Novel Approach for Image Compression Using Curvelet Transform

Kamlesh Gupta¹ and Ranu Gupta²

¹ RustamJi Institute of Technology, Boarder Security Force, Tekanpur, MP, India ² Jaypee University of Engg. & Technology, Guna, MP, India kamlesh.gupta@juet.ac.in, ranu.gupta@juet.ac.in

Abstract

This paper proposes a novel image compression algorithm using curvelet transform. The proposed research decomposed the original image into curvelet coefficients by using fast discrete curvelet transform, after that the different scales of quantized curvelet coefficients were selected for lossy compression by the help of cutoff threshold. The proposed method was compared with image compression method based on wavelet transform. Experimental results show that the compression performance of our method gains much improvement based on PSNR, MSE and memory size. Moreover, the algorithm works fairly well for declining block effect at higher compression ratios.

Keywords: Curvelet Transform, Wavelet Transform, Peak Signal to Noise Ratio and Mean Square Errors

1. Introduction

The basic objective of image compression is the reduction of size for transmission or storage while maintaining suitable quality of reconstructed images. Since the lossy schemes can produce much higher compression ratios than the lossless ones, for this purpose many compression techniques have been introduced in [1]. Most of them are efficient for low bit rates.

In the past few years, wavelets [2, 3] and related multi-scale representations pervade all areas of signal processing. The reason for the success of wavelets is the fact that wavelet bases represent well a large class of signals, and therefore allow us to detect roughly isotropic features occurring at all spatial scales and locations. However, there has been a growing awareness to the observation that wavelets may not be the best choice for presenting natural images recently. This observation is due to the fact that wavelets are blind to the smoothness along the edges commonly found in images. In other word, the wavelet can't provide the 'sparse' representation for an image, so the second generation curvelet transform are use for image compression.

2. Curvelet Transform

The E. J Candes and D.L Donoho were introduced new transform called curvelet transform. The curvelet transform is a special member of the multi-scale geometric transforms [5-7]. It is a transform with multi-scale pyramid with many directions at each length scale. Curvelets will be superior over wavelets in following cases:

- Optimally sparse representation of objects with edges.
- Optimal image reconstruction in severely will posed problems.
- Optimal sparse representation of wave propagators.
- Reduce redundancy smartly from the image.

The curvelet transform based coding performance is better for compression over wavelet transform based coding for gray scale and color images. If the coefficient neglect large, the curvelet transform has better reconstruction over wavelet transform and remove the redundancy from the image, so it is beneficial for image encryption.

2.1 Continuous-Time Curvelet Transforms

In 2000, Candes and Donoho introduced the curvelet transform [7, 4]. The continuous curvelet transform can be defined by a pair of windows W(r) (a radial window) and V(t) (an angular window), with variables W as a frequency domain variable, and r and θ as polar coordinates in the frequency-domain.

$$\sum_{j=-\infty}^{\infty} w^2 \left(2^j r \right) = 1, \quad r \in \left(\frac{3}{4}, \quad \frac{3}{2} \right) \dots \dots \dots \dots (\mathbf{1})$$

$$\sum_{l=\infty}^{\infty} v^2 \left(t - 1 \right) = 1, \quad t \in \left(-\frac{1}{2}, \quad \frac{1}{2} \right) \dots \dots (\mathbf{2})$$

A polar 'wedge' represented by U_j is supported by W and V, the radial and angular windows. U_j is defined in the Fourier domain by

$$U_{j}(r,\theta) = 2^{-\frac{3j}{4}} W(2^{j}r) V\left(\frac{2^{\lfloor j/2 \rfloor}\theta}{2\pi}\right) \dots \dots \dots$$
(3)

The curvelet transform can be defined as a function of x = (x1, x2) at scale 2^{-j} , orientation θ_l , and position $x_k^{(j,l)}$ by



Figure 1. Curvelet Tiling in the Frequency Domain

Where R_{θ} is the rotation in radians. Figure 1 illustrates the polar 'wedges' represented by U_i . Further details are presented in [5].

