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MPEC Model and Feasible Direction Method for Asymmetric Signal-Controlled Network Design Problem

Aihua Luo[1] and Zhengshun Ruan[2*]

[1]*School of Mathematics and Statistics, South-Central University for Nationalities, Wuhan 430074, China*
[2]*School of Science, Wuhan Institute of Technology, Wuhan 430073, China*
[1]*luoah@foxmail.com,* [*]*xiaoruan55@163.com*

Abstract

In this paper, asymmetric traffic network design problem is formulated as mathematical program with equilibrium constraints instead of bilevel program, where user equilibrium traffic assignment problem is expressed as variational inequality problem the solution of which follows Wardrop user equilibrium principle. Due to sensitivity analysis of parametric variational inequality, Mathematical program with equilibrium constraints can be written as an implicit optimization problem and the generalized gradient of objective function can be received. A feasible and descent direction method based on generalized gradient is addressed where the direction can be computed with the simple formula and the optimal step size can be received by comparison operation. Finally, numerical experiments are conducted and calculation results show high efficiency of the proposed method in solving asymmetric equilibrium network design problem.

Keywords: *Network design problem; User equilibrium; Mathematical program with equilibrium constraints; Feasible direction method*

1. Introduction

For an urban traffic road network of signal-controlled junctions, the network flows and travel cost are strongly influenced by the operation of signals in a linked signal system. Therefore, the Jacobi matrix of link flow travel cost function should be asymmetric and it is more reasonable for considering signal-controlled network design problem (SCNDP) on an asymmetric traffic road network. A SCNDP problem is that the set of link capacity expansions and signal setting variables needs to be simultaneously determined where the sum of total travel time and investment cost is minimized. Meng *et. al.,* [1] presented a bilevel program for SCNDP and transferred it into a single level continuously differentiable optimization problem by augmented Lagrangian method. Luo *et. al.,* [2] formulate SCNDP as a special case of the mathematical program with equilibrium constraints problem (MPEC) which had been extensively studied. Ban, Liu, Ferris and Ran [3] furthermore formulated a mathematical program with complementarity constraints (MPCC) for a general network design problem.

For the algorithm, Abdulaal and LeBlanc [4] were the first ones who proposed the algorithm for continuous network design problems with link capacity expansions. Yang and Yagar [5] linearized sub-problem which was solved by the simplex method for the objective function of SCNDP and proposed an algorithm for solving the CNDP. In order to receive a global signal setting variables with user equilibrium network flows, Cipriani and Fusco [6] presented a new general projected gradient algorithm with Armijo step length rule and investigated algorithm properties via numerical calculations on a small test

[*] Corresponding Author

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network. Chou and Teng [7] analyzed an urban road network with signal settings by utilizing a fuzzy control approach. Ceylan and Bell [8] also proposed a genetic algorithm (GA) to deal with an area traffic control problem. Chiou [9] formulated SCNDP as MPEC problem where the signal setting plan was defined by the common cycle time, the start and duration of greens and the performance index was defined as the sum of a weighted linear combination of rate of delay and number of stops per unit time and a bundle method combined with sub-gradient projection was proposed because of non-differentiability of the perturbed solutions in the equilibrium constraints with respect to the decision variables.

This paper will research asymmetric signal-controlled network design problem with linked signal setting and link capacity expansion variables which consequentially effect on the network flows. This problem can be formulated as a nonlinear program subject to Wardrop user equilibrium where the sum of a weighted linear combination of rate of delay and investment cost is minimized. The gradient of variable of interest with respect to link capacity expansion can be conveniently computed by the first order sensitivity and a feasible and descent direction method (FDDM) based generalized gradient is presented for MPEC where the direction can be received by simple formula and the optimal step can be received by comparison operation. The character for computing direction and step size in brief makes algorithm win high efficiency which is embodied with numerical experiments and comparative analysis on an example network.

The organization of this paper is as follows, in the next section, a VI model for UE and an MPEC for SCNDP are given. The first order sensitivity analysis for obtaining the gradient is conducted. In the Section 3, a feasible and descent direction method is proposed for solving MPEC. In Section 4, numerical calculations are conducted and good computational results are obtained on an example road network. Conclusion and further research work are summarized in Section 5.

