

Mathematical Analysis of Different Noises for 3D Perfect Difference Codes based OCDMA System

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Abstract

The perfect difference codes minimize the Phase Induced Intensity Noise (PIIN) and cancel the multiple access interference. Earlier researchers analyze the performance of OCDMA systems considering three noises namely PIIN, shot and thermal noise for 3D Perfect Difference Codes (PDC) for OCDMA system. In this research paper we have considered two more noises i.e. $1/f$ and generation recombination noise for 3D PDC based OCDMA system. Mathematical analysis has been done and the performance of the system has been evaluated for higher data rates i.e. 1Gbps to 10Gbps for minimum BER e^{-9} .

Keywords: 3D Perfect Difference Codes (PDC), Optical Code Division Multiple Access (OCDMA), shot noise, thermal noise, PIIN, $1/f$ noise

1. Introduction

Optical code division multiple access system came into existence as it allows more number of users to access the channel simultaneously at high security. System performance is affected mainly by interference among adjacent users known as Multiple User Interface (MUI). Various researchers are working on 3D codes by considering noises. In [13] the authors analyze the performance of OCDMA systems considering three noises namely PIIN, shot and thermal noise.

In this research paper we have considered two more noises i.e. $1/f$ and generation recombination noise for 3D perfect difference codes. The $1/f$ noise occurs in almost all of the electronic devices and it increases without any limit as frequency decreases. $1/f$ noise alternately referred to as pink or flicker noise can be found in a wide range of systems. A $1/f$ power spectrum can arise from very different time traces (sharp bursts versus slower baseline drifts of the system). The origins of these noise sources are not the focus of this research. In the context of our analysis of performance evaluation, it is sufficient to quantitatively characterize $1/f$ noise based on empirical data without seeking the exact nature of the noise source. Interestingly, the typical noise characteristics specified for high sensitivity optical detectors, including the noise equivalent power reflect only the PIIN, shot and thermal noise. We note that such a characterization is incomplete, as the detector circuitry, among other potential sources, necessarily contributes dark $1/f$ noise. In the presence of dark $1/f$ noise, such devices will deviate from their predicted performance in which only dark white noise is accounted for. Our goal in this manuscript is to quantify this deviation. In particular, we will study the impact of dark $1/f$ noise on the receiver performance. Generation-Recombination noise is a type of electrical signal noise caused statistically by the fluctuation of the generation and recombination of electrons in the semiconductor-based photon detectors [14].

The performance has been evaluated for higher data rates i.e. 1Gbps, 2.5Gbps, 5Gbps and 10Gbps. The paper is organized as follows. Mathematical model of all the noises is

described in Section II. The results are given in Section III. Finally, conclusion is explained in Section IV and future scope in section V followed by references.

2. Mathematical Model

3D PD codes are constructed using 1D PD codes [13].

The photocurrent is given by:

$$\langle i_{\text{noise}}^2 \rangle = \langle i_{\text{PIIN}}^2 \rangle + \langle i_{\text{Shot}}^2 \rangle + \langle i_{\text{thermal}}^2 \rangle$$

$$= I_r B_r \tau_r + 2e I_{\text{total}} B_r + \frac{4K_b T_r B_r}{R_L} \quad (1)$$

Where, I_r and I_{total} are the average and the total photocurrents, which will be derived latter. B_r is the electrical bandwidth, τ_r is the coherence time of the light incident to the photodiode, e is electron's charge, K_b is Boltzmann's constant, T_r is the absolute noise temperature, and R_L is the load resistance.

