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Dynamic Behavior Analysis of a Class of Neurons Network Model

Kaijun Wu[a,*], Hesheng Shi[b], Huaiwei Lu[c] and Yazhou Shan[a]

[a]School of Electronic and Information Engineering, LanZhou JiaoTong Unversity, Lanzhou 730070, China
[b]School of Computer Science and Technology Department, Pingdingshan University, pingdingshan 467000,China
[c]School of Physics and Software Engineering, LanZhou JiaoTong Unversity, Lanzhou 730070, China
E-mail: wkj@mail.lzjtu.cn

Abstract

A mathematical model of neurons network cells was studied. Firstly the study conducted numerical calculation by the C language program, and then simulated drawings with the grapher to analyze the model's complex dynamic conduct under different control parameters in the change interval of bifurcation parameter. Period-adding bifurcation and period doubling bifurcation must exist in the graph of ISI for different control parameters. As the external electric fields join in, the dynamic conduct of neuron model will change under corresponding parameters. The research results indicate external electric fields can change the discharge range of the model, which is helpful to explore the influence of external electric field on the excitability of biological neural system and pathogenic mechanism of neural system disease.

Keywords: *bursting, ISI, period-adding bifurcation, period-doubling bifurcation, external electric fields*

1. Introduction

Neuron is the basic function unit of the nervous system, which activity expresses as the generation of biological signal, changes and dissemination in the electrical signal and materials of neuronal membrane mainly work as information exchange in the Tran membrane transport process [1]. Biological nervous system is the extremely complex multi-level information network system interconnected by a huge number of neuron cells, therefore discharge activities of neurons and information encoding process exist complex nonlinear dynamical behavior [2-7] Neuron dynamics is the combination of neurophysiology and nonlinear dynamics, which becomes the hot spot in the current world cutting-edge research. Biology research shows that different discharge modes have different biological meanings and studies on the discharge of neurons have important meaning on dynamics and biology. In order to analyze the nonlinear dynamics characteristics of neurons, mathematical models of various different nonlinear neurons are built by experiments such as HH model, HR model, Chay model and ML model [4-7].

Numerical simulation calculation of neuron model can imitate the discharge activities of real neurons and discharge rhythm bifurcation diagram and let people understand and comprehend the dynamic characteristics of biological neuron system through simulated neuron models. In the analyzing process, neuron model can imitate the dynamic characteristics of normal or lesions-occurred neurons, which plays a very important role

[*] Corresponding author.
E-mail address: wkj@mail.lzjtu.cn

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in analyzing the distinction and solutions of neurons discharge behavior. Therefore, many scholars make analysis to above-mentioned different neuron models in theoretical or numerical simulated way, such as Qiao Zeng [8] analyzed the chaotic dynamic behavior of neuron model brought up by Rulkov under the changing of parameters through the way combining with theoretical proof and numerical simulation; such as Pei Lijun [9] found out the equilibrium point of system under a value of control parameters and proved the uniqueness of model equilibrium point through hopf bifurcation theory to indicate the decisive role of control parameters to model dynamic behavior; such as Wang Haixia analyzed and found that neurons under different parameters had different discharge way through making numerical simulation analysis of complex dynamic behavior of ML neuron model [10]. Many experts and scholars have made deep research of these neuron models and achieved lots of theoretical results that have a positive guiding meaning on biology study [11-15].

The object of this study, which is the neuron model brought up by Sherman in 1988 [16], analyzed the influence of different parameters to model discharge rhythm through changing parameters in the model to analyze the model dynamic characteristics.

2. Dynamic Model

This paper mainly conducted a study on descried excitable neuron discharge model [15]; the mathematical model is shown in Figure (1).

$$\begin{aligned}
&\tau \frac{dV}{dt} = -I_{Ca}(V) - I_k(V,k) - g_S S(V - V_k) \\
&\tau \frac{dK}{dt} = \sigma[K_\infty(V) - K] \\
&\tau_S \frac{dS}{dt} = S_\infty(V) - S \\
&I_{Ca}(V) = g_{Ca} m_\infty (V - V_{Ca}) \\
&I_k(V,k) = g_k K(V - V_k) \\
&m_\infty(V) = 1/[1 + e^{(V_m - V/\theta_m)}] \\
&K_\infty(V) = 1/[1 + e^{(V_n - V/\theta_n)}] \\
&S_\infty(V) = 1/[1 + e^{(V_S - V/\theta_S)}]
\end{aligned} \tag{1}$$

Among the system, V represents membrane potential, K represents open rate of Potassium ion channel, and S represents a slow subsystem. Other parameters of the system, such as g_S, g_{Ca} and g_k represent inward current of corresponding ion, V_k and V_{Ca} are relevant reverse voltage, τ and τ_S represent loose time constant, the rest parameters of the system are shown as Table 1.