2. Problem Formulations

In this section, SCNDP is presented with a mathematical program with equilibrium constraints where UE is expressed in terms of variational inequality (VI). Due to the first-order sensitivity analysis for VI, the gradient of variables of interests is conducted. Finally an MPEC formulation can be written as an implicit program.

The following Notation will be used.

$G(N, A)$: directed road network, where N is set of nodes and A is set of links.

W : set of OD pairs.

R_w :set of paths between OD w .

y_a : link capacity expansion on link a .

l_a, u_a : bounds of link capacity expansion on link a .

$G_a(y_a)$: investment cost on link a .

η : conversion factor from investment cost to travel time.

f : vector of link flow.

h : vector of path flow.

t : vector of link flow travel cost.

C : vector of path flow travel cost.

q : vector of travel demand.

Δ : link-path incidence matrix.

Γ : OD-path incidence matrix.

ζ :signal setting variables for the reciprocal of cycle time.

$\zeta_{\min}, \zeta_{\max}$:bounds for cycle time.

g_{jm} :minimum green time for signal group j at junction m.

θ_{jm} :vector of green starts time for signal group j at junction m.

ϕ_{jm} :vector of green duration's time for signal group j at junction m.

$\psi = (\zeta, \theta, \phi)$: set of signal setting variable.

$\overline{c}_{jlm}$:clearance time between the end of green for signal group j and the start of green for signal group l at junction m.

$\Omega_m(j,l)$: collection of numbers 0 and 1 for each pair of incompatible signal groups at junction m. If the start of green for signal group j proceeds that of l, $\Omega_m(j,l) = 0$; otherwise $\Omega_m(j,l) = 1$.

d_a :delay cost on link a ..

For traffic assignment problem with user equilibrium, a variational inequality model can be expressed as follows. Determine values f such that

$$c^T(f)(\overline{f} - f) \geq 0, \forall \overline{f} = \left(\cdots, \overline{f}_a, \cdots\right)^T \in K_1 = \left\{ f \,\middle|\, f = \Delta h,\ \Gamma h = q, h \geq 0 \right\} \tag{1}$$

The solution of (1) can be proved to follow Wardrop user equilibrium principle.

In term of (1), user equilibrium traffic assignment problem with link capacity expansions and signal setting parameters can be concluded as the following parametric variational inequality. Determine values $f(y,\psi)$ such that

$$t(f(y,\psi))^T(\overline{f}(y,\psi) - f(y,\psi)) \geq 0,\ \forall \overline{f}(y,\psi) = \left(\cdots, \overline{f}_a(y,\psi), \cdots\right)^T \in K(y,\psi) \tag{2}$$

Where $K(y,\psi) = \left\{ f(y,\psi) \,\middle|\, f(y,\psi) = \Delta h(y,\psi), \Gamma h(y,\psi) = q, h(y,\psi) \geq 0 \right\}$.

Assume $S(y,\Psi)$ be locally Lipschitz which is the solution set of (2), the first order sensitivity analysis of (2) can be carried out by the conclusion of Patriksson [10] in the following way. Given y^*, ψ^*, let f^* be the link flow, π^* be the minimum path flow and h^* be arbitrary one of the path flows which are not unique. Let the changes in link and path flow travel time denoted by $g_{f(y^*,\psi^*)}$ and $g_{h(y^*,\psi^*)}$ which can be received by the following affine variational inequality when the changes in link capacity expansion and signal setting parameters g_{y^*,ψ^*} are specified.

