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Wideband DOA Estimation with Interpolated Focusing KR Product Matrix

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Abstract

A new direction-of-arrival estimation method for uncorrelated wideband sources is proposed. Firstly, the covariance matrices of different frequency bins are transformed into higher dimensional vectors through Khatri-Rao product to produce the new models of the array outputs. Secondly, the higher dimensional noiseless signal subspace at each frequency is focused to a certain reference frequency and a single correlation matrix is constructed, where the focusing matrices are designed by interpolating the virtual steering matrices of the new model. Finally, the DOA estimates are acquired through using narrowband MUSIC method. The proposed method is able to increase the degrees of freedom of a uniform linear array and enable us to handle more sources than sensors. Numerical results demonstrate the performance of the proposed method.

Keywords: *direction-of-arrival estimation; wideband signal; Khatri-Rao product; interpolated array*

1. Introduction

The estimation of the direction-of-arrival (DOA) of wideband signals has been widely applied in radar, sonar and wireless communication. The phase difference of the array output depends on not only the direction of arrival, but also the frequency. Therefore, the time delay is not simply deduced from the phase shift. The common method for wideband DOA estimation is to decompose the wideband signal into narrowband components through FFT or filtering, and use the multiple correlation matrices at different frequencies to get accurate DOA estimates, such as the incoherent signal subspace method (ISSM) [1]. It estimates the source DOAs separately at each frequency and then constructs the final estimate by taking an average. This method is simple but suffers severely at low SNR. In order to acquire a high spatial resolution, various methods have been proposed. For example, the coherent signal subspace methods (CSSM) [2-4], the signal subspace at each frequency is focused to a certain reference frequency to construct a single correlation matrix, and the narrowband DOA estimation algorithms are applied to obtain the DOA estimates. These methods can work well in the condition of low SNR, but need the preliminary estimates of DOA to design the focusing matrices, and their performances are sensitive to the preliminary estimates. Recently, based on the fact that the positions of signal sources are sparse in whole space, the sparse representation algorithms are proposed to estimate the spatial spectrum by using sparse representation [5-7]. Although these algorithms possess some salient characteristics, such as high resolution and improved robustness to noise, they have a degree of difficulties to obtain the optimal solutions.

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Because the spatial resolution is limited to the aperture of array, in order to improve the spatial resolution and even resolve more sources than sensors case, the virtual array structure should be constructed to increase the degrees of freedom of array. Lately, a Khatri-Rao (KR) product was proposed to expand the array structure [8]. The extended array is similar to the virtual array established by the higher order cumulant method [9-10]. After excluding the overlapping elements in the virtual array structure, the number of virtual sensors can reach $2N-1$, approximately 2 times of the actual array elements N. Based on CSSM and KR product, a new DOA estimation method for uncorrelated wideband sources named FKR-RSS was proposed [11]. Compared with the conventional CSSM, FKR-RSS transforms the covariance matrices into the high dimensional matrices, and achieves a high resolution and smaller root mean square error even if the number of sensors is about half of the number of sensors. However, the performance of this method also depends on the preliminary estimates. In our opinion, it seems more reasonable that the focusing matrices will be constructed according to the new steering matrices and array outputs built by KR product, rather than the original steering matrices and array outputs.

In this paper, we propose a new estimation method for wideband DOA estimation named KR-I-F/KR-I-F-A based on the KR product and interpolated focusing transformation. Similar to CSSM, it consists of two primary steps: 1)KR product. The covariance matrices of different frequency bins are transformed and combined into higher dimensional vectors through KR product to produce the new array outputs and virtual array steering matrices. 2)Interpolated focusing transformation. We develop the focusing matrices in the light of the interpolation principal and focus the higher dimensional noiseless signal subspace at each frequency to a certain reference frequency and constructs a single correlation matrix, after deleting the noise terms from the new array output signals. Finally, we obtain the DOA estimates by MUSIC method. Compared with FKR-RSS, due to the fact that the focusing process is implemented after the KR product transformation, the focusing error will be alleviated. The simulation results show that this method achieves better performance under the overdetermined or under-determined cases.

