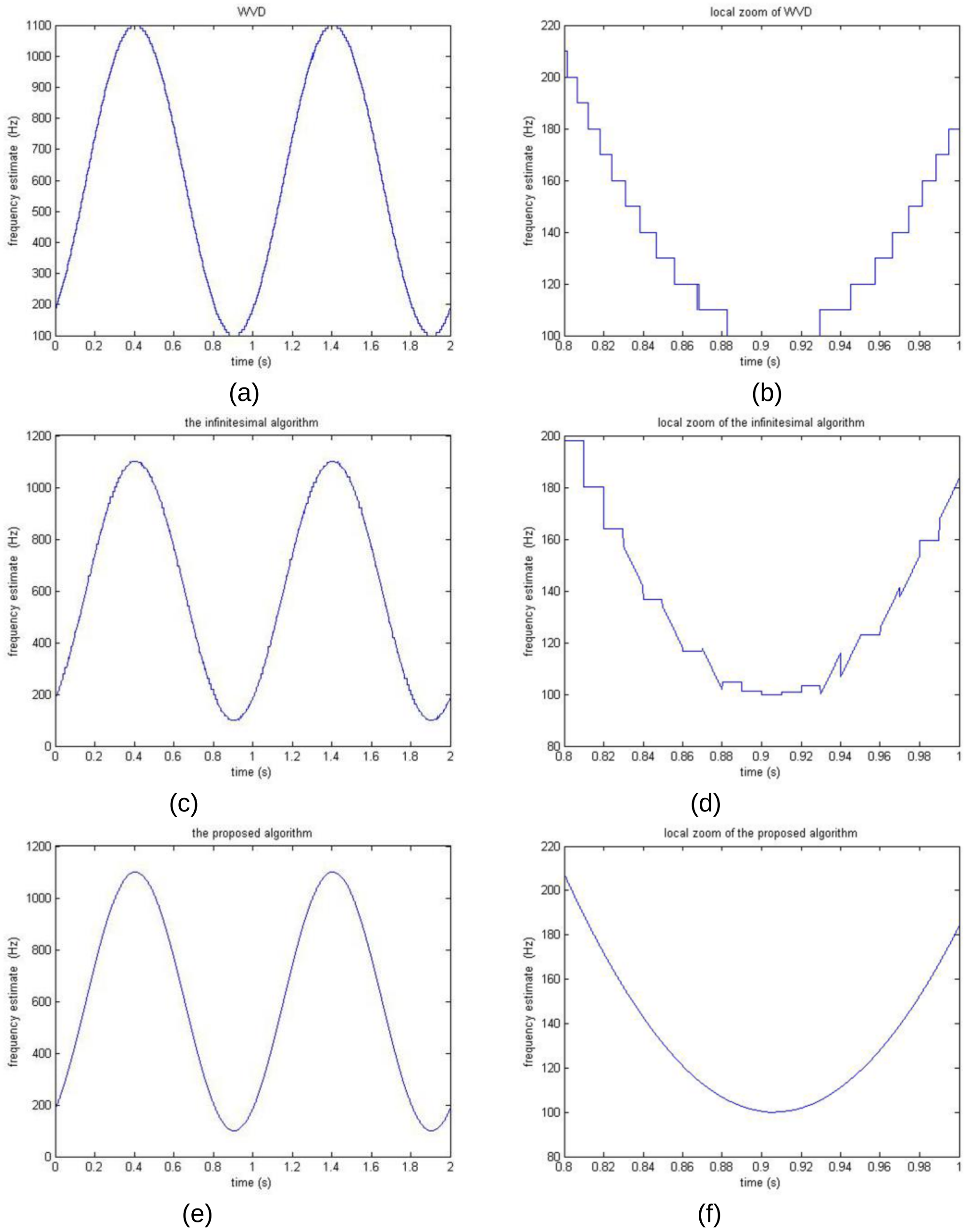


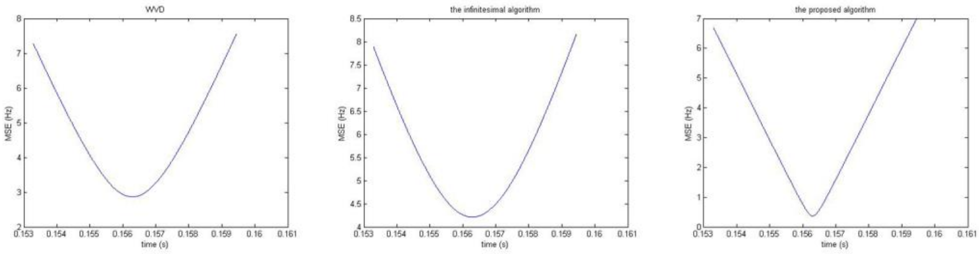
Figure 8. The Pictures of Time-frequency Distribution

In the Figure 8, picture (a), (c) and (d) are the time-frequency distribution images of WVD, infinitesimal algorithm and the proposed algorithm. The picture (b), (d) and (f) are the corresponding local zoom images. From the local zoom images, we can find that the time-frequency distribution of WVD is as stair-step and the time-frequency distribution of infinitesimal algorithm is choppy. Therefore, the time-frequency distributions of two pre-existing algorithms are not smooth, and the time-frequency distribution of the proposed algorithm in this article is smooth.

From the equation (3.3), the time-frequency representation of the produced signal is

$$f(t) = 600 + 500\sin(2\pi \cdot (1 : Fs \cdot T) / (Fs \cdot T)) \tag{3.4}$$

In order to compare the three algorithms further, we shift the equation (3.4) and let it match with the above three calculated time-frequency distribution and calculate the MSE (mean square errors) as Figure 9.



(a) (b) (c)

Figure 9. MSE Picture of the Three Algorithms

The smallest MSE in each image is the final MSE corresponding to each algorithm. The MSE of WVD is 2.8656 as picture (a) in Figure 9. The MSE of infinitesimal algorithm is 4.2126 as picture (b) in Figure 9. The MSE of proposed algorithm is 0.3656 as picture (c) in Figure 9. Therefore, the proposed algorithm is better than the WVD and infinitesimal algorithm.

4. Conclusion

In this article, a new method of time-frequency analysis is to be submitted and this method can achieve a high precision. This method sections the signal into short-time pieces and uses FRFT to calculate the FM rates of the every piece signal. An ideal chirp signal is produced to mix with the short-time pieces signal and the IF signal can be obtained. Some spectrum-zooming methods are used to calculate the frequency of the IF signal and we can obtain the unbiased estimation frequency of midpoints. Linear interpolation methods can connect these unbiased estimation points and the high precision time-frequency distribution can be obtained. The simulation and measurement compare the proposed algorithm with WVD and infinitesimal algorithm, which testify the proposed algorithm is better than the two pre-exciting algorithms.

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