2.2 Digital Curvelet Transforms

In the continuous time definition (2.1), the window U_j smoothly extracts frequencies near the dyadic corona and near the angle. Coronae and rotations are not especially adapted to Cartesian arrays. Instead, it is convenient to replace these concepts by Cartesian equivalents here, "Cartesian coronae" based on concentric squares (instead of circles) and shears, as shown in figure 2. Define the "Cartesian" window

$$\tilde{U}_{j}(\omega) = \tilde{W}_{j}(\omega)V_{j}(\omega)\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
(5)

 $\widetilde{W}_{l}(\omega)$ is a window of the form.

$$\widetilde{W}_{j}(\omega) = \sqrt{\varphi_{j+1}^{2}(\omega) - \varphi_{j}^{2}(\omega)} \quad J \ge 0 \dots \quad (6)$$

Where φ is defined as the product of low-pass one-dimensional windows.

$$\varphi_j(\omega_1, \omega_2) = \varphi(2^j \omega_1) \varphi(2^j \omega_2) \dots \dots \dots$$
(7)
The function φ obeys $0 \le \varphi \le 1$, might be equal to 1 on [-1/2, 1/2], and vanishes
outside of [-2, 2]. The digital curvelet transform coefficient is obtained by

c(j, l, k) =
$$\int f(\omega) \widetilde{U}_j \left(S_{\theta_l}^{-l} \omega \right) e^l \langle S_{\theta_l}^T b, \alpha x \rangle d\omega \dots$$
 (8)



Figure 2. Digital Curvelet Tiling of Space and Frequency

3. Flowchart for Image Compression Based On Curvelet Transforms



Figure 3. Curvelet based Image Compression

4. Algorithm for Image Compression Based On Curvelet Transforms

This research proposes a novel image compression algorithm using curvelet transform. The original image was decomposed into curvelet coefficients using fast discrete curvelet transform, after that the different scales of quantized curvelet coefficients were selected for lossy compression and arranged in descending order. Then we set the cutoff threshold for curvelet coefficients. The proposed method was compared with image compression method based on wavelet transform. Experimental results show that the compression performance of our method gains much improvement based on PSNR and MSE. Moreover, the algorithm works fairly well for declining block effect at higher compression ratios.

Step 1: Calculate the cuvelet coefficient of the image planes using following equations

$$c(j,l,k) = \int_{\mathbb{R}^2} f(x) \overline{\psi_{j,l,k}(x)} \, dx \qquad \dots \dots \dots$$
 (9)

Where R denote the real line.

Step 2: Calculate the size of compressed image according to given compression ratio (CPR).

Step 3: Arrange the Curvelet coefficients in descending order C.

Step 4: Find out the cutoff threshold for Curvelet coefficients (CL) as given below

$$N = CPR * Image Size \dots \dots \dots \dots (10)$$

Where CL is the curvelet coefficients array arrange in descending order.

Step 5: Remove all the coefficients below cutoff

C1 = C > Cutoff

Step 6: Perform inverse curvelet transform of C1 to get compressed image.

5. Simulation Result

5.1 PSNR and MSE for Image Compression

The proposed algorithm is evaluated on color images and compared with traditional image compression methods based on wavelet transform, which pervade all areas of signal processing. As an objective measure of reconstructed image quality by the PSNR (Peak Signal to Noise Ratio) and MSE (Mean Square Error).

A lower value for MSE means lesser error, and as seen from the inverse relation between the MSE and PSNR, this translates to a high value of PSNR. Logically, a higher value of PSNR is good because it means that the ratio of Signal to Noise is higher. Here, the 'signal' is the original image, and the 'noise' is the error in reconstruction. So, a compression scheme having a lower MSE (and a high PSNR), can be recognized as a better one. We evaluated the performance of image compression with an un-optimized MATLAB 7.0 code. Performance was measured on a machine with Intel Core 2 Duo 2.00 GHz CPU with 2 GB of RAM running on Windows XP.

Table 1, 2 and 3 shows the simulation results of various standard images (from Signal Image Processing Institute SIPI) of different size. In these tables we simulate the result on 120×120 , 256×256 , and 512×512 standard 24-bit color, 96 dpi JPEG images. It is clear that the PSNR and MSE value on 120×120 in table 1 and figure 4, 5 and 256×256 in table 2 and figure 6, 7 shows that the image reconstruction quality for curvelet transform is better than that of wavelet transform. The figure 4, 5, 6, 7, 8 and 9 indicate that curvelet show better PSNR and MSE for 120×120 , 256×256 and 512×512 Lena 24-bit color image compare to wavelet transform respectively. The higher PSNR and lower MSE value show the better reconstruction quality of the image. The image data are highly redundant, so in the case of curvelet the redundancy of the image is highly reduced which in turn gives the advantage in the image encryption.