Table1. Model Parameters for which All Results are Calculated Unless Otherwise Stated

parameter	τ	τ_S	g_S	g_{Ca}	g_k	V_k	V_{Ca}	V_m	V_n	θ_m	θ_n	θ_S	σ
value	0.02s	35s	4	3.6	10	-75mv	25mv	-20mv	-16mv	12mv	5.6mv	10mv	0.85

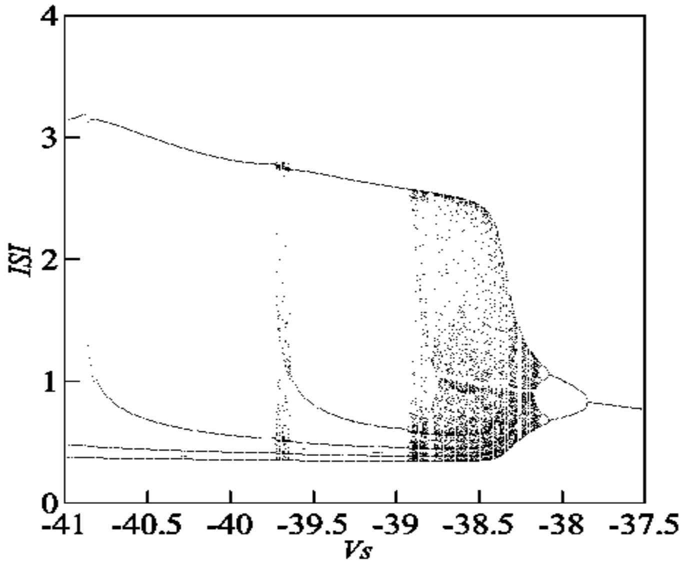
3. Analysis of Dynamic Behavior

3.1 V_S as Control Parameter

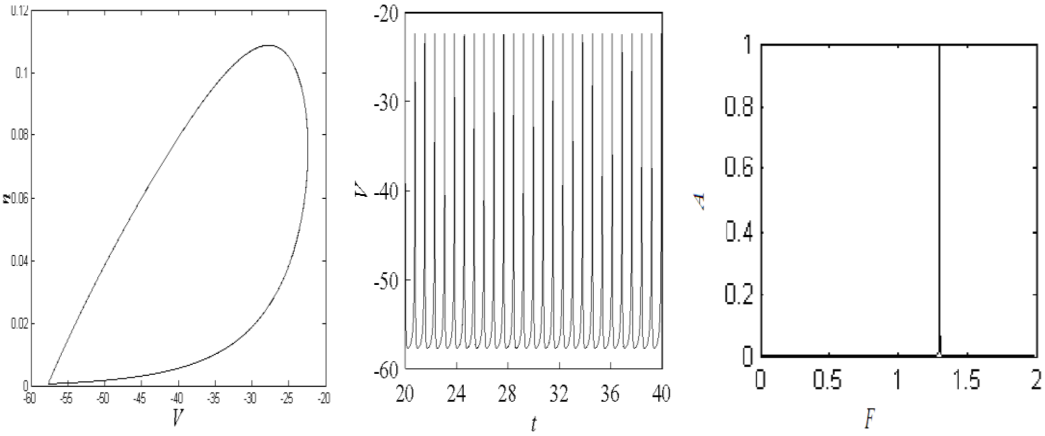
When analyzing the dynamic behavior of the model under different parameters, firstly, considered other parameters are given while reverse voltage V_S is the control parameter, we analyze the dynamic behavior through ISI figure of the system. When the control parameter changes in the range [-41, 37.5], because neurons discharge model is coded based on peak-to-peak intervals, we use C language to program and variable-step Runge-Kutta method to make numerical simulation and grapher to draw global bifurcation and ISI Figure. ISI figure of system is shown in Figure 1 (a). In Figure a, with the increasing of V_S, the global trend of dynamic behavior of system changes from period discharge activity to chaotic interval, and then degenerates to stable discharge form with period 1 through reverse doubling bifurcation. With the parameter changing from big to small, it is known from bifurcation figure that when the value of V_S is -37.5, discharge activity of system is the condition of period 1. With the decreasing of V_S discharge status of system changes from period 1 to period 2 and if the value of V_S is -38 system lies in period 2. System continues doubling bifurcation with the decreasing of parameter. We can observe the discharge form of period 4 and discharge activity of system becomes chaotic discharge status finally. If amplifying bifurcation figure we can observe interior crisis similar to literature [14] that is two chaotic attractor merge together gradually to form a piece of chaotic attractor. When the value of parameter is about -38.8, chaotic discharge form changes from reverse doubling bifurcation to period 5 bursting activity. It is seen from the figure that discharge form changes to chaotic bursting activity after system has experienced certain period 5 and then turns into period 4 bursting activity window from ending chaotic discharge by saddle-note bifurcation, and through a little chaotic window period 4 degenerates to period 3 bursting activity. The bursting activity here from period 5 to period 3 becomes reverse plus period bifurcation process accompanied with chaotic.