Find $g_{f(y^*,\psi^*)} \in K'(y^*,\psi^*)$ such that

$$(\nabla_{(y,\psi)} t(f^*, y^*, \psi^*) g_{y^*,\psi^*} + \nabla_f t(f^*, y^*, \psi^*)\ g_{f(y^*,\psi^*)})^T (z - g_{f(y^*,\psi^*)}) \geq 0 \tag{3}$$

$$\forall z \in K'(y^*,\psi^*) = \left\{ g_{f(y^*,\psi^*)} \,\middle|\, \exists g_{h(y^*,\psi^*)} \ \ such\ that\ g_{f(y^*,\psi^*)} = \Delta g_{h(y^*,\psi^*)}, \Gamma g_{h(y^*,\psi^*)} = 0 \right\},$$

where

$$K_0'(y^*,\psi^*) = \left\{ g_{h(y^*,\psi^*)} \,\middle|\, ① \ if\ h_k^{w*} > 0, g_{h_k^w(y^*,\psi^*)} is\ free. ② \ if\ h_k^{w*} = 0\ \ and\ C_k^{w*} = \pi^{w*}, \right.$$

$g_{h_k^w(y^*,\psi^*)} \geq 0.$ ③ $if\ h_k^{w*}=0\ and\ C_k^{w*} > \pi^{w*},\ \ g_{h_k^w(y^*,\psi^*)} = 0.\}$

According to Rademacher's theorem, the solution set $S(\cdot)$ is differentiable almost everywhere and the generalized gradient for $S(\cdot)$ can be denoted as follows.

$$\partial S(y^*,\psi^*) = conv\Big\{g_{f(y^*,\psi^*)} = \lim_{k\to\infty} \nabla f(y^k,\psi^k)$$

$$\Big|(y^k,\psi^k) \to (y^*,\psi^*), \nabla f(y^k,\psi^k) exists\Big\}(4) \tag{4}$$

An optimization model for SCNDP can be formulated as

$$\begin{aligned} \min\quad & Z = Z(f,y,\psi) = f^T(t(f,y,\psi)+d(y,\psi))+\eta G(y,\psi) \\ s.t.\quad & \xi_{\min} \leq \xi \leq \xi_{\max} \\ & g_{jm} \leq \phi_{jm} \leq \xi, \forall j,m \\ & \theta_{jm} + \phi_{jm} + \overline{c}_{jlm} \leq \theta_{lm} + \Omega_m(j,l) \forall j,l,m \\ & l \leq y \leq u \\ & f \in S(y,\psi) \end{aligned} \tag{5}$$

where $S(y,\psi)$ is the solution set of parametric variation inequality (2).

Due to the sensitivity analysis of (4), the model (5) can be re-expressed as a single-level problem:

$$\begin{aligned} \min\quad & Z = Z(f(y,\psi),y,\psi) \\ s.t\quad & B\psi \leq b \\ & l \leq y \leq u \end{aligned} \tag{6}$$

Considering the difficulty of achieving $S(y,\psi)$, the objective function $Z(y,\psi)$ of (6) has no specific form and is non-smooth and non-convex function with respect to the decision variable. However, suppose $Z(y,\psi)$ is semi-smooth and locally Lipschitz, the directional derivatives can be characterized by the following generalized gradient.

$$\partial Z(y^*,\psi^*) = conv\Big\{\lim_{k\to\infty} \nabla Z(y^k,\psi^k) \Big| (y^k,\psi^k) \to (y^*,\psi^*), \nabla Z(y^k,\psi^k) \text{exists}\Big\}.$$

$$\nabla Z(y^k,\psi^k) = \nabla_{(y,\psi)} Z(f^k,y^k,\psi^k) + \nabla_f Z(f^k,y^k,\psi^k) g_{f(y^k,\psi^k)}. \tag{7}$$

3. FDDM for SCNDP

Due to the sensitivity analysis, a feasible and descent direction method for traffic network design with capacity expansions and signal setting variables in (5) can be established.