Table 1. PSNR and MSE for Different Compression Ratio of Different Color
Image

			Len	a 120×1	20 (PSNF	R)					
Compression	1:20	1:30	1:40	1:5	0	1:60	1:	70	1:	80	1:90	1:100
Ratio												
Curvelet	40.01	32.41	24.0	3 14.7	'9	11.46		9.06		34	7.38	6.64
Wavelet	37.87	29.31	22.20	5 12.4	4	4 8.97		12	5.	96	5.16	4.57
			Lei	na 120×	120 ((MSE)					
Compression Ratio	1:20	1:30	1:40	1:5	0	1:60	1:'	70	1:	80	1:90	1:100
Curvelet	8.11	15.71	24.0	3 33.3	33	36.67	38.	52	39	.78	40.74	41.48
Wavelet	10.25	18.81	25.8	5 35.6	58	39.15	41.	.00	42	.16	42.96	43.55
			Babo	on 120>	(120	(PSN	R)					
	1.00	1 2 2 2	1 4 40		0	1 (0		-0	_	00	1.00	4 4 0 0
Compression	1:20	1:30	1:40	1:5	U	1:60	1:	/0	1:	80	1:90	1:100
Ratio												
Constant	25.00	20.54	22.07	2 160	0	12 (0	10	72	0	40	9.50	7.07
Curvelet	35.99	29.54	23.9.	5 16.0	0	12.60	10.	13	9.	48	8.56	/.8/
					-							
Wavelet	32.16	25.90	21.3	7 12.6	52	9.56	7.9	90	6.	84	6.08	5.50
			Bab	oon 120	×120) (MS	E)					
Compression	1:20	1:30	1:40	1:5	0	1:60	1:	70	1	:80	1:90	1:100
Ratio												
Curvelet	12.13	18.58	24.1	9 32.1	2	35.52	37.	.39	3	8.64	39.56	40.25
Wavelet	15.96	22.23	26.7	5 35.5	50	38.56	40.	22	4	1.28	42.04	42.62
Pepper 120×120 (PSNR)												
Compression	1:20	1:30	1:40	1:50	1:0	60	1:70	1:8	30	1:90	1:	100
Ratio												
Curvelet	38 85	31 59	25.23	19 14	15	42.	13 07	11 4	47	10.35	9	49
Wavelet	37.25	29.30	23.18	14.58	11.	32	9.47	8.1	2	7.12	6	.35
,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,, ,,	020	_>.00	Pen	per 120	<120	(MSI	<u>E)</u>	0.1	-			
Compression	1:20	1:30	1:40	1:50	1:6	60	<u>-</u> / 1:70	1:8	80	1:90	1.	100
Ratio	1.40	1.00	1.10	1.00	1.0		1 07 U	1.0		1.70	1.	100
	0.27	16.52	22.00	20.00	22	70 /	25.05	26	<u>(</u> 5	27.77		
Curvelet	9.27	16.53	22.89	28.99	32.	/0	35.05	36.0	65	57.77	38	3.63
wavelet	10.87	18.82	24.94	33.54	36.	80 3	58.65	40.0	00	41.01	4	.//



Figure 4. MSE of Curvelet vs. Wavelet for Different Compression Ratio



Figure 5. MSE of Curvelet vs. Wavelet for Different Compression Ratio

		L	ena 256	×256 (F	PSNR)					
Compression	1:20	1:30	1:40	1:50	1:60	1:70	1:80	1:90	1:10	
Ratio									0	
Curvelet	45.18	38.6	31.9	24.9	21.8	19.8	18.4	17.2	16.38	
		8	3	7	2	3	2	9		
Wavelet	38.63	28.2	21.6	18.0	15.8	14.3	13.8	12.6	12.11	
		5	1	8	2	8	3	6		
Lena 256×256 (MSE)										
Compression	1:20	1:30	1:40	1:50	1:60	1:70	1:80	1:90	1:10	
Ratio									0	
Curvelet	2.94	9.44	16.1	23.1	26.3	28.2	29.7	30.8	31.74	
			9	5	0	9	1	3		
Wavelet	9.49	19.8	26.5	30.0	32.3	33.7	34.7	35.4	36.02	
		8	1	4	0	4	4	7		
Baboon 256×256 (PSNR)										
Compression	1:20	1:30	1:40	1:50	1:60	1:70	1:80	1:90	1:10	

