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Variational Bayesian Inference Based Image Inpainting using Gamma Distribution Prior

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Abstract

Variational Bayesian (VB) inference is the latest iterative method for prediction of data in machine learning. It provides the solution for intractable integration in Bayesian methodology. In this paper, a simple VB linear regression is applied for prediction of the damaged pixels in an image. Bayesian linear regression model is used for prediction of the pixels. For this neighbor pixels are used as training data to generate the parameters of the prediction function. Now using this prediction function, damaged pixels are predicted and incorporated into the image. Proposed method is linear while image is a non-linear object, generally. Hence, for linearity, a small image window size is used to avoid the nonlinearities in image.

Keywords: *Bayesian linear regression, Variational approximation, Gamma Distribution, Image Inpainting*

1. Introduction

Image restoration is a new and very important topic of research. It is also referred as inverse problem in image processing [1]. Image inpainting is a part of the image restoration in which lost or missing pixels are restored using an algorithm. Image restoration helps in providing better quality images which are degraded by some means like during communication.

Image inpainting removes the effects of the broken and missing portions. Image inpainting was first introduced by Bertalmio, Sapiro and Caselles [2]. In this method, A partial differential equations method is used for the image inpainting. Area of a contour is used to repair and filling purpose. This method was proposed to provide the continuity for the linear structure. If this method is applied on a nonlinear image then the image is blurred. Problem of this method is overcome by Alexandru [3]. Fast Marching Method (FMM) was proposed as an extension of the method used in [2]. This method was simple, practical and efficient as compared to the previous method. This method can filled small scratches, crack and texts but fails to deliver when applied on large cracks. Another diffusion based algorithm is used by Chan and Shen [4]. Here, a curvature-driven diffusions (CDD) based method is used for non-texture inpainting. Tauber *et al.*, gave an overview of the image inpainting and defined that image inpainting could also provide a framework for increasing the accuracy of the depth recovery of an image [5]. Recently, Liangtian *et al.*, proposed a method for wavelet frame based image inpainting [6]. In this inpainting, an iterative method is used which is based on support detection based split Bergman method. Recently, Tijana and Aleksandra used an algorithm based upon context-aware for patched based image inpainting [7]. In this, Markov random field modeling is used to use textural descriptors for improvement and guide the process of inpainting.

In this paper, a latest iterative method of image inpainting is introduced which is known as Variational Bayesian inference. It uses an approximate analysis which is faster

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as compared to classical methods. Chantas *et al.*, used Variational Bayesian method in image restoration using product of t-distributions image prior [8]. They degraded the image by blur and adding noise. Now a variational methodology is applied for restoring the blurred image. Babacan *at al.*, used the variational Bayesian method with total variation (TV) prior for blind deconvolution and parameter estimation of an image [9]. For this, a hierarchical Bayesian model is used to insert the noise in image and then restored the image solving as inverse problem. A sparse kernel based blind image deconvolution (BID) is proposed by Tzikas *et al.*, using VB inference. In this methodology, student's-t distribution is used as prior information for image restoration [10].

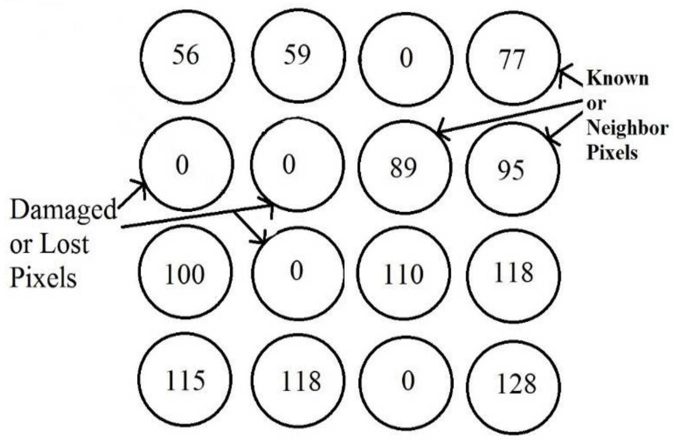

Figure 1. Lost and Neighbor Pixels

VB inference is a fully automatic algorithm for prediction. Prior distribution can change the accuracy of the algorithm. For instance, Chantas *et al.*, used another prior with same variational algorithm which is used in [8], but this time, the prior is based on products of the spatially weighted total variations [11]. VB technique is also used in other applications of image processing. For example, this method is also used in ensemble registration of the multisensor images by Hao Zhu *et. al.,* [12]. He used an infinite Gaussian mixture model (IGMM) for modeling of the joint intensity scatter plot (JISP) and Bayesian clustering for ensemble registration. IGMM is a combination of a Dirichlet process (DP) and a joint Gaussian mixture model. Recently, VB approach is applied on subspace optimization using TV priors by Zheng *et. al.*, [13]. They show how the VB method can be efficient and converges for large computational problems. In proposed method, VB inference technique is used for image inpainting. This type of technique, to the best of authors' knowledge, is not used for image inpainting before. In image inpainting, only knowledge of the neighbor pixels is available to reconstruct the lost pixels as shown in Figure 1. With the help of neighbor pixels, a function, known as regression function, is generated.