Periodic and chaotic can also be judged by power spectrum generally, if corresponding FFT diagrams are some mutually incommensurable frequency and distribution of frequency density is regular, system lies in quasi-periodic activity; if power spectrum are some mutually commensurable frequency, system lies in periodic activity; system lies in chaotic activity if broadband appears. Shown in Fig. b-f, when V_S =-37.5 system is in peak discharge of period 1 through ISI and phase diagram in Fig. b is a closed circle composed of a curve and the time response diagram and power spectrum testify the peak discharge phenomena of period 1.

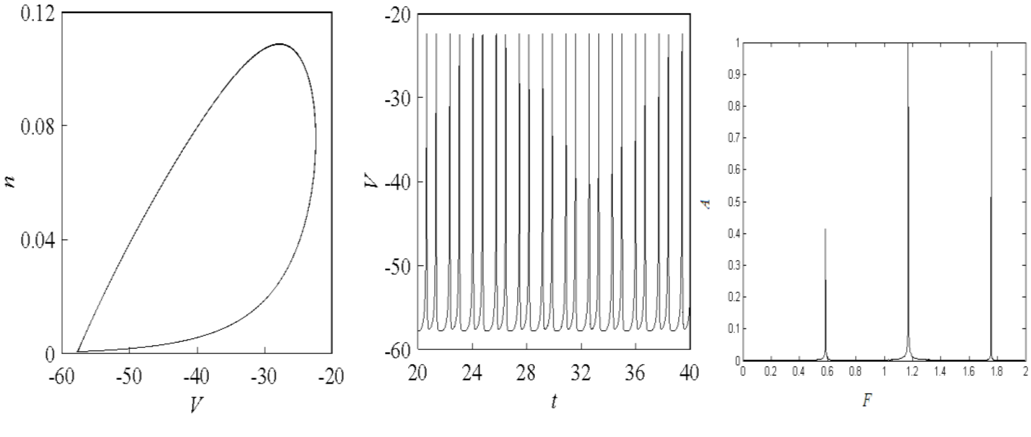
Figure c is the phase diagram, time response diagram and power spectrum when V_S =-38 and system is in period 2. Figure d is the phase diagram, time response diagram and power spectrum when V_S =-40 and system is in bursting discharge of period 4. Figure e is the phase diagram, time response diagram and power spectrum when V_S =-39 and system is in bursting discharge of period 5. Figure f is the phase diagram, time response diagram and power spectrum when V_S =-38.5 and system is in chaotic bursting discharge, here power spectrum and time response diagram obviously testify the result of chaotic bursting discharge. Through ISI and corresponding phase diagrams, it is obtained that control parameter V_S has important influence to dynamic behavior of the model.