Table 2. PSNR and MSE for Different Compression Ratio of Different Color Image

Ratio															0
Curvelet	42	7 4	34.9	20) 1	22.7	10) 1	17	0	15	6	14	6	13.94
Curvelet	+2.	~ .	8		5	8		7.1 6	17	.∪ ≥	15).U)	14	r.u)	13.74
Wavalat	34	69 ′	26.9 1		27	15.4	13	36	12	$\frac{1}{2}$		117		<u>,</u> 0	10.59
vv avelet	54.		3		4	7	1.	9.0	12	3	1	1	11	3	10.57
			Ba	iboo	n 256	x 256	(MS	SE)		-	-	-		-	L
Compression	1:2	20 1	1:30	1:	40	1:50	1:	<u>60</u>	1:'	70	1:	80	1:	90	1:10
Ratio										-					0
Curvelet	5.8	38	13.1	18	3.9	25.3	28	3.9	31	.0	32	2.4	- 33	5.4	34.19
			4		7	4	,	7	5	5	3	3	3	3	
Wavelet	13.	43 2	21.2	29	9.3	32.6	34	1.4	35	.6	36	5.4	37	0.'	37.53
			0	(9	5		3	()	2	2	4	1	
		1	Pe	ppei	r 256×	256	(PSN	NR)						1	
Compressio	1:20	1:30	1:	40	1:50) 1	:60	1:	70	1:8	80	1:	90	1	1:100
n Ratio															
Curvelet	48.0	39.6	32	2.5	25,1	1 2	0.9	18	8.4	16	.8	15	5.6		1.71
	3	9		3	1		9	7	7	0)	()		
Wavelet	43.0	32.3	3 19.8 16.3 14.4 1		13	0.0	11	.9	11	1]	0.57			
	8	2		/	6		3	4	ł	9)	ļ)		
Pepper 256×256 (MSE)							100								
Compressio	1:20	1:30	1:	40	1:50) I	:60	1:	/0	1:0	SU	1:	90	J	1:100
n Katio	0.12	0 12	14	5 5	22 (2	71	20	6	21	2	20) 5	_	22.40
Curvelet	0.12	0.45).J D	25.0		7.1 3	29	.0	21	.3	52	2.5	-	55.42
Wavelet	5.04	15.8	25	ז זי	31.7	7 3	36	35	, ; ()	36	. 1	36	<u> </u>	-	37 55
wavelet	5.04	15.0	20	5. <i>2</i> 5	6		9.0 9	55	2	30	.1	50	3)1.55
		Ŭ	Air	_ nlan	e 256	×256	PS	NR)	,	5	,		,		
Compressio	1:20	1:30	1:	40	1:50) 1	:60	1:	70	1:8	80	1:	90	1	:100
n Ratio						-			-				-		
Curvelet	49.0	40.1	32	2.2	24.6	5 2	0.8	18	3.0	16	.0	14	1.7]	13.97
	6	3	4	4	7		7	1	L	7	7	8	3		
Wavelet	43.9	30.8	20).2	16.1	l 1	4.1	12	.9	11	.7	11	.3]	10.72
	3	7		9	3		4	6	5	4	ŀ	1	1		
		1	Ai	rpla	ne 25	6×256	6 (M	SE)						1	
Compressio	1:20	1:30	1:	40	1:50) 1	:60	1:	70	1:8	80	1:	90	1	:100
n Ratio								_				_			
Curvelet	-0.92	8.00	15	5.8	23.4	1 2	7.2	30).1	32	.0	33	3.3		34.15
XX 7 1 4	04.1	17.0		8	6		5			5)		1		77.40
Wavelet	04.1	17.2	27	/.8	31.9	1 3	3.9 0	35	.1	36	.3	36	0.8 1	-	57.40
	9	5		3	9		9	6)	8	5		l		