In this paper, missing terms are predicted using neighbor pixels with the help of VB inference. Here, Bayesian linear regression model is used for prediction. A fast marching method is used for comparison purpose. The remaining paper is arranged as: in Section 2, fast marching method is reviewed. In Section 3, the basics of Variational Bayes are given. In Section 4, Bayesian linear regression model, used for image inpainting, is given. In Section 5, the experiments are performed and shown the results on images. Finally, in Section 5, we provide conclusions and direction for future research work.

2. Review of Fast Marching Method

Fast marching method (FMM) is an algorithm in which Eikonal equation is solved for inpainting which is given as

$$|\nabla T| = 1 \text{ on } \Omega, \quad \text{with} \quad T = 0 \text{ on } \partial\Omega \tag{1}$$

where Ω represents pixels and $\partial\Omega$ represents boundary of the pixels. T is known as distance map of the pixels. In this method, value of T is stored with image gray value I and flag f. This is done in five steps. In first step, band point for smallest T is extracted. In second step, marching of the boundary inward is done by the addition of new points in it. In third step, inpainting takes place. In the fourth step, propagation of the values from point (i, j) to the neighbor point (k, l) takes place which is given by the solution of (1) as

$$\max(D^{-x}T, -D^{+x}T, 0)^2 + max(D^{-y}T, -D^{y}T, 0)^2 = 1 \tag{2}$$

where $D^{-x}T(i,j) = T(i,j) - T(i-1,j)$ and $D^{+x}T(i,j) = T(i+1,j) - T(i,j)$. Similarly for y can be calculated. In final step, value of new (k, l) are inserted with its new T [3].

3. Elements of Variational Bayes

Variational Bayes method is based upon the variational approximation of the posterior density function which is given by the Bayes' rule

$$p\ (x \mid y) = \frac{p\ (y,x)}{p\ (y)} = \frac{p\ (y \mid x) p\ (x)}{p\ (y)} \tag{3}$$

where $x \in \varphi,$ and y represents the parameter vector and observed data vector, respectively. Numerator term in middle is known as joint distribution function. Denominator is known as marginal likelihood or normalizing function as it ensures that the posterior density function is a probability density function. First and second term in numerator of right side represents the likelihood function and prior distribution, respectively.

Let assume an arbitrary density function q over φ. Now the marginal likelihood satisfies the condition $p\ (y) \geq p(y; q)$ where

$$\mathcal{L}\ (q) = p\ (y; q) \equiv exp \int_{\varphi} q\ (x) ln \left\{\frac{p\ (y,x)}{p\ (y)}\right\} dx \tag{4}$$

is known as lower bound. Kullback-Leibler divergence is defined as the difference between log $\{p\ (y)\}$ and log $p\ (y; q)$. When the lower bound is maximized then the KL divergence is minimized. KL divergence is minimized using posterior density function as$q_{exact}(x) = p\ (x \mid y)$. However, the calculation of $q_{exact}(x)$ is intractable and cannot be calculated directly for most of the models. Some type of restrictions on q can make it tractable. If an appropriate choice of q is chosen which can maximized the lower bound then the KL divergence is reduced to minimum value. Variational Bayes uses the restrictions by product density restrictions:

$$q\ (x) = \prod_{i=1}^{M} q_i\ (x_i) \tag{5}$$

This approximate form is based upon the mean field theory in physics. Now we are to select an appropriate distribution for q which maximizes the lower bound.