2. Problem Formulation

Consider an N elements uniform linear array (ULA) with spacing d, and suppose that wideband signals $\{s_k(t)\}_{k=1}^{K}$ impinge on the ULA from distinct directions of arrival $\{\theta_k(t)\}_{k=1}^{K}$. The signal received at p-th sensor is given by

$$x_p(n)=\sum_{k=1}^{K}s_k(n-\tau_p(\theta_k))+n_p(n),\ \ 1\le p\le N \tag{1}$$

where $\tau_p(\theta_k)$ is the propagation delay associated with the k-th source and p-th sensor, and $n_p(n)$ is the additive noise at the p-th sensor. The propagation delay $\tau_p(\theta_k)$ is expressed as

$$\tau_p(\theta_k)=\frac{(p-1)d\sin(\theta_k)}{c} \tag{2}$$

where c is the propagation speed. Suppose that the frequency range is $[\omega_L,\omega_H]$, and the total observation time T can be divided into M nonoverlapping segments and each segment has a duration of $J=T/M$. We apply the J-point discrete Fourier transform (DFT) to each segment and combine the components from each segment at the same frequency point, write the output of the array as

$$\boldsymbol{X}(\omega_j) = \boldsymbol{A}(\omega_j, \boldsymbol{\theta})\boldsymbol{S}(\omega_j) + \boldsymbol{N}(\omega_j),\ j = 1, 2, \cdots, J \tag{3}$$

where $\boldsymbol{A}(\omega_j, \boldsymbol{\theta}) = [\boldsymbol{a}(\omega_j, \theta_1), \boldsymbol{a}(\omega_j, \theta_2), \cdots, \boldsymbol{a}(\omega_j, \theta_K)]$ is the steering matrix with the steering vector of the form $\boldsymbol{a}(\omega_j, \theta_k) = [1, e^{-i\omega_j d \sin\theta_k / c}, \cdots, e^{-i\omega_j (N-1) d \sin\theta_k / c}]^T$. $\boldsymbol{X}(\omega_j) = [X_1(\omega_j), \cdots, X_N(\omega_j)]^T$, $\boldsymbol{S}(\omega_j) = [S_1(\omega_j), \cdots, S_K(\omega_j)]^T$ and $\boldsymbol{N}(\omega_j) = [N_1(\omega_j), \cdots, N_N(\omega_j)]^T$ are the DFT coefficients of array outputs, source signals and additive noises, respectively, $(\cdot)^T$ is the transpose operation. We assume that the signals are spatially uncorrelated and temporally white. As a consequence, the source covariance matrix at frequency ω_j is computed as

$$\boldsymbol{R}_s(\omega_j) = \mathrm{E}[\boldsymbol{S}(\omega_j)\boldsymbol{S}^H(\omega_j)] = diag(\boldsymbol{\sigma}_s(\omega_j)) = diag(\sigma_{s1}(\omega_j), \sigma_{s2}(\omega_j), \cdots, \sigma_{sK}(\omega_j))$$

where $(\cdot)^H$ is the conjugate transpose. Additionally, we assume that the noise are spatially uniform white and independent of each other and of the signals. Consequently, the observation covariance matrix is

$$\boldsymbol{R}(\omega_j) = \mathrm{E}[\boldsymbol{X}(\omega_j)\boldsymbol{X}^H(\omega_j)] = \boldsymbol{A}(\omega_j, \boldsymbol{\theta})\boldsymbol{R}_s(\omega_j)\boldsymbol{A}^H(\omega_j, \boldsymbol{\theta}) + \sigma_{nj}^2 \boldsymbol{I}_N \tag{4}$$

where $\sigma_{nj}^2 = \mathrm{E}[\,|N_n(\omega_j)|^2\,] = \sigma_n^2$.

The problem at hand is to infer the DOAs of the K wideband signals from $\boldsymbol{R}(\omega_j)$.