Variance of PIIN is given by

$$\langle i_{\text{PIIN}}^2 \rangle = \frac{M B_r}{2\Delta f} \left\{ \frac{(I_0 - I_1 - I_4 + I_5)^2}{K_1} + \frac{(I_2 - I_3 - I_6 + I_7)^2}{(K_1 - 1)^2} \right\} \quad (2)$$

Variance of shot noise is given by:

$$\langle i_{\text{Shot}}^2 \rangle = e B_r I_{\text{total}}$$

$$= e B_r (I_0 + I_1 + I_2 + I_4 + I_5 + I_6 + I_7) \quad (3)$$

Variance of thermal noise is given by:

$$\langle i_{\text{thermal}}^2 \rangle = \frac{4K_b T_n B_r}{R_L} \quad (4)$$

We have considered two more noises i.e. 1/f and generation recombination noises to further evaluate the performance of OCDMA system.

Variance of 1/f noise is given by:

$$\langle i_{1/f}^2 \rangle = \frac{t^2 I_{\text{Total}}^2 \Delta f}{f} \quad (5)$$

Value of $t=2$

Variance of generation recombination noise is given by:

$$\langle i_{g-r}^2 \rangle = 4\eta q^2 R^2 h f \Delta f \quad (6)$$

where, η is efficiency, q is electron charge, R is photoconductive gain, h is Planck's constant, f is frequency.

$$I_0 = \frac{R P_{sr}}{K_3 K_2 M} \left\{ K_1 K_2 K_3 + \frac{K_2 K_1 (W-1)(P-1)}{(MNP-1)} + \frac{K_1 K_3 (W-1)(N-1)}{(MNP-1)} \right.$$

$$+ \frac{K_1 (W-1)(N-1)(P-1)}{(MNP-1)} + \frac{K_2 K_3 (W-1)(M-1)}{(MNP-1)}$$

$$+ \frac{K_2 (W-1)(M-1)(P-1)}{(MNP-1)} + \frac{K_3 (W-1)(M-1)(N-1)}{MNP-1}$$

$$\left. + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_1 = \frac{R P_{sr}}{K_3 K_2 M} \left\{ \frac{K_1 K_3 (W-1)(N-1)}{(MNP-1)} + \frac{K_1 (W-1)(N-1)(P-1)}{(MNP-1)} \right.$$

$$+ \frac{K_3 (W-1)(M-1)(N-1)}{MNP-1}$$

$$\left. + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_2 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{K_2K_3(W-1)(P-1)}{(MNP-1)} + \frac{K_2(W-1)(M-1)(P-1)}{(MNP-1)} \right. \\ \left. + \frac{K_3(W-1)(M-1)(N-1)}{MNP-1} + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_3 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{K_3(W-1)(M-1)(N-1)}{MNP-1} + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_4 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{K_2K_1(W-1)(P-1)}{(MNP-1)} + \frac{K_2(W-1)(M-1)(P-1)}{(MNP-1)} \right. \\ \left. + \frac{K_1(W-1)(P-1)(N-1)}{MNP-1} + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_5 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{K_1(W-1)(P-1)(N-1)}{MNP-1} + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_6 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{K_2(W-1)(P-1)(M-1)}{MNP-1} + \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

$$I_7 = \frac{RP_{sr}}{K_3K_2M} \left\{ \frac{(W-1)(M-1)(N-1)(P-1)}{(MNP-1)} \right\}$$

Consequently SNR is given as:

$$SNR = \frac{I_f^2}{\langle i_{PIN}^2 \rangle + \langle i_{shot}^2 \rangle + \langle i_{thermal}^2 \rangle + \langle i_{1/f}^2 \rangle + \langle i_{g-r}^2 \rangle} \quad (7)$$

Using the values of I_0 to I_7 variance of noises is calculated and furthermore, the BER can then be estimated from SNR.

$$BER = \frac{\text{erfc}\left(\sqrt{\frac{SNR}{8}}\right)}{2} \quad (8)$$

Where,

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-z^2) dz$$

The design parameters are listed as follows in table 1.