Figure 6. PSNR of Curvelet vs. Wavelet for Different Compression Ratio



Figure 7. MSE of Curvelet vs. Wavelet for Different Compression Ratio

When the size of the image increases the results are contradictory for some images like baboon 512×512 and Peeper 512×512 as shown in table 3 and figures 10, 11, 12 and 13 i.e. the wavelet transform gives higher PSNR and lower MSE for baboon 512×512 image for the compression ratio 1:20, 1:30 and 1:40 as shown in figures 5.10 and 5.11 i.e. for low compression ratio PSNR are 41.80, 38.43 and 27.64 respectively but in the case of curvelet transform it is 40.56, 32.90, 26.39 and the MSE are 6.32, 13.29, and 20.48 but in curvelet it is 7.56, 15.22 and 21.73. As the compression ratio increases such as 1:50 or above the curvelet give higher PSNR (18.13, 14.29, 12.16, 10.75, 9.75, 8.99) and lower MSE over wavelet PSNR (16.66, 11.00, 9.07, 7.84, 6.97, 6.31) and MSE as shown in Figures 10 and 11.

Table 3. PSNR and MSE for Different Compression Ratio of Different Color Image

Lena 512×512 (PSNR)									
Compressio	1:20	1:30	1:40	1:50	1:60	1:70	1:80	1:90	1:10
n Ratio									0
Curvelet	47.2	39.40	30.5	15.7	12.1	10.2	8.97	8.02	7.26
	4		9	4	8	5			

Wavelet	45.2	36.8	1	27	.7	13	5.1 7	9.:	35	7.	35	6.	13	5.2	8	4.66
	5			Len	a 51	12×4	512	MS)	SE)							
C	1.20	1.20		1.	40	1.	50	1.	(A)	1./	70	1.0	00	1.0	<u> </u>	1.10
n Ratio	1:20	1:50	,	1:4	40	1:	50	1:0	DU	1:	/0	1:0	DU	1:9	U	0
Curvelet	0.88	8.72	2	17	.5	32	.3	35	.9	37	.8	39	.1	40.1	1	40.8
				3	3	8	3	4	5	7	7	5	5			6
Wavelet	2.89	113	2	20	1	34	9	38	7	40	7	41	9	42.8	84	43.4
··· u · cice	2.07	11.5	_	20		4	5	50	2		7	0)	12.0	,	6
	l		B	apu	on 5	12×	, 512		, NR	<u>'</u>		,				0
Compressio	1.20	1.30)	1.	40	1.	50	1.	60	1.	70	1.	80	1.0	0	1.10
n Ratio	1.20	1.50	,	1.	TU	1.	50	1.	00	1.	/0	1.	50	1.7	U	0
Curvelet	40.5	32.9	0	26	3	18	1	14	. 2	12	1	10	7	97	5	8 99
ourverer	6	020	v	-0)	10	3	Ċ)	6	ί. Γ		5			0.77
Wavalat	41.8	34.8	3	27	6	14	6	11	0	91)7	7 9	, 84	69	7	631
wavelet	-1.0	54.0	5	4	 L	17 6	ί. ί	()).	0/ /.		54 0.3		/	0.51
Baboon 512×512 (MSE)																
Compressio	1:20	1:30)	1:4	40	1.	50	1:	60	1.	70	1	:80	1:	90	1:10
n Ratio	1.20	1.0.	,		••				00			-			20	0
Curvelet	7.56	15.2	2	21	.7	29	.9	33	.8	35	.9	3'	7.37	38	3.3	39.1
				3	3	9)	3	3	7	7			,	7	3
Wavelet	6.32	13.2	9	20	.4	33	.4	37	.1	39	0.0	40	0.28	4	1.1	41.8
			-	- 8	3	6	5	2	2	4	5				5	1
			P	eppe	er 5	12×:	512	(PS	SNR	()						
Compressio	1:20	1:30	1:	40	1:	50	1:	60	1:	70	1:	80	1:	:90	1	:100
n Ratio																
Curvelet	47.1	38.3	31	.1	21	.8	17	0.'	14	.1	12	2.3	11	.04]	10.10
	9	4	9)	Z	1	1	l]	l]	l				
Wavelet	47.6	39.1	30).2	15	5.5	11	.7	9.	80	8.	33	7.	.24		6.43
	2	6	()	Z	1	Ģ)								
Pepper 512×512 (MSE)																
Compressio	1:20	1:30	1:	40	1:	50	1:	60	1:	70	1:	80	1:	:90	1	l:100
n Ratio																
Curvelet	0.93	9.78	16	5.9	26	5.2	31	.1	34	0.	35	5.8	37	.08		38.02
				3	8	3	4	2	1	l	1	l				
Wavelet	0.50	8.96	17	.9	32	2.5	36	5.3	38	3.3	39	0.7	40	.88	4	41.69
				2	8	3		3	2	2	ç)				
		•														