3. Proposed Method

$\boldsymbol{R}(\omega_j)$ in (4) is stacked into column vector, and can be written as

$$\begin{aligned}\boldsymbol{y}(\omega_j) &\triangleq \mathrm{vec}(\boldsymbol{R}(\omega_j)) = \mathrm{vec}(\boldsymbol{A}(\omega_j, \boldsymbol{\theta})\boldsymbol{R}_s(\omega_j)\boldsymbol{A}^H(\omega_j, \boldsymbol{\theta})) + \sigma_n^2 \mathrm{vec}(\boldsymbol{I}_N) \\ &= (\boldsymbol{A}^*(\omega_j, \boldsymbol{\theta}) \odot \boldsymbol{A}(\omega_j, \boldsymbol{\theta}))\boldsymbol{\sigma}_s(\omega_j) + \sigma_n^2 \mathrm{vec}(\boldsymbol{I}_N)\end{aligned} \tag{5}$$

where $\boldsymbol{A}^*(\omega_j, \boldsymbol{\theta}) \odot \boldsymbol{A}(\omega_j, \boldsymbol{\theta}) = [\boldsymbol{a}^*(\omega_j, \theta_1) \otimes \boldsymbol{a}(\omega_j, \theta_1), \cdots, \boldsymbol{a}^*(\omega_j, \theta_K) \otimes \boldsymbol{a}(\omega_j, \theta_K)] \in \mathbb{C}^{N^2 \times K}$ is KR product. Compared with model of (3), $\boldsymbol{y}(\omega_j)$ in (5) can be viewed as the new array output signals. Thus, $\boldsymbol{A}^*(\omega_j, \boldsymbol{\theta}) \odot \boldsymbol{A}(\omega_j, \boldsymbol{\theta})$ is a new virtual array steering matrix, and $\boldsymbol{\sigma}_s(\omega_j)$ is the source signal vector. The virtual array dimension given by N^2 is greater than the physical array dimension N. Consequently, it can enhance the equivalent aperture of the array and provide us with the capability of processing cases where there are less sensors of array than sources ($N < K$). In order to remove the redundancy of the steering matrix, a column orthogonal matrix $\boldsymbol{G}$ [8] is introduced as $\boldsymbol{G} = [\mathrm{vec}(\boldsymbol{J}_{N-1}), \cdots, \mathrm{vec}(\boldsymbol{J}_1), \mathrm{vec}(\boldsymbol{J}_0), \mathrm{vec}(\boldsymbol{J}_1^T), \cdots, \mathrm{vec}(\boldsymbol{J}_{N-1}^T)]$, where

$$\boldsymbol{J}_m = \begin{bmatrix} \boldsymbol{0}_{N-m,m} & \boldsymbol{I}_{N-m} \\ \boldsymbol{0}_{m,m} & \boldsymbol{0}_{m,N-m} \end{bmatrix}, m = 0, 1, \cdots N-1.$$

Thus we will get $\boldsymbol{A}^*(\omega_j, \boldsymbol{\theta}) \odot \boldsymbol{A}(\omega_j, \boldsymbol{\theta}) = \boldsymbol{G}\boldsymbol{B}(\omega_j, \boldsymbol{\theta})$ with $\boldsymbol{B}(\omega_j, \boldsymbol{\theta}) = \left[\boldsymbol{b}(\omega_j, \theta_1), \cdots, \boldsymbol{b}(\omega_j, \theta_K)\right]$ and $\boldsymbol{b}(\omega_j, \theta_k) = [e^{i\omega_j (N-1) d \sin\theta_k / c}, \cdots, e^{i\omega_j d \sin\theta_k / c}, 1, e^{-i\omega_j d \sin\theta_k / c}, \cdots, e^{-j\omega_j (N-1) d \sin\theta_k / c}]^T$, $1 \le k \le K$.

After KR product processing, the steering matrix is $\boldsymbol{B}(\omega_j, \boldsymbol{\theta})$, and the array output signals is $\boldsymbol{y}(\omega_j)$. Therefore, the focusing matrix should be computed by the steering matrix $\boldsymbol{B}(\omega_j, \boldsymbol{\theta})$ not by $\boldsymbol{A}(\omega_j, \boldsymbol{\theta})$. We will discuss the design method of focusing matrix, and formulate it as an interpolation problem in the following section.

