An Image Quality Assessment Algorithm on the Basis of Edge Information and Singular Value decomposition

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Abstract

Due to the existing image quality assessment algorithm does not take the visual information and the essential features of the image into account and cannot meet the actual need, a new method of objective quality assessment which related to both cases was proposed in this paper. The singular value information of the image shows the essential information of image and human eyes are sensitive to the edge information of image. Theoretically, the algorithm of image quality assessment based on edge information and Singular Value Decomposition is better than traditional methods. The simulation experiment results show the proposed algorithm is more consistent with human subject scores and has greater stability than traditional methods. Through comparison with the time efficiency, the proposed algorithm can basically be able to meet the practical demand, and the algorithm is more usability.

Keywords: The edge information of image; Singular value decomposition; Image quality assessment

1. Introduction

The purpose of image quality assessment is to measure the quality of distorted image. It is indispensable to estimate the image quality in such as image compression, transport, display fields [1]. Image quality assessment mainly consists of subjective quality assessment and objective quality assessment. The subjective quality assessment mainly depends on that the observers watch the image directly and give the image score according to a certain image evaluation criterion. Theoretically, the subjective image quality assessment is very accurate and reliable, but it has many drawbacks including the professional background of observers, mental factors and so on. In addition, the subjective quality assessment is time consuming and cannot be applied to the real-time processing. The objective image quality assessment is a method which measures the image quality from the perspective of mathematics [2-5]. This method is the most widely used and could be applied in the Real-time processing.

The current image quality assessment method mainly measures the image quality in two aspects, the one is the method from image itself, and another is method from image features. The first one, that measures the image quality from the pixel level, includes Mean Square Error (MSE) and Peak Signal-to-Noise Ratio method (PSNR), etc. It has the advantage of lower computing complexity and physical meaning clearly. But this method measures the image quality only on the aspect of pixel and there are some deviations with the actual quality of image. The second one is the method that measures the image quality with the features of image [6-7]. Due to the human eye being the ultimate recipient of image quality, the best image quality assessment method is the subjective quality assessment method. The objective quality assessment is a method which uses a
mathematical model to approximate the subjective quality assessment. An image quality assessment from the aspect of image features is proposed in this paper.

2. The Edge Information and Singular Value Information

Due to the human eye is sensitive to the edge and contour information of the image, the outline and the edge information can well indicate the structure information of image. Therefore, the outline of image and edge information can make people easily capture the image scene [8-13]. The Singular Value Decomposition (SVD) is a concept in matrix theory. The Singular Value Decomposition (SVD) can be applied to the field of image quality assessment, which is because the singular value vector of the image presents essential features of the image when the image is decomposed by the Singular Value Decomposition (SVD) [14]. Therefore, the Singular Value Decomposition can be applied to image quality assessment.

3. The Singular Value Decomposition of the Edge Information of the Image

The image quality assessment on the basis of edge information and Singular Value Decomposition consists of two parts. The first part is the edge information of the image which is extracted by Sobel operator. The second part is the singular value information which is extracted by making the edge information of image Singular Value Decomposition.

3.1. Sobel Operator

The Sobel operator is composed of two groups 3×3 matrix which is the horizontal and vertical matrix respectively, then which are made the convolutions of the image A and will obtain the horizontal and vertical of edge information of image. The horizontal and vertical edge information can be computed as:

\[
S_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix} \ast A \quad S_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} \ast A
\]

(1)

The symbol ‘\ast’ indicates convolution, and the gradient of every pixel is defined as:

\[g(x, y) = \sqrt{S_x^2 + S_y^2}\]

(2)

Choosing a suitable threshold value \(t\), if \(g(x, y) > t\), the pixel point is regarded as edge point [5].

3.2. The Singular Value Decomposition (SVD)

The Singular Value Decomposition theory of the image can be described as: the real matrix \(A\) with \(M\) rows and \(N\) columns can be decomposed as: \(A = USV^T\).

Where the matrix \(U\) and \(V\) is orthogonal matrix, \(S\) matrix is diagonal matrix. The orthogonal matrix is defined as: the matrix \(I\) and its transposed matrix \(I'\), if \(I*I'=E\), the matrix \(I\) is orthogonal matrix. The \(E\) is unit matrix.

3.3. Singular Value Decomposition of Edge information of the image

When the matrix of the image is decomposed by Singular Value Decomposition, the diagonal elements (singular value vector) indicate the essential features of the image. The image quality assessment based on singular value decomposition is the method that decomposed the original images and the distorted images respectively, and calculating the
intersection angle between the original images and distorted images, the smaller the angle, the smaller the essential features of the difference between two images, that is to say, the original image and the distorted image is similar.