Table 1. Parameters

S.No.	Parameters	Value
1	Efficiency	$\eta=0.6$
2	Spectral width	$\Delta\lambda=40\text{nm}$
3	Wavelength Location	$1.55\mu\text{m}$
4	Data Transmission Rate	1Gbps, 2.5Gbps, 5Gbps and 10 Gbps
5	Noise Temperature	$T= 300\text{K}$
6	Receiver resistor	$R=1030\Omega$
7	Power	-70dbm to 10dbm
8	Code set (M,N,P)	(3,21,3),(7,21,3),(13,21,3) & (3,21,7),(3,31,7),(7,21,7)
9	Electron charge	1.6×10^{-19}
10	Planck's constant	6.623×10^{-34}
11	Frequency	$1.93 \times 10^{14} \text{ Hz}$

M, N, P used in above equations are three parameters that define the code sets of 3D perfect difference codes. M implies spectral code length, N is time spreading code length and P is spatial code length.

3. Results

The analysis has been done for various numbers of users for BER e^{-9} . Even the performance has been analyzed for minimum received power i.e. 10dBm to -70dBm. Code set considered are (3,21,3), (7,21,3), (13,21,3) and (3,21,7), (3,31,7), (7,21,7).

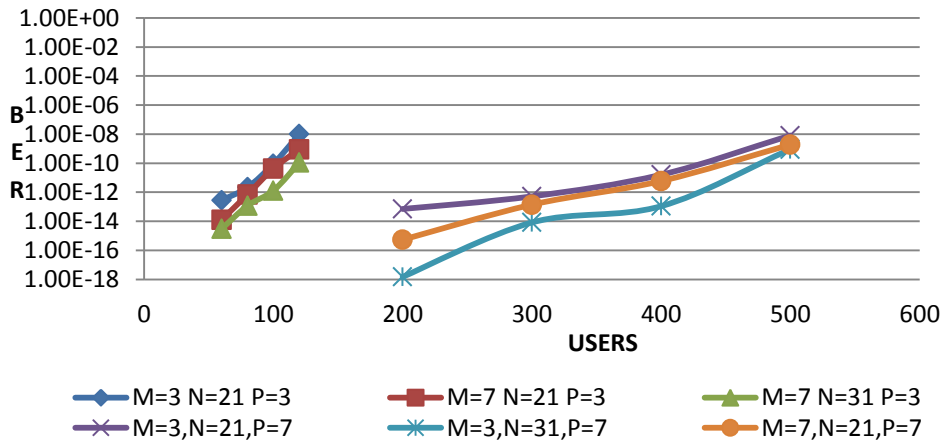


Figure 1. Number of Users vs BER at 1Gbps, power is -10dBm

Initially, results are obtained in Figures 1, 2 and 3 for the number of users versus BER. Received power is set to -10dBm. Noises considered while result analysis are PIIN, shot, thermal, 1/f and generation recombination noise. Data rate considered is 1Gbps in figure 1, 5Gbps in figure 2 and 10Gbps in figure 3. M, N and P are spectral, time spreading and spatial code length of 3D perfect difference code set respectively.