Both sides of (5) are multiplied by $\boldsymbol{W}^{-1}\boldsymbol{G}^T$, the new array output signals is rewritten as

$$\bar{\boldsymbol{y}}(\omega_j) = \boldsymbol{W}^{-1}\boldsymbol{G}^T \boldsymbol{y}(\omega_j) = \boldsymbol{W}^{-1}\boldsymbol{G}^T\boldsymbol{G}\boldsymbol{B}(\omega_j, \boldsymbol{\theta})\boldsymbol{\sigma}_s(\omega_j) + \sigma_n^2 \boldsymbol{W}^{-1}\boldsymbol{G}^T \mathrm{vec}\left(\boldsymbol{I}_N\right) = \boldsymbol{B}(\omega_j, \boldsymbol{\theta})\boldsymbol{\sigma}_s(\omega_j) + \sigma_n^2 \boldsymbol{e} \tag{6}$$

where $\boldsymbol{e}=\Big[\underbrace{0,\cdots,0}_{N-1},1,\underbrace{0,\cdots,0}_{N-1}\Big]^T$, $\boldsymbol{W}=\boldsymbol{G}^T\boldsymbol{G}$. The specific form of (6) is elaborated as

$$\bar{\boldsymbol{y}}(\omega_j)=\boldsymbol{B}(\omega_j,\boldsymbol{\theta})\boldsymbol{\sigma}_s(\omega_j)+\sigma_n^2\boldsymbol{e}=\begin{bmatrix} e^{i\omega_j(N-1)d\sin\theta_1/c} & \cdots & e^{i\omega_j(N-1)d\sin\theta_K/c} \\ \vdots & \vdots & \vdots \\ e^{i\omega_j d\sin\theta_1/c} & \cdots & e^{i\omega_j d\sin\theta_K/c} \\ 1 & \cdots & 1 \\ e^{-i\omega_j d\sin\theta_1/c} & \cdots & e^{-i\omega_j d\sin\theta_K/c} \\ \vdots & \vdots & \vdots \\ e^{-i\omega_j(N-1)d\sin\theta_1/c} & \cdots & e^{-i\omega_j(N-1)d\sin\theta_K/c} \end{bmatrix}\boldsymbol{\sigma}_s(\omega_j)+\sigma_n^2\begin{bmatrix}0\\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0\end{bmatrix}.$$

In the above expression, the noise term is emerged in the N-th row. Accordingly, we delete the N-th row from $\bar{\boldsymbol{y}}(\omega_j)$, and obtain the noiseless array output signals $\hat{\bar{\boldsymbol{y}}}(\omega_j)$, as presented below

$$\hat{\bar{\boldsymbol{y}}}(\omega_j)=\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\boldsymbol{\sigma}_s(\omega_j)=\begin{bmatrix} e^{i\omega_j(N-1)d\sin\theta_1/c} & \cdots & e^{i\omega_j(N-1)d\sin\theta_K/c} \\ \vdots & \vdots & \vdots \\ e^{i\omega_j d\sin\theta_1/c} & \cdots & e^{i\omega_j d\sin\theta_K/c} \\ e^{-i\omega_j d\sin\theta_1/c} & \cdots & e^{-i\omega_j d\sin\theta_K/c} \\ \vdots & \vdots & \vdots \\ e^{-i\omega_j(N-1)d\sin\theta_1/c} & \cdots & e^{-i\omega_j(N-1)d\sin\theta_K/c} \end{bmatrix}\boldsymbol{\sigma}_s(\omega_j)\ . \tag{7}$$

From the virtue steering matrix $\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})$ in (7), we learn that the array is nonuniform linear structure which does not satisfy the half-wavelength constraint and prohibits us to correctly align the observation for DOA estimation. Thereby, we design the focusing matrix according to the interpolation principal [12] to meet the constraint.