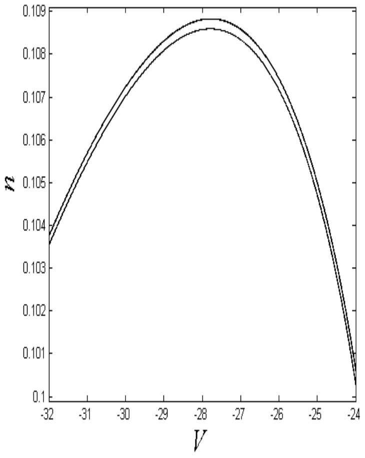


a

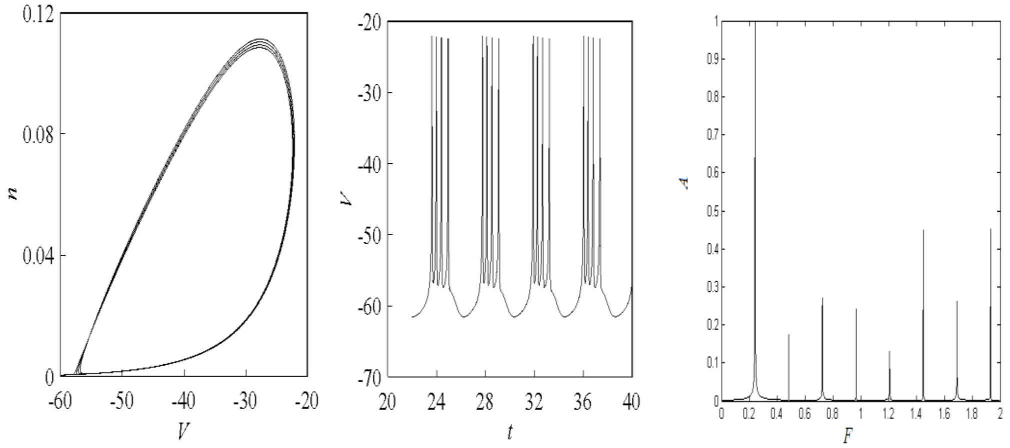


b V_s =-37.5

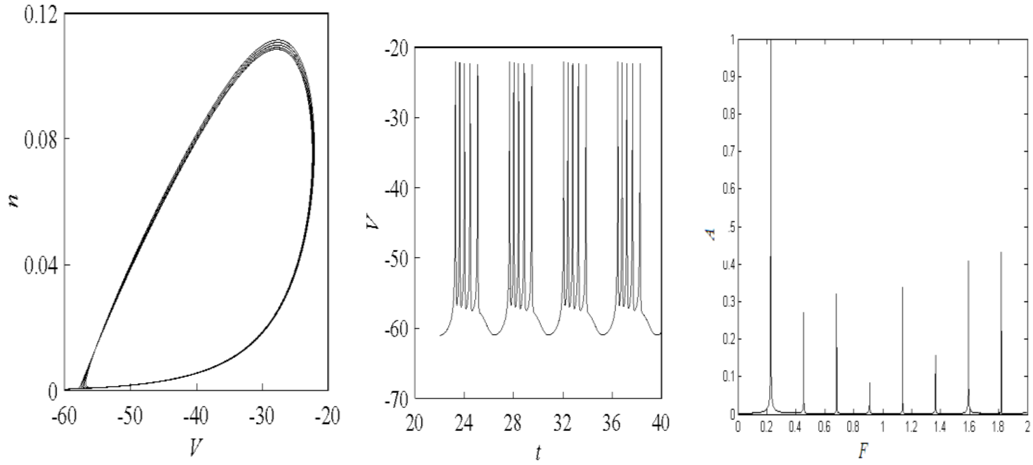




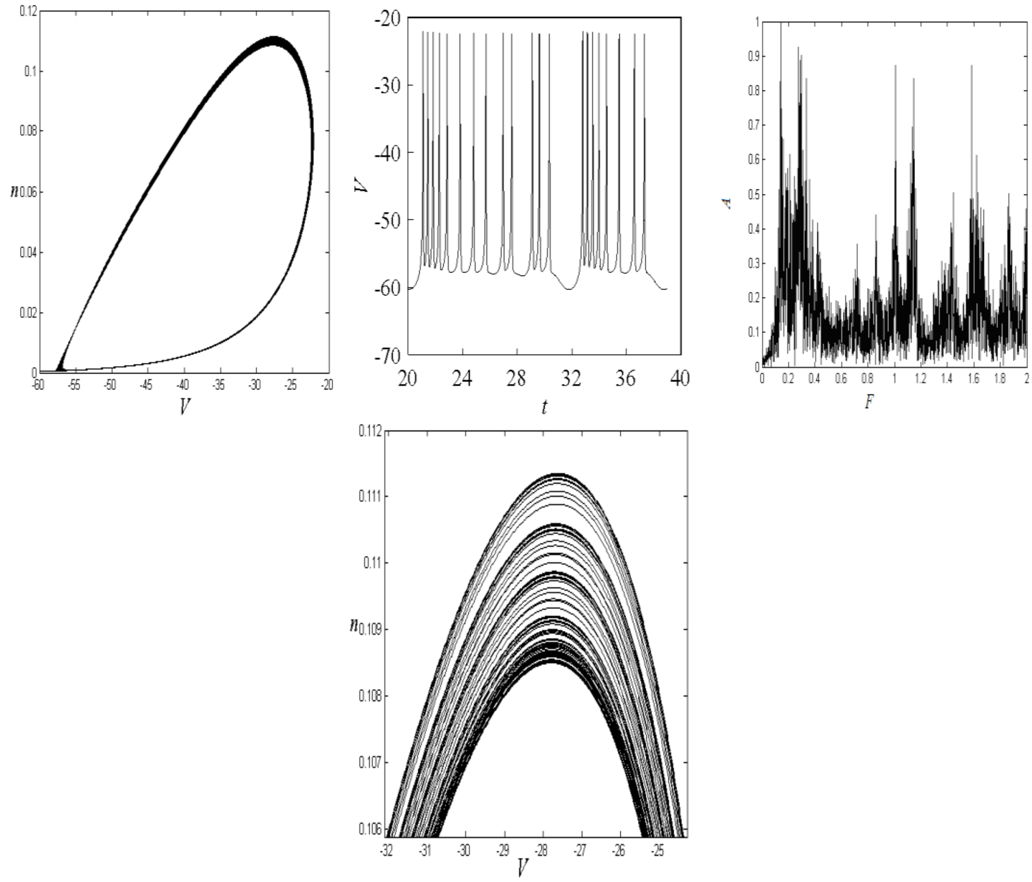
c V_S =-38



d V_S =-40



e V_S =-39



f V_S =-38.5

Figure 1 (a) ISI, (b-f)the Time Courses and the FFT

Figure 1(a) is the ISI, (b-f) are the corresponding phase diagrams, time response diagrams and power spectrum when control parameters are different values.

3.2 consider Micro External Electric Fields

External electric fields on the organism especially the mechanism of the human body has become a very concerned research spot of experts and scholars. On one hand, the electromagnetic radiation of organism environment is strengthening rapidly and the phenomenon of induced neurological disease becomes increasingly common because external electric affects the function of nervous system; on the other hand, electrical stimulation has become one of the main experimental means of neurological diseases treatments [16]. Many experiments confirmed that external electric field can affect the dynamic characteristics of neurons [17-18]. Dynamic behavior analysis on the model of a pancreatic beta in external electric is still rare. In the paper, the model considers the influence of small external