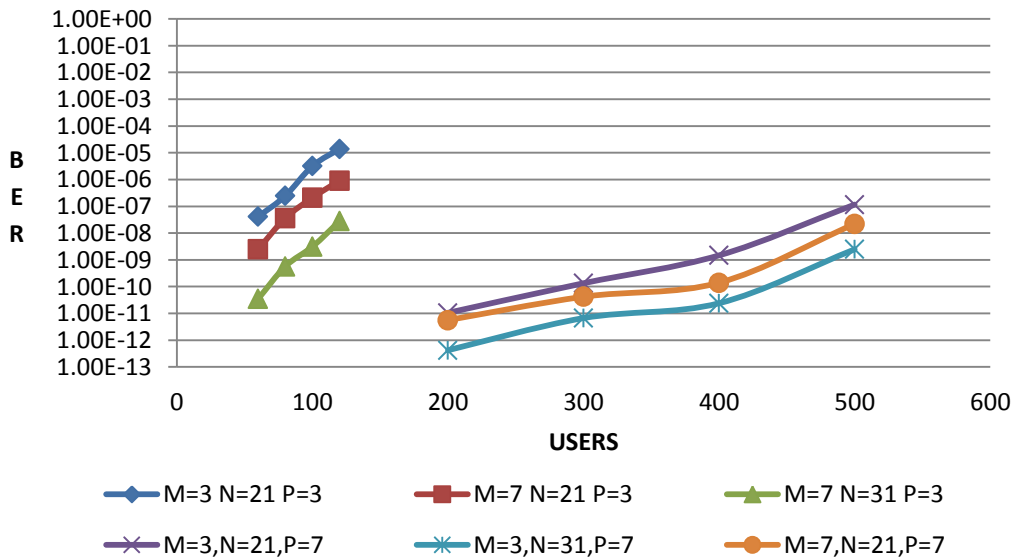


Figure 2. Number of Users vs BER at 5Gbps power is -10dbm

It is analyzed that as spatial code length increases as the number of users increases. Though increasing data rate increases the value of BER also but to meet with high pacing

technology higher data rates has to be switched over. At 1Gbps code set (3,21,7), (3,31,7), (7,21,7) can support 100 more users as compared to code set (3,21,3), (7,21,3), (13,21,3). With increasing data rate from 1Gbps, 5Gbps to 10Gbps the BER has been increased from e-18 to e-11 for (7, 21, 7) and e-15 to e-9 for (7, 31, 3).