Supposed the focusing matrix $\boldsymbol{T}_j\in\mathbb{C}^{(2N-2)\times(2N-1)}$, the focusing process is expressed as

$$\boldsymbol{\Upsilon}_j=\boldsymbol{T}_j^H\hat{\bar{\boldsymbol{y}}}(\omega_j)=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{\sigma}_s(\omega_j) \tag{8}$$

where $\boldsymbol{B}(\omega_0,\boldsymbol{\theta})=\left[\boldsymbol{b}(\omega_0,\theta_1),\cdots,\boldsymbol{b}(\omega_0,\theta_K)\right]\in\mathbb{C}^{(2N-1)\times K}$ is the interpolation virtue steering matrix at the desired reference frequency, and k-th column is $\boldsymbol{b}(\omega_0,\theta_k)=[e^{i\omega_0(N\text{-}1)d_0\sin\theta_k/c},\cdots,e^{i\omega_0 d_0\sin\theta_k/c},1,e^{-i\omega_0 d_0\sin\theta_k/c},\cdots,e^{-i\omega_0(N\text{-}1)d_0\sin\theta_k/c}]^T$. In other words, the deleted row is interpolated by other rows.

The focusing matrix $\boldsymbol{T}_j$ is constructed by minimizing the Frobenius norm of the mismatches between the actual steering matrix and the desired virtue steering matrix, viz.,

$$\min_{\boldsymbol{T}_j}\left\|\boldsymbol{B}(\omega_0,\boldsymbol{\theta})-\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\right\|_F^2\ . \tag{9}$$

From (9), we can get $\min_{\boldsymbol{T}_j}\left\|\boldsymbol{B}(\omega_0,\boldsymbol{\theta})-\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\right\|_F^2=\min_{\boldsymbol{T}_j}\mathrm{tr}\left\{\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\bar{\boldsymbol{B}}^H(\omega_j,\boldsymbol{\theta})\boldsymbol{T}_j-\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta})-\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\bar{\boldsymbol{B}}^H(\omega_j,\boldsymbol{\theta})\boldsymbol{T}_j+\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta})\right\}$, where $\mathrm{tr}(\cdot)$ denotes the trace operation. The optimal estimation in the least squares (LS) sense is given as

$$\boldsymbol{T}_j=(\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\bar{\boldsymbol{B}}^H(\omega_j,\boldsymbol{\theta}))^{\dagger}\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta}) \tag{10}$$

where † is Moore-Penrose pseudoinverse. The target area will be divided into serve sub regions supplied for interpolated angle segments. We define a set of interpolated angles as $\boldsymbol{\theta}=[\theta^{(1)},\theta^{(1)}+\Delta\theta,\theta^{(1)}+2\Delta\theta,\cdots,\theta^{(2)}]$, where $\Delta\theta$ is the interpolation step, $\theta^{(1)}$ and $\theta^{(2)}$ determine the range of the current sector and the interpolation accuracy. The interpolated angle segment is subdivided so that the value of the ratio of the Frobenius norm of $\boldsymbol{B}(\omega_0,\boldsymbol{\theta})-\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})$ and $\boldsymbol{B}(\omega_0,\boldsymbol{\theta})$, i.e., $\|\boldsymbol{B}(\omega_0,\boldsymbol{\theta})-\boldsymbol{T}_j^H\bar{\boldsymbol{B}}(\omega_j,\boldsymbol{\theta})\|_F^2 \Big/ \|\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\|_F^2$ can be reduced as low as possible. For example, when the value of the ratio is less than 10^{-3}, the calculation of the focusing matrix is completed. Moreover, the condition $d_j f_j = d_0 f_0$ must be attained during the computation processing, where f_0 is the centre frequency, and d_0 is the wavelength correspondingly. Coarse grid results in interpolation error, but fine grid and will increase the computation complexity. In order to reduce the interpolation error in the actual application, the focusing matrix can be acquired by off-line calculation.