Supposing y^k,ψ^k is current iterative point of (11), introduce the following sets:

$$A' = \{a \mid y_a^k = l_a, \forall a \in A\}, \quad A'' = \{a \mid y_a^k = u_a, \forall a \in A\}.$$

At the same time, assume $B_b\psi^k = b_b$ and $B_{nb}\psi^k < b_{nb}$ .Let $d = \begin{pmatrix} d_y \\ d_\psi \end{pmatrix} = \begin{pmatrix} y - y^k \\ \psi - \psi^k \end{pmatrix}$, a linear program can be concluded by using linear approximation of the objective function

$$
\begin{aligned}
\min \quad & g_y(y^k, \psi^k)^T d_y \\
s.t. \quad & d_{y_a} \ge 0, \quad \forall a \in A' \\
& d_{y_a} \le 0, \quad \forall a \in A'' \\
& -1 \le d_{y_a} \le 1, \quad \forall a \in A
\end{aligned} \tag{8}
$$

Where $\left(g_y(y^k,\psi^k)^T, g_\psi(y^k,\psi^k)^T\right)^T \in \partial Z(y^*,\psi^*)$. The aim of appending constraints $-1 \le d_a \le 1, \quad \forall a \in A$ is to ensure that (8) has optimal solution.

Theorem 1 The solution of LP (8) is $d_y = (\cdots, d_{y_a}^k, \cdots)$, where

$$
d_{y_a}^k = \begin{cases} 0, & if\,(g_y(y^k,\psi^k))_a > 0, a \in A' \\ & or\,(g_y(y^k,\psi^k))_a \le 0, a \in A'' \\ 1, & (g_y(y^k,\psi^k))_a \le 0, a \in A \setminus A'' \\ -1, & otherwise \end{cases} \tag{9}
$$

Proof Considering (8), we can receive the solution d_y by analyzing the sign of components of $g_y(y^k,\psi^k)$. If $(g_y(y^k,\psi^k))_a > 0, \quad 0 \le d_{y_a} \le 1$, then $d_{y_a}^k = 0, \forall a \in A'$.

If $(g_y(y^k,\psi^k))_a \le 0, \quad 0 \le d_{y_a} \le 1$, then $d_{y_a}^k = 1, \quad \forall a \in A'$;

If $(g_y(y^k,\psi^k))_a > 0, -1 \le d_{y_a} \le 0$, then $d_{y_a}^k = -1, \forall a \in A''$;

If $(g_y(y^k,\psi^k))_a \le 0, -1 \le d_{y_a} \le 0$, then $d_{y_a}^k = 0, \forall a \in A''$;

If $(g_y(y^k,\psi^k))_a > 0, -1 \le d_{y_a} \le 1$, then $d_{y_a}^k = -1, \forall a \in A \setminus A' \cup A''$;

If $(g_y(y^k,\psi^k))_a \le 0, -1 \le d_{y_a} \le 1$, then $d_{y_a}^k = 1, \forall a \in A \setminus A' \cup A''$.

Summarize the above analysis; we can conclude (9).

Theorem 2 Let $d_\psi = -H_k g_\psi(y^k,\psi^k)$, $H_k = I - B_b^T(B_b B_b^T)^{-1} B_b$ (10)

If $d_\psi \ne 0$, then $B_b d_\psi = 0$ and $g_\psi(y^k,\psi^k)^T d_\psi < 0$.

Proof Because $B_b d_\psi = -B_b H_k g_\psi(y^k,\psi^k) = -B_b[I - B_b^T(B_b B_b^T)^{-1} B_b] g_\psi(y^k,\psi^k)$

$$= -B_b g_\psi(y^k,\psi^k) + B_b B_b^T (B_b B_b^T)^{-1} B_b \ ,$$

$g_\psi(y^k,\psi^k) = 0$, $H_k^T H_k = H_k$, then

$$g_\psi(y^k,\psi^k)^T d_\psi = -g_\psi(y^k,\psi^k)^T H_k g_\psi(y^k,\psi^k) = -g_\psi(y^k,\psi^k)^T H_k^T H_k g_\psi(y^k,\psi^k)$$

$$= -\left\| H_k g_\psi (y^k, \psi^k) \right\|_2^2 = -\left\| d_\psi \right\|_2^2 \le .$$

The above analysis means $B_b d_\psi = 0$ and $g_\psi (y^k, \psi^k)^T d_\psi < 0$ when $d_\psi \neq 0$.

Theorem 3 If $g_{(y,\psi)} (y^k, \psi^k)^T d = g_y (y^k, \psi^k)^T d_y + g_\psi (y^k, \psi^k)^T d_\psi \neq 0$, where $d = \begin{pmatrix} d_y \\ d_\psi \end{pmatrix}$, then d is a descent and feasible direction.