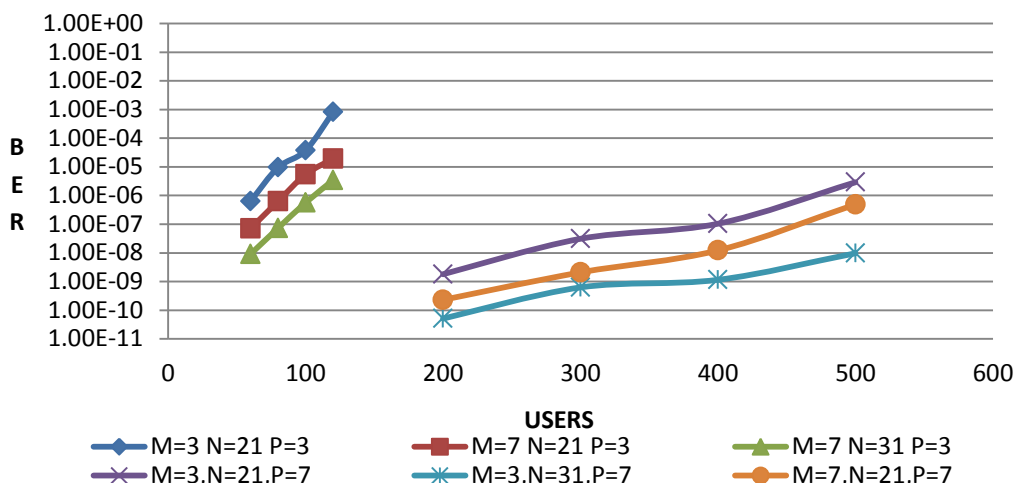


Figure 3: Number of Users vs BER at 10Gbps for power -10dBm

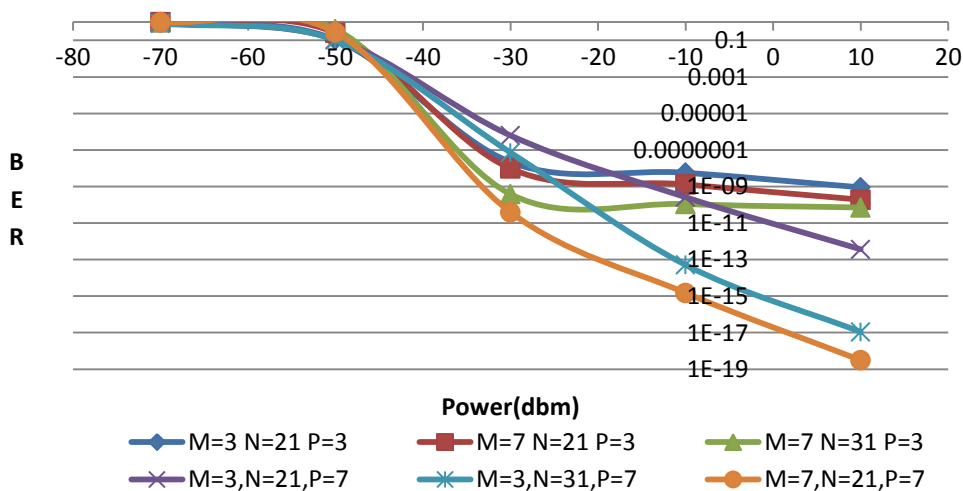


Figure 4. Power (dBm) vs BER at 1Gbps

So we have done the calculation for different number of users versus BER with received power fixed to -10dBm. Now, the number of users are fixed to 117 for (3,21,3), (7,21,3), (13,21,3) and 300 for (3,21,7), (3,31,7), (7,21,7) whereas received power is varied from 10dBm to -70dBm.

Figure 4, 5 and 6 shows the power in dBm versus BER for 1Gbps, 5Gbps and 10Gbps respectively. It is analyzed that with increased value of spatial code length i.e. BER is improved. Though increasing the value of spectral and time spreading code length also gives improved results but increased value of P yields better results.