With the interpolated focusing matrices, we transform the steering matrices of the NLA (nonuniform linear array) at each frequency to a common steering matrix of the virtual ULA at the centre frequency, which allow us to estimate the DOAs just like that in the narrow-band scenario. To this end, by combining the noiseless focused array output signals to yield the following matrix

$$\boldsymbol{R}_{\Upsilon}=\left[\boldsymbol{\Upsilon}_1,\boldsymbol{\Upsilon}_2,\cdots,\boldsymbol{\Upsilon}_J\right]=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{\Psi} . \tag{11}$$

We perform SVD on $\boldsymbol{R}_{\Upsilon}$ to obtain the noise subspace matrix, and the DOA estimates are calculated by MUSIC method. This method is named KR-I-F, where “I” denotes interpolation, and “F” indicates focusing.

In order to further de-correlate the sources, we also estimate the covariance matrix of $\boldsymbol{R}_{\Upsilon}$ as

$$\boldsymbol{R}_v=(1/J)\boldsymbol{R}_{\Upsilon}\boldsymbol{R}_{\Upsilon}^H=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\left(\boldsymbol{\Psi}\boldsymbol{\Psi}^H/J\right)\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta})=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{R}_{\Psi}\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta}) . \tag{12}$$

According to $\boldsymbol{\Gamma}\boldsymbol{B}^*(\omega_0,\boldsymbol{\theta})=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})$, where $\boldsymbol{\Gamma}$ is the $(2N-1)\times(2N-1)$ exchanging matrix with one on its anti-diagonal and zero elsewhere, we can calculated the spatially smoothed covariance matrix as

$$\boldsymbol{\Gamma}\boldsymbol{R}_v^*\boldsymbol{\Gamma}=\boldsymbol{\Gamma}[\boldsymbol{B}^*(\omega_0,\boldsymbol{\theta})\boldsymbol{R}_{\Psi}^*(\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta}))^*]\boldsymbol{\Gamma}=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{R}_{\Psi}^*(\boldsymbol{\Gamma}\boldsymbol{B}^*(\omega_0,\boldsymbol{\theta}))^H=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})\boldsymbol{R}_{\Psi}^*\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta}) .$$

Consequently, the averaged covariance matrix is computed by

$$\hat{\boldsymbol{R}}=(\boldsymbol{R}_v+\boldsymbol{\Gamma}\boldsymbol{R}_v^*\boldsymbol{\Gamma})/2=\boldsymbol{B}(\omega_0,\boldsymbol{\theta})[(\boldsymbol{R}_{\Psi}+\boldsymbol{R}_{\Psi}^*)/2]\boldsymbol{B}^H(\omega_0,\boldsymbol{\theta}) . \tag{13}$$

We can acquire the estimation $\hat{\boldsymbol{\theta}}$ by MUSIC algorithm of which the noise subspace matrix is obtained by perform SVD on $\hat{\boldsymbol{R}}$. We call this method KR-I-F-A, where “A” denotes averaging operation.

In the follows, the steps of KR-I-F/ KR-I-F-A are summarized:

1) Divide the sensor output into M identical segments, and perform FFT at each segment to acquire $\boldsymbol{X}(\omega_j)$.
2) Estimate the covariance matrix $\boldsymbol{R}(\omega_j)$, and generate $\boldsymbol{y}(\omega_j)$.
3) Premultiply $\boldsymbol{y}(\omega_j)$ by $\boldsymbol{W}^{-1}\boldsymbol{G}^T$ to yield $\bar{\boldsymbol{y}}(\omega_j)=\boldsymbol{W}^{-1}\boldsymbol{G}^T\boldsymbol{y}(\omega_j)$.
4) Remove the N-th element from $\bar{\boldsymbol{y}}(\omega_j)$ to get the noiseless vector $\hat{\bar{\boldsymbol{y}}}(\omega_j)$.
5) Compute $\boldsymbol{\Upsilon}_j=\boldsymbol{T}_j^H\hat{\bar{\boldsymbol{y}}}(\omega_j)$ to construct the matrix $\boldsymbol{R}_{\Upsilon}=\left[\boldsymbol{\Upsilon}_1,\boldsymbol{\Upsilon}_2,\cdots,\boldsymbol{\Upsilon}_J\right]$, or calculate $\boldsymbol{R}_v=(1/J)\boldsymbol{R}_{\Upsilon}\boldsymbol{R}_{\Upsilon}^H$ to get $\hat{\boldsymbol{R}}=(\boldsymbol{R}_v+\boldsymbol{\Gamma}\boldsymbol{R}_v^*\boldsymbol{\Gamma})/2$.