electric field to dynamic behavior of system when V_S is the control parameter. When V_e =0.5, V_S is the control parameter and other parameters remain unchanged, ISI is drawn and shown in Figure 2. Compared ISI in Figure 1 with Figure 2, it is discovered that the adding of external electric field changes discharge interval of system. The peak discharge interval of system becomes longer from Figure 2, and when V_S =-38, system still in peak discharge condition of period 1, but in Figure 1 system changes to peak discharge of period 2 through doubling bifurcation. Another different point between Figure 2 and Figure 1 is that chaotic bursting discharge window in Figure 2 degenerates to peak discharge of period 4 through reverse doubling bifurcation and then experiences reverse period-adding bifurcation. It is also completely testified the influence of external electric field to neuron system and indicated the importance of external electric field in real life.

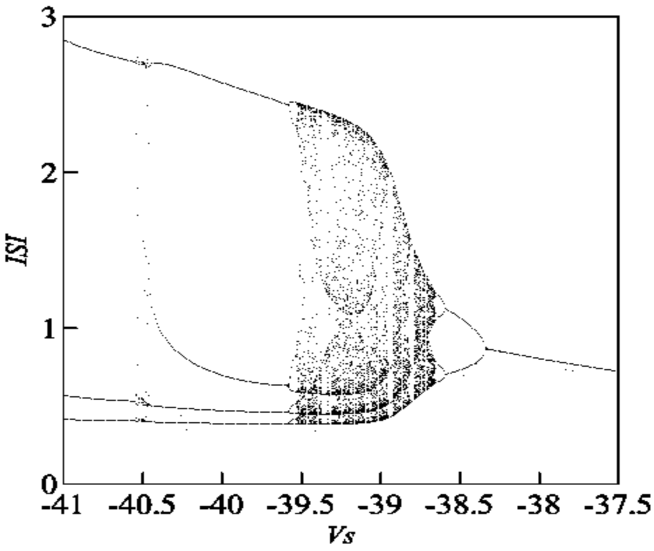


Figure 2. ISI Figure

3.3 consider Reverse Voltage V_n as Control Parameter

In the system, V_S =-38.4 is settled and considers the influence of dynamic behavior of system with the change of V_n while other parameters remain unchanged. The ISI of system is shown in Figure 3.