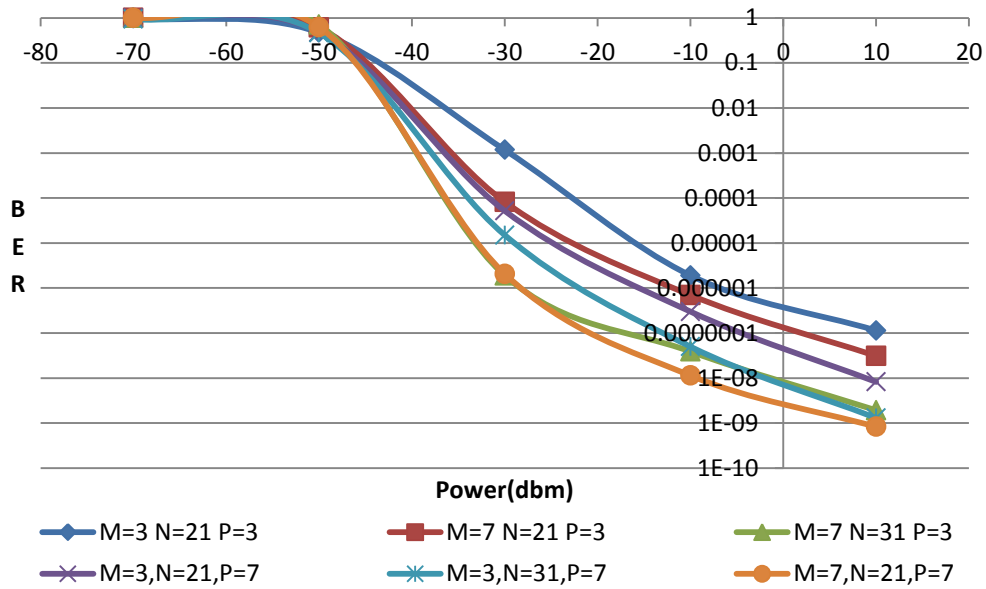


Figure 5. Power (dBm) vs BER at 5Gbps

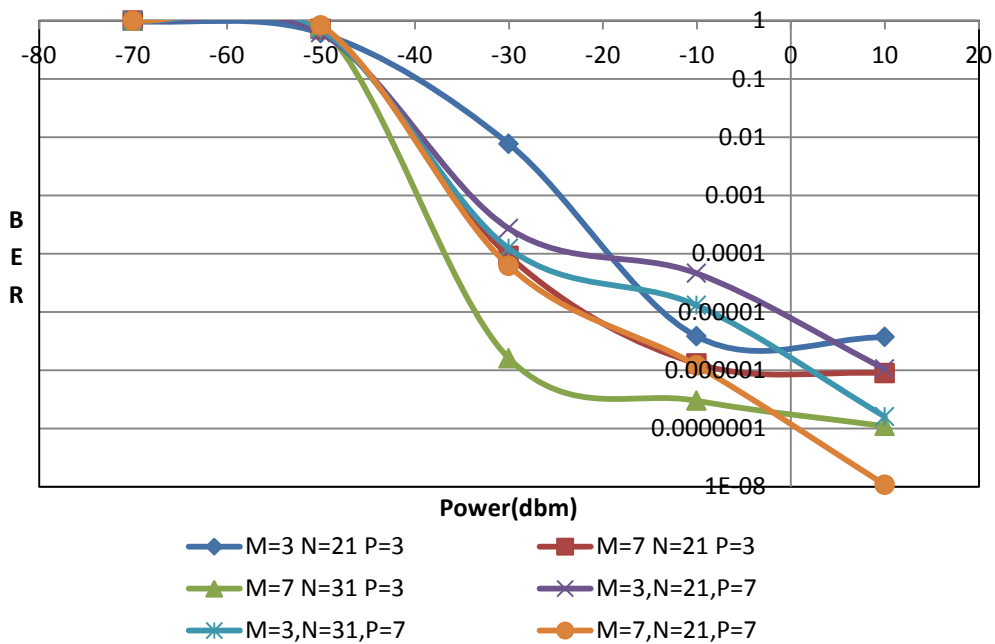


Figure 6. Power (dBm) vs BER at 10Gbps

Table 2 gives an overview of number of users supported by OCDMA system at different data rates considering three noises (PIIN, shot and thermal noise) and five noises (PIIN, shot, thermal, 1/f and generation recombination noise).

Table 2. Result Tabulation

Data Rate (Gbps)	(PIIN, shot and thermal noise) (Users Supported)		(PIIN, shot, thermal, 1/f and generation recombination noise) (Users Supported)	
Code Set	(3,21,3), (7,21,3),	(3,21,7), (3,31,7),	(3,21,3), (7,21,3),	(3,21,7), (3,31,7),

	(13,21,3)	(7,21,7)	(13,21,3)	(7,21,7)
1Gbps	110,120, more than 125	425,500, 440	100,110, more than 115	400,480,435
2.5Gbps	78,80,95	365,420,382	70,80,90	360,400,325
5Gbps	70,85,90	180, 300, 240	45, 60, 85	165, 280, 220
10Gbps	45, 60,75	175	35, 55, 75	160

4. Conclusion

Results are obtained in various forms for various data rate and power. With high speeding technology our aim is to accommodate more and more number of users at higher data rates. Though as data rate is enhanced BER is increased but to increase the number of users is also necessary so a trade off is required. All noises considered which exists in the electronic devices so they will play an important role as OCDMA utilizes number of electronic devices. Some noise exists on receiver side while some on transmitter side. There are noise that exists on both the receiver as well as transmitter side. It is analyzed from results that increasing the value of P gives better results. It means that increasing the spatial encoding implementation complexity can greatly improve the system performance. However, unfortunately, increasing the spatial code length P is the most difficult part practically. It is because that the numbers of fibers and star couplers will be increased in proportion to the spatial code length P. If the spatial code length and the value of MN are fixed, increasing the time-spreading code length N yields better performance than increasing the spectral code length M. However it needs more high speed electronics. With (3,21,7), (3,31,7), (7,21,7) system can accommodate 100 more users in the system. For better performance, system will be cost effective but it will support more users and yields better results. In future, data rate can be increased beyond 10 Gbps. In this paper only one type of codes *i.e.* 3D Perfect difference codes are considered. System can be modeled for orthogonal or some other codes. Even code set can be increased.

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