6) Perform SVD on $\boldsymbol{R}_\Gamma$ or $\hat{\boldsymbol{R}}$ to obtain the noise subspace matrix, and calculate the spatial spectrum by MUSIC algorithm, the estimation $\hat{\boldsymbol{\theta}}$ of is the local maxima in the spatial spectrum.

When $[\boldsymbol{\Psi}^T \ \boldsymbol{1}_J] \in \mathbb{C}^{J\times(K+1)}$ is of full column rank, this method will be used for the underdetermined case, and the virtual array $\bar{\boldsymbol{B}}(\omega_j, \boldsymbol{\theta}) \in \mathbb{C}^{(2N-2)\times K}$ can offer $2N-2$ degrees of freedom, thereby being able to handle $2N-3$ sources.

4. Experiment Results

In this section, the performances of the proposed algorithms are evaluated and compared against FKR-RSS through computer simulation examples.

The wideband Gaussian sources have the same center frequency $f_0 = 100\text{Hz}$ and the same bandwidth 40Hz. The noise are white Gaussian signals with zeros means. The spacing between adjacent sensors is $d = c/(2f_0) = 340/(2*100) = 1.7\text{m}$. The total time is $T_0 = 51.2\text{s}$, the sampling frequency is 80Hz. The array outputs are divided into 33 narrow band components and each components set has 64 snapshots. The SNR is defined as

$$\text{SNR} = \sum_{k=0}^{K-1} \text{E}\{\| \boldsymbol{s}_k(t) \|_2^2\} / (K \times \text{E}\{\| \boldsymbol{v}(t) \|_2^2\}) .$$

In the first example, the resolution is compared between FKR-RSS and KR-I-F methods, where the array is a uniform linear array with 5 sensors. Two uncorrelated wideband sources impinge on the array from $\theta_1 = 5°$ and $\theta_2 = 5° + \delta$, where δ is the degree interval with a step of $1°$. The initial angles $\{2°, 7°, 12°\}$ are selected for FKR-RSS algorithm. The SNR is 5dB. $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are the estimation values. The two signals are achieved to be resolved if both $|\tilde{\theta}_1 - \theta_1|$ and $|\tilde{\theta}_2 - \theta_2|$ are less than $|\theta_2 - \theta_1|/2$. The number of times that each method resolves the two sources is counted to calculate the probability of resolution per 500 times. 100 Monte Carlo runs are performed to calculate the average result. Figure 1 shows how the probabilities of resolution change with separation of the two sources. When the degree is $3°$, the probability of resolutions of the two methods are about 0.5. KR-I-F is the better than the FKR-RSS with 100% probability of resolution at $\delta = 6°$.

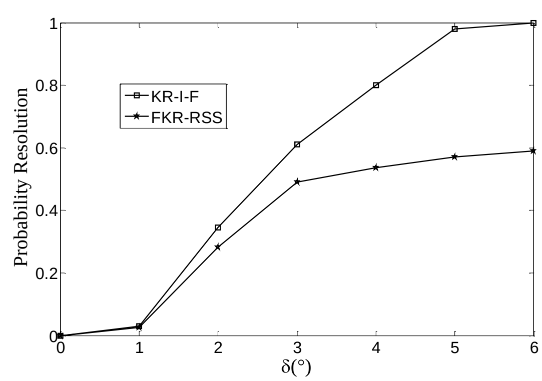

Figure 1. Probabilities of Resolution versus Separation of the Two Sources

For the second experiment, the estimation accuracy is investigated among FKR-RSS, KR-I-F and KR-I-F-A methods. The root mean square error is defined as