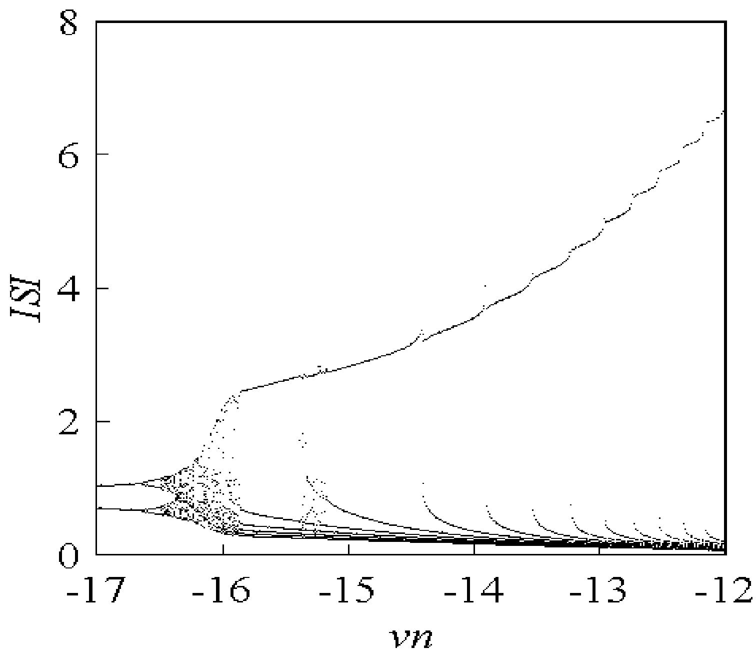


Figure 3. ISI Figure

The dynamic behavior of system when the control parameter changes in [-17,-12] is shown in the figure. System represents discharge of period 2 when V_n is smaller, with V_n changing to -17.5, system occurs doubling bifurcation and turns to chaotic bursting discharge through doubling bifurcation. With the increasing of V_n, system experiences an interior cataclysm. As the value of V_n reached -16.2, discharge condition of system degenerates to bursting discharge of period 5 from chaotic discharge. Afterwards, a process of period-adding bifurcation accompanied with chaotic is observed. It is indicated in Figure 3 that, in the global period-adding bifurcation process, with the gradually increasing of periods of periodic bursting discharge, the width of mutually adjacent chaotic discharge interval decreases gradually and the chaotic interval is scarcely observed in the later periods, and with the number of periods increased, the chaotic interval in the bifurcation gets smaller and smaller. Window of period 6 is obviously wider than discharge window of period 7, which is wider than period 8. Through analyzing, it is discovered that the influence of the changing of V_n to dynamic behavior of system is also obvious; therefore the select of the value of V_n should be emphasized in the experiment.

3.4 Consider Heart Electric Current g_S as Control Parameter

Consider the dynamic behavior of system when taking another parameter g_S as the control parameter. When V_n=-16, V_S=-37.8 is confirmed, g_S changes in [3.95, 4.6], the corresponding ISI bifurcation diagram is shown in Figure 4.