Proof According to the conclusion of theorem 1 and theorem 2, d is a feasible direction. Because $d = 0$ is feasible solution of (8), $g_y (y^k, \psi^k)^T d_y \le 0$. With

$$g_\psi (y^k, \psi^k)^T d_\psi == -\frac{1}{\beta} \left\| d_\psi \right\|_2^2 \le 0 ,$$

from the proving process of theorem 2,

$$g_{(y,\psi)} (y^k, \psi^k)^T d = g_y (y^k, \psi^k)^T d_y + g_\psi (y^k, \psi^k)^T d_\psi \le 0 .$$

If $g_{(y,\psi)} (y^k, \psi^k)^T d \neq 0$, then $g_{(y,\psi)} (y^k, \psi^k)^T d < 0$, which means d is a descent and feasible direction.

Theorem 4 Considering (6), do linear search with d^k at $\begin{pmatrix} y^k \\ \psi^k \end{pmatrix}$, the optimal step size is $\bar{\beta} = \min \{ \beta_1, \beta_2 \}$, where $\beta_1 = \min \left\{ \min_{a \in A' \cup A''} \{ u_a - l_a \}, \min_{a \in A \setminus A' \cup A''} \{ y_a^k - l_a, u_a - y_a^k \} \right\}$,

$$\beta_2 = \min_{\left\{ j \left| (B_{nb} d_k)_j > 0 \right. \right\}} \left\{ \frac{\left(b_{nb} - B_{nb} \psi^k \right)_j}{\left(B_{nb} d_k \right)_j} \right\} . \quad (11)$$

Proof (1) When $y_a^k = l_a, \forall a \in A'$, $l_a \le y_a^k + \beta d_{y_a} \le u_a$,then $0 \le \beta d_{y_a} \le u_a - l_a$;If $d_{y_a} = 0$, $\beta \ge 0$;if $d_{y_a} = 1, 0 \le \beta \le u_a - l_a$.

When $y_a^k = u_a$, $\forall a \in A''$, $l_a \le y_a^k + \beta d_{y_a} \le u_a$, then $l_a - u_a \le \beta d_{y_a} \le 0$;if $d_{y_a} = 0$, $\beta \ge 0$;if $d_{y_a} = -1, 0 \le \beta \le u_a - l_a$.

When $l_a < y_a^k < u_a$, $\forall a \in A \setminus A' \cup A''$, $l_a \le y_a^k + \beta d_{y_a} \le u_a$, then $l_a - y_a^k \le \beta d_{y_a} \le u_a - y_a^k$; if $d_{y_a} = 1, 0 \le \beta \le u_a - y_a^k$;if $d_{y_a} = -1, 0 \le \beta \le y_a^k - l_a$.

Conclude the above analysis, we can receive

$$0 \le \beta \le \min \left\{ \min_{a \in A' \cup A''} \{ u_a - l_a \}, \min_{a \in A \setminus A' \cup A''} \{ y_a^k - l_a, u_a - y_a^k \} \right\} = \beta_1 .$$

(2) Because $B_b \psi^k = b_b$, $B_{nb} \psi^k < b_{nb}$, then

$B_b(\psi^k + \beta d_\psi) \le b_b, \beta B_b d_\psi \le 0$.

Due to $B_b d_\psi$ =0, β is free.When $B_{nb}(\psi^k + \beta d_\psi) \le b_{nb}$, then $\beta B_b d_\psi \le b_b - B_b \psi^k$. If $\left(B_{nb} d_\psi\right)_j \le 0$, β >0. If $\left(B_{nb} d_\psi\right)_j > 0$, then $\beta \le \frac{\left(b_{nb} - B_{nb}\psi^k\right)_j}{\left(B_{nb} d_\psi\right)_j}$.

So $0 \le \beta \le \min_{\left\{ j \middle| \left(A_{nb} d_k\right)_j > 0 \right\}} \left\{ \frac{\left(b_{nb} - B_{nb}\psi^k\right)_j}{\left(B_{nb} d_\psi\right)_j} \right\} = \beta_2$.