Firstly, Sobel operator was adopted to extract the edge information of the distorted image and the original image, secondly, the edge information of the distorted image and the original image is decomposed by the Singular Value Decomposition. Finally, the intersection angle of the distorted image and the original image is calculated to measure the distorted image quality. If the smaller is the intersection angle, the more similar between the distorted and original images. The intersection angle of the singular value of two images is defined as:

$$EQSVD = \arccos \left( \frac{\sum_{i=1}^{n} (s_i \times s'_i)}{\sqrt{\sum_{i=1}^{n} (s_i \times s_i) \sum_{i=1}^{n} (s'_i \times s'_i)}} \right)$$  (3)

The $s_i$ is the singular value of the edge information of the original image and the $s'_i$ is the singular value of the edge information of distorted image.

The Singular Value Decomposition of edge information is shown in figure 1.

**Figure 1. Singular Value Decomposition of Edge Information of Image**

### 4. The Experiment Method

#### 4.1. The Selection of Image Database

This paper conduct experiments on the JPEG2000 of the LIVE database released by the university of Texas and video engineering lab [15]. This database involves 29 original images, and there are 227 distorted images through different compression ratio. This LIVE database is shown in the figure 2 as follows. In addition, the database includes the MOS value of each images, the MOS value is given objectively through observing the images. If the larger is the MOS value, the better is the quality of image.

**Figure 2. LIVE Image Database**

#### 4.2. The Selection of Image Database

Due to nonlinear relationship exists between the objective assessment and subjective assessment value, it is necessary to map the objective quality score to subjective
assessment value through the nonlinear function. Nonlinear regression Gaussian function is adopted in this paper as shown in the formula 4.

\[ f(x) = a_1 \cdot e^{-(x-b_1)^2/c_1} + a_2 \cdot e^{-(x-b_2)^2/c_2} \]  

(4)

The \( a_1, a_2, b_1, b_2, c_1, c_2 \) is the coefficient under test. After nonlinear regression, it is necessary to measure the performance among the various methods. The root mean square error (RMSE) method is used in this paper to measure the performance of the quality assessment algorithm. RMSE is used to forecast the error between the data and original data corresponding points. The function is defined as:

\[ MSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} w_i \left( y_i - \hat{y}_i \right)^2} \]  

(5)

Where the \( y_i \) is the predicted value which makes the objective assessment value nonlinear regression, and \( \hat{y}_i \) is the MOS value.

The RMSE expresses the error between the predicted value and the MOS value, if the smaller is the RMSE value, the smaller is the error between the predicted value and the MOS value.

4.3. Experimental Procedures

In order to test the performance of our quality assessment algorithm, the experiments are made according to the following steps.

a) Firstly, MATLAB software is used to read the original image and distorted image through different compression ratio at the same time.

b) Reading without distortion image respectively and the corresponding after JPEG2000 compression upon Sobel operator to extract the edge of image.

c) Calculating the value of MSE, PSNR, SSIM and EQSVD.

d) Using the MATLAB software to draw the scatter diagram between these values and corresponding MOS value.

e) Using the nonlinear regression Gaussian function to make the objective quality assessment of MSE, PSNR and EQSVD back to the corresponding value in subjective quality assessment.

f) Finally, using the fitting toolbox of MATLAB to fit these values and the MOS, and analysis the advantages and disadvantages of these algorithms.

4.4. Experimental Results and Analysis

According to the experimental steps, we will get the values of MSE, PSNR, SSIM and EQSVD for 227 images. Start the fitting toolbox in MATLAB, import the experimental data, use the Gaussian function to Nonlinear Curve Fit, calculate the RMSE and compare the advantages and disadvantages of each algorithm. The experimental results are shown as:

(a) MSE RMSE= 8.874  
(b) PSNR RMSE=9.17
As shown in the above figure, the value of RMSE through the singular value decomposition is smaller than the value of RMSE for MSE, PSNR and SSIM, this indicates that the difference between the objective quality assessment value under nonlinear regression and MOS value is the smallest, therefore, the algorithm of singular value decomposition is better than MSE, PSNR and SSIM.

5. Conclusion

A kind of image quality assessment method based on edge information singular value decomposition is put forward in this paper. In the procedure of image quality assessment, the edge information of image is of importance. Therefore, the image on the basis of the edge information is decomposed through the singular value information. Firstly, edge information of image is extracted by Sobel operator, secondly, the edge information of the original image and the distorted image are decomposed by Singular Value Decomposition. Finally, the Angle of the singular values of two images is calculated, if the smaller is the angle, the more similar the two images. The simulation experiments show that the Image quality assessment on the basis of edge information and singular value decomposition is better than traditional methods and is closer to the subjective quality assessment.

Acknowledgements

The research leading to these results has received funding from Jilin province department of science and technology of natural fund project (201215107) and Jilin province department of education (the class word [2013] no. 264).

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