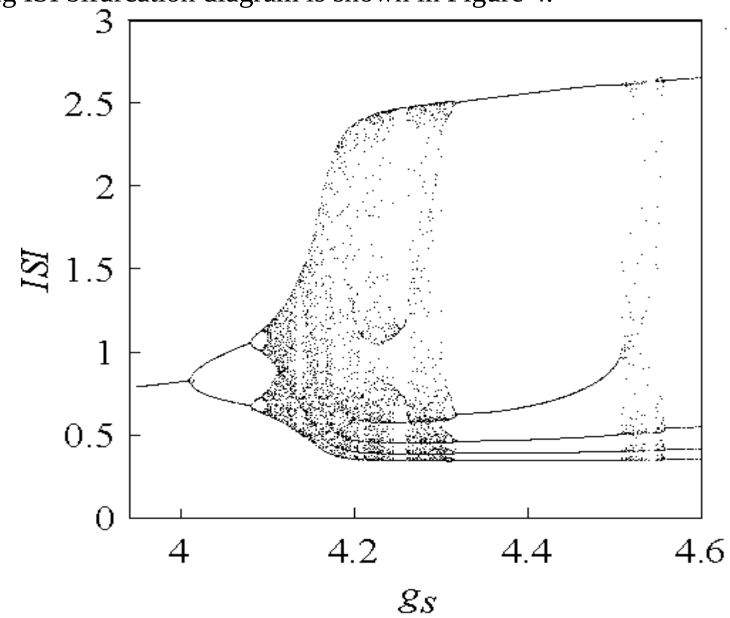


Figure 4. ISI Figure

With the increasing of the control parameter, system still experiences the way from peak discharge doubling bifurcation to chaotic, and chaotic window contains an interior cataclysm and the chaotic discharge window transforms to periodic bursting discharge status through reverse doubling bifurcation. The control parameter continues to increase and system experiences the process of reverse period-adding bifurcation accompanied with chaotic. In the control parameter interval, we can only observe bursting discharge of period 5 and discharge activity of period 4, if considered wider control parameter interval the number of periods is observed to decrease gradually. In addition, we can find in the chaotic discharge interval, discharge activity of system is not chaotic but contains many periodic windows.

4. Conclusions

Period-adding bifurcation and doubling bifurcation are the normal bifurcation structure in dynamic behavior of neuron model. In this paper, we consider multiple parameters as control parameters and different control parameters have corresponding different ISI bifurcation diagrams, which mean the changing of values of these control parameters would have impact on dynamic behavior of system and more emphasis need to be put in realistic experimental or clinical process. In fact, the chaotic motion is extremely sensitive to parameter change. In the chaotic interval, a very small parameter space corresponds to an infinite number of chaotic motions, therefore, a small parameter perturbation makes the chaotic systems experience many chaotic states, and thus the chaotic state will have a very big gap before and after the disturbance. Moreover, we can observe period-adding bifurcation and doubling bifurcation from the corresponding bifurcation diagrams, which testifies these two bifurcation structure are the normal bifurcation types of neuron model. So we can draw a conclusion that cycle the bifurcation between periodic discharge and chaotic discharge, chaos discharge interval bifurcation and sensitive dependence on parameter of chaotic motion lead to produce more sensitive and frequent dynamics mechanism for chaotic discharge cells than ordinary cycles to external stimuli.

The actual neuron model discharge is generally carried under multi-parameter continuous dynamic adjustment, but analysis of fixed parameter obtains ideal bifurcation structure and it provides the basis of the theoretical framework for deep research. However, discussing neurons discharge under multi-parameter more accords the objective laws so we should put more focus on multi-parameter in the subsequent analysis of the study.

With the rapidly strengthen of the electromagnetic radiation in the organism survival environment, the influence of external electric field to the function of nervous system becomes increasingly serious. External electric filed can't be ignored considering the dynamic behavior of neuron discharge model. Of course, the influence of external electric field to neuron model also has positive aspect.

Acknowledgements

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Authors

Kaijun Wu, born in Shandong, P.R. China, March 1, 1978, associate professor and master tutor in the Lanzhou Jiaotong University, PhD student of automation and electrical engineering, Lanzhou Jiaotong University, P.R.

Scientific interests: nonlinear dynamics, neural network, *etc.*

Publications: 35 papers, 2 textbooks.

Hesheng Shi, born in Henan, P.R. China, August 14, 1978, a lecturer in Pingdingshan University, School of Computer Science and Technology Department, Pingdingshan University.

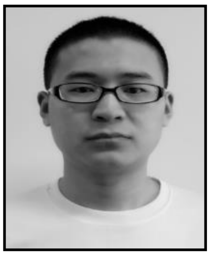
Scientific interests: Software engineering, artificial intelligence, *etc.*

Publications: 15 papers, 1 textbook.

Huaiwei Lu, born in Gansu, P.R. China, March 1, 1959, professor and doctoral tutor in the Lanzhou Jiaotong University, master degree in the University of electronic science and technology of China in 1989.

Scientific interests: optical fiber communication, optical fiber passive components, *etc.*

Publications: 48 papers.



Yazhou Shan, born in Henan, P.R. China, June 27, 1990, master student in the Lanzhou Jiaotong University, master degree will be received in the school of Electronic and Information Engineering, LanZhou JiaoTong in 2016.

Scientific interests: nonlinear dynamics, neural network, *etc.*