With the conclusion of (1)and (2), we can receive $0 \le \beta \le \min\{\beta_1, \beta_2\} = \bar{\beta}$.

Owing to theorem 1-4, a feasible and descent direction scheme for SCNDP is established in the following steps.

Step 1. Set initial parameters $\begin{pmatrix} y^1 \\ \psi^1 \end{pmatrix}$, $k = 1$.

Step 2. Solve (2) and let h^k be the solution, $f^k = \Delta h^k$.

Step 3 According to (4), $\nabla_{(y,\psi)} f$ is obtained. Compute

$$g_{(y,\psi)}(y^k, \psi^k) = \left(g_y(y^k, \psi^k)^T, g_\psi(y^k, \psi^k)^T \right)^T \in \partial Z(y^*, \psi^*)$$

with (7) and $d^k = \begin{pmatrix} d_y^{\ k} \\ d_\psi^{\ k} \end{pmatrix}$ with (9),（10）.

Step 4. If $g_{(y,\psi)}(y^k, \psi^k)^T d^k = 0$, then stop, y^k, ψ^k, f^k is optimal solutions; otherwise continue.

Step 5 Compute optimal step size $\bar{\beta}$ with (11). Let $\begin{pmatrix} y^{k+1} \\ \psi^{k+1} \end{pmatrix} = \begin{pmatrix} y^k \\ \psi^k \end{pmatrix} + \bar{\beta} \begin{pmatrix} d_y \\ d_\psi \end{pmatrix}, k = k + 1,$ and then go to step 2.

4. Numerical Calculations

In this section, numerical computations are conducted by feasible and descent direction method in signal-controlled network where example network is shown in Figure 1. In this traffic network, the capacity of links 1-4 need adjustment and the green light proportions of intersections 2, 4, 5, 6, 8 need to be assigned. Computational results with two kinds of initial parameters are concluded in Table 1, Table 2.

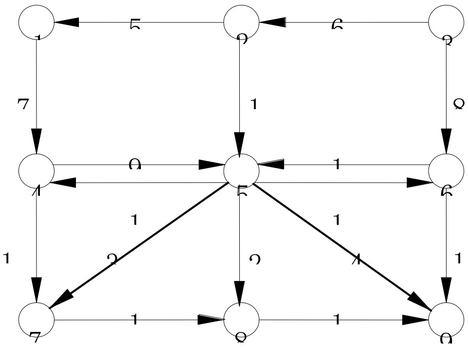

Figure 1.Testing Network

Table 1. Computational Results for Example Network with Initial Sets of
$\xi_{\min} = 75, \xi_{\max} = 100, \; g_{jm} = 15, \; \bar{c}_{jlm} = 3, l_a = 0, u_a = 10$

Variables	ζ	θ 12	Φ 12	θ 14	Φ 14	θ 15	Φ 15	θ 16	Φ 16	θ 18	Φ 18	y_1	y_2	y_3	y_4
η =1	83	17	29	21	24	13	17	23	21	19	27	8.7	6.7	4.5	4.2
η =2	85	18	31	23	24	15	18	25	22	20	27	8.3	5.8	4	4.2
η =4	88	18	31	24	25	14	17	25	24	21	29	7.7	5.5	3.3	3.1
η =8	87	20	32	24	23	16	21	24	26	21	30	7.1	4.9	3.3	2.9
η =16	90	21	30	27	24	16	23	27	27	23	31	6.3	4.3	2.9	2.3
η =32	91	21	31	26	25	17	24	28	27	22	31	4.9	3.5	2.1	1.7

Table 2. Computational Results for Example Network with Initial Sets of
$\xi_{\min} = 90, \xi_{\max} = 120, \; g_{jm} = 20, \bar{c}_{jlm} = 3, l_a = 0, u_a = 15$