$$\text{RMSE}=\sqrt{\frac{1}{K\times P}\sum_{k=1}^{K}\sum_{m=1}^{P}(\hat{\theta}_k^m-\theta_k)^2}$$

where $K=2$ and $P=500$. Two uncorrelated wideband sources impinge on the array from $\theta_1=8°$ and $\theta_2=13°$. The number of sensors is 8. The SNR varies from -6 to 15dB with a step of 1dB. The initial angles $\{5°,10°,16.5°\}$ are selected for FKR-RSS algorithm. The results are shown in Figure 2. The proposed algorithms are found to outperform FKR-RSS with respect to RMSE. It is also to be noted that the interpolation transformation will inevitably produce errors, which result in the disturbances of eigenvalue vector. These errors and disturbances will affect the performance of the proposed algorithms.

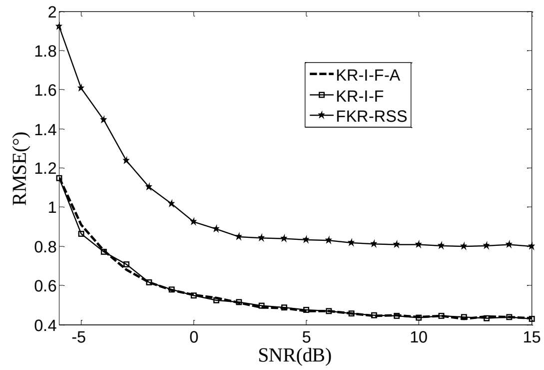

Figure 2. RMSE of the DOA Estimation versus SNR

In the third experiment, we set up a underdetermined case where $(N,k)=(4,4)$ and the SNR is 5dB, the true DOAs are $\{2°,11°,20°,28°\}$. The settings are as follows: the sampling frequency is $f_s=8000\text{Hz}$, the inter sensor spacing is $d=4.25\text{cm}$ under a sound propagation speed of 340m/s, the FFT window length is 64, the frame length is 200, the total frame number is 450. The initial angles $\{-2°,10°,15°,22°,35°\}$ are selected for FKR-RSS algorithm, and the experiment results are plotted in Figure 3. Obviously, KR-I-F is better than FKR-RSS, and the former can distinguish the four sources.

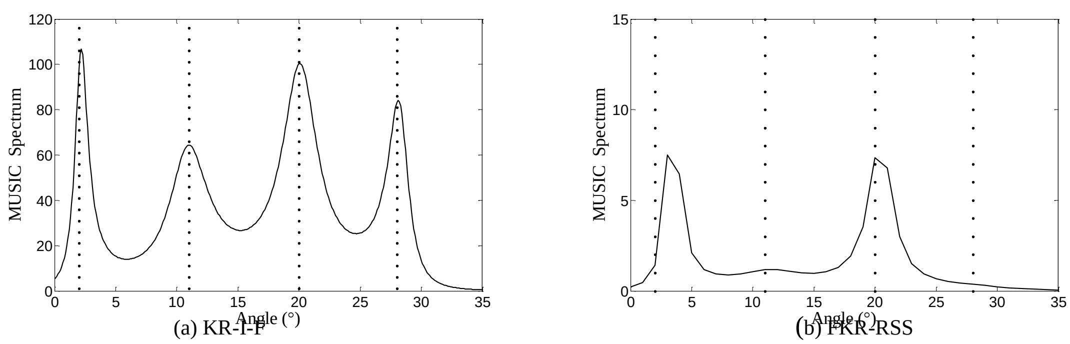

Figure 3. DOA Spectra of the Various Algorithms in the Wideband Underdetermined Case

5. Conclusion

In this paper, we propose a new wideband DOA estimation algorithm based on the KR product and interpolated focusing transformation. We are able to increase the degrees of freedom of a ULA of N physical sensors from N to $2N-2$. Meanwhile, as the noise power can be efficiently eliminated, the accuracy and robustness are significantly enhanced. Nevertheless, the proposed method cannot tackle the coherent wideband sources. We will address it as our future works.

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