Figure 9. MSE of Curvelet vs. Wavelet for Different Compression Ratio



Figure 10. PSNR of Curvelet vs. Wavelet for Different Compression Ratio



Figure 11. MSE of Curvelet vs. Wavelet for Different Compression Ratio



Figure 12. PSNR of Curvelet vs. Wavelet for Different Compression Ratio





Table 5.	Image Co	mpressed	in Byte	for Different	Compression	Ratio

	1:20	1:30	1:40	1:50	1:6	50	1:7	0	1:8	0	1:90)	1:10	0
	Lena 120×120													
	Plain image size 43200 bytes													
				-							1			
Curvelet	21600	14400	10800	8640	7	7200	61	171	540	0	4800	0	4320)
Transform														
Wavelet	24221	17009	13415	11261	98	812	87	789	802	1	741	5	6932	2
Transform														
Lena 256×256														
	Plain image size 196608 bytes													
Curvelet	98304	65536	49152	39321	327	768	280)87	245	76	2184	45	1966	50
Transform														
Wavelet	102589	69997	53632	43822	372	255	325	575	29080		9080 2631		24124	
Transform														
				Lena	512×	<512								
Plain image size 786432 byte														
Curvelet	393216	262144	196608	15728	6	13107	72	1123	347	9830)4	873	81	78643
Transform														
Wavelet	465458	336836	271655	23230	1	20600)3	1872	289	1733	342	162	233	153356
Transform														

The compression and decompression time is measured for different size of color image by using curvelet transform is depicted in table 5.

The size of the compressed image are more in wavelet case as compared to curvelet as shown in table 5 and figures 14, 15 and 16, so its take less time for encryption and less bandwidth for transmission.

 Table 4. Time for Compression / Decompression Time Using Curvelet

 Transform

Lena (24-bit color image)	Comp Time in sec.	Decomp Time in sec.
102×102	0.436	0.416
120×120	0.452	0.367
180×180	0.889	0.689
204×204	1.156	1.018
256×256	1.332	1.930
512×512	5.716	4.127



Figure 15. Performance of Compress Image for Lena 256×256 Curvelet vs Wavelet



Figure 16. Performance of Compress Image for Lena 512×512 Curvelet vs Wavelet



Figure 14. Performance of Compress Image for Lena 120×120 Curvelet vs Wavelet

The Figure 17(a) show the original 256×256 , 24-bit JPEG Lena image. The figure 17 (b-j) and figure 18(b-j) shows the reconstruction quality of the compressed image by using curvelet transform and wavelet transform respectively with different compression ratio. In this case curvelet transform base coding give better reconstruction quality or low error for higher compression ratio compare to wavelet transform base coding.







(c) 1:30







(e) 1:50



Figure 18. (b-j) Reconstructed Lena Image for Different Compression Ratio in Wavelet Transforms

6. Summary

In this paper, with the help of simulation results it is clear that curvelet transform gives the better performance for PSNR and MSE over the wavelet transform. More over the curvelet transforms are more suitable for the image data to represent the singularities over geometric structures in the image, than the wavelet counterpart because cuevelet are designed to handle the singularities on the curves, where as wavelets are effective for point singularities; hence curvelet transforms can be effectively used for the image compression with quality reconstruction. Thus curvelet based image compression, eliminate the redundancy from the image and achieve higher compression that reduces the encryption time.

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Authors



Kamlesh Gupta, he is an Associate professor in Department of Information Technology, RustamJi Institute of Technology, Boarder Security Force, Gwalior, M.P., India. He has published 15 papers in International and National journals and conferences. His research interest includes cryptography and image processing. He is a life member of IETE.



Ranu Gupta, he Assistant professor in Department of Electronics and Engg. Jaypee University of Engg. & Technology, Guna, MP, India. He has published 5 papers in International and National journals and conferences. His research interest includes cryptography and image processing.