Variables	ζ	θ 12	Φ 12	θ 14	Φ 14	θ 15	Φ 15	θ 16	Φ 16	θ 18	Φ 18	y_1	y_2	y_3	y_4
η =1	108	23	31	22	27	19	21	25	28	21	30	13.8	12.7	7.9	9.1
η =2	111	23	34	23	26	19	23	27	27	23	31	13.1	12.3	6.3	7.9
η =4	115	25	32	23	28	21	20	26	29	22	33	12.7	11.9	5.1	7.2
η =8	115	27	35	24	31	20	25	27	30	25	34	11.9	11.1	4.9	5.8
η =16	116	29	36	23	29	21	23	28	31	26	33	11.3	10.8	3.1	4.7
η =32	118	29	37	25	30	22	27	29	31	27	34	10.5	10.3	2.4	3.6

As it observed in experiments result, with the increasing of η, investing cost become higher which leads to deceasing of link capacity expansions. Feasible and descent scheme for SCNDP receives promising results and shows inspiring character of solving equilibrium network design problem in asymmetric signal- controlled traffic road network.

5. Conclusions

This paper presents SCNDP based on UE as an MPEC program. A feasible and descent scheme based on the generalized gradient is proposed to effectively search for optimal solution. In each iteration, the feasible and descent can be easily concluded by sign judgment and the optimal step size can be receive by comparison operation which make algorithm possess high efficiency. Numerical experiments are conducted on example network, where good performance shown in solving SCNDP.

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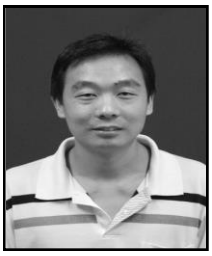
References

[1] Q. Meng, H. Yang and M.G.H. Bell, "An Equivalent Continuously Differentiable Model and a Locally Convergent Algorithm for the Continuous Network Design Problem", Transportation Research Part B, vol.35, no. 1, **(2001)**, pp. 83-105.
[2] Z.Q. Luo, J. S. Pang and D. Ralph, "Mathematical Programs with Equilibrium Constraints", Cambridge University Press, Cambridge, UK, 1996.
[3] J.X. Ban, H.X. Liu, M.C. Ferris and B. Ran, "A General MPCC Model and Its Solution Algorithm for Continuous Network Design Problem", Mathematical and Computer Modeling, vol. 43, no. 5, **(2006)**, pp. 493-505.
[4] M. Abdulal and L. LeBlanc, "Continuous Equilibrium Network Design Models", Transportation Research Part B, vol. 13, no. 1, **(1979)**, pp. 19-32.
[5] H. Yang and S. Yagar, "Traffic Assignment and Signal Control in Saturated Road Networks", Transportation Research Part A, vol. 29, no. 2 **(1995)**, pp. 125-139.
[6] E. Cipriani and G. Fusco, "Combined Signal Setting Design and Traffic Assignment Problem", European Journal of Operational Research, vol. 155, no. 3, **(2004)**, pp. 569-583.
[7] C. H. Chou and J. C. Teng, "A Fuzzy Logic Controller for Traffic Junction Signals", Information Sciences, vol. 143, no. 1, **(2002)**, pp. 73-97.
[8] H. Ceylan and M.G.H. Bell, "Traffic Signal Timing Optimization Based on Genetic Algorithm Approach", Including Drivers Routing. Transportation Research Part B, vol. 38, no. 4, **(2004)**, pp. 329-342.
[9] S. W. Chiou, "A Non-smooth Model for Signalized Road Network Design Problems", Applied Mathematical Modelling, vol. 32, no. 7, **(2008)**, pp. 1179-1190.
[10] M. Patriksson, "Sensitivity Analysis of Traffic Equilibria", Transportation Science, vol. 38, no. 3, **(2004)**, pp. 258-281.

Authors

Aihua Luo, a lecturer in the School of Mathematics and Statistics, South-Central University for Nationalities, Wuhan, China. She obtained her PhD in applied mathematics from Wuhan University in 2012. Her research work has centered on optimization theory and methods.



Zhengshun Ruan, a vice professor in the School of Science, Wuhan Institute of Technology, Wuhan, China. His research fields are carbon emission reduction, clean development etc.