On substituting (3) into (2) gives the solution as

$$ln\, q_i^*\,(x_i) = \mathbb{E}_{-x_i}\{ln\, p\,(y, x)\} + const. \quad (6)$$

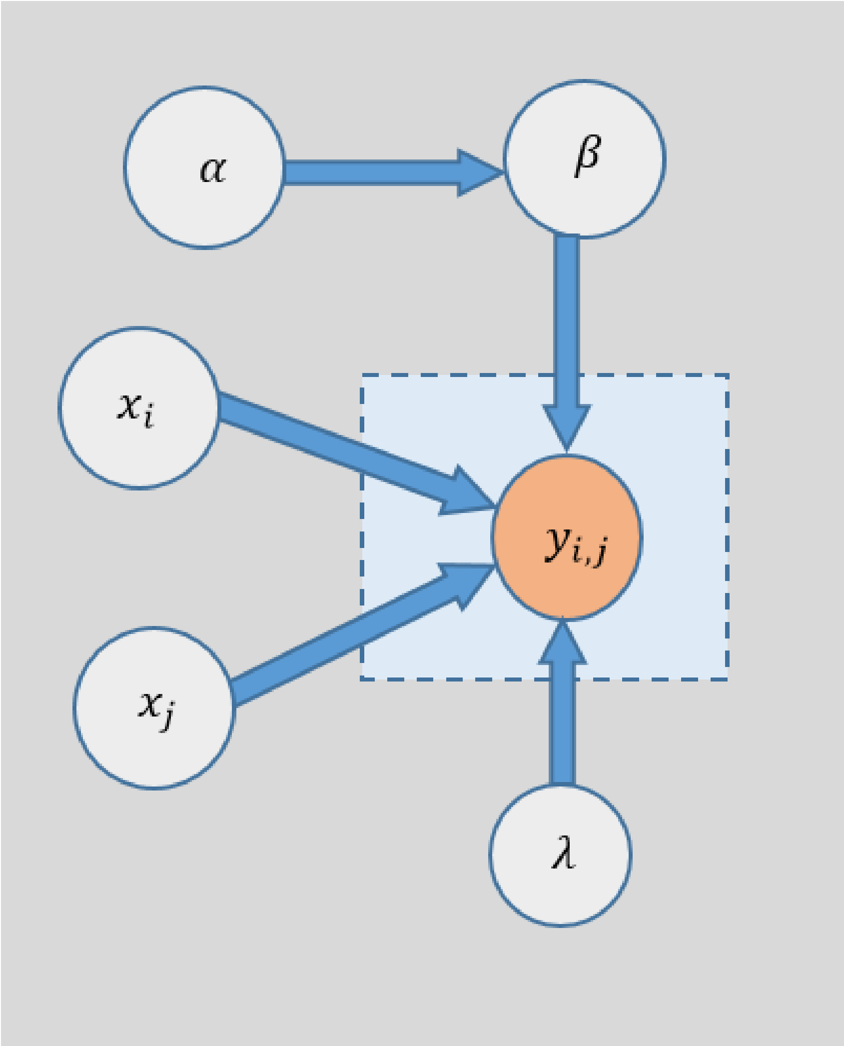

Figure 2. Graphical Presentation of Proposed Bayesian Linear Regression Model [17]

where $\mathbb{E}_{-x_i}$ represents the expectation for the density related to $\prod_{j\neq i} q_j\,(x_j)$. (4) defines the condition which maximizes the lower bound with respect to (3). In variational method the lower bound is calculated and then it is updated by iteration. In each iteration the lower bound is maximized as a result the KL distance is minimized [14-16].

Algorithm: VB Inference based Image Inpainting

1. Initialize algorithm with initial parameter values of $a_0, b_0, c_0,\ d_0,\ m_R$ and λ_R
2. Update the values of a_0, b_0, c_0 and d_0 using following expressions

$$a_R = a_0 + \frac{C}{2},$$

$$b_R = b_0 + \frac{1}{2}\{m_R m_R^T + tr(\lambda_R^{-1})\},$$

$$c_R = c_0 + \frac{R}{2},$$

$$d_R = d_0 + \frac{1}{2}\left[(\mathbf{Y} - \mathbf{X} * m_R)' * (\mathbf{Y} - \mathbf{X} * m_R) + tr\left\{\frac{X'X}{\lambda_R}\right\}\right]$$

3. Update the values of m_R and λ_R using following expression

$$m_R = \frac{c_R}{d_R}\left\{\frac{\mathbf{X'Y}}{\lambda_R}\right\},$$

$$\lambda_R = \frac{a_R}{b_R} + \frac{c_R}{d_R}(\mathbf{X}' * \mathbf{X}),$$

4. Use the updated parameter values in prediction of missing or lost pixels.

$$Y_{new} = \frac{1}{\frac{d_R}{c_R} + diag\left\{\frac{\lambda_R}{X_{new}} * X'_{new}\right\}}$$

4. Bayesian Linear Regression Model

In proposed method, a multiple linear regression model with two independent variables is used. The predictor values, x_i and x_j, are two independent variables while $y_{i,j}$ is the response variable, thus the regression model is

$$y_{i,j} = \beta_0 + \beta_1 x_i + \beta_2 x_j + \varepsilon_{i,j} \tag{7}$$

where β_0, β_1 and β_2 are the parameters and $\varepsilon_{i,j}$ is independent additive noise. The values of i and j are varies as $1 \le i \le R$ and $1 \le j \le C$, respectively. Here $R \times C$ is the size of the window used for prediction.

Let us assume a prior distribution function α over the parameter β for Bayesian approach. Now defining the matrices as

$$\mathbf{X} = \begin{bmatrix} 1 & x_1 & x_1 \\ \vdots & \vdots & \vdots \\ 1 & x_1 & x_C \\ 1 & x_2 & x_1 \\ \vdots & \vdots & \vdots \\ 1 & x_R & x_C \end{bmatrix}, \qquad \mathbf{Y} = \begin{bmatrix} y_{1,1} \\ \vdots \\ y_{1,C} \\ y_{2,1} \\ \vdots \\ y_{R,C} \end{bmatrix}, \qquad \beta = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{bmatrix}$$

and the model as

$$y_{i,j} | x_i, x_j, \beta, \lambda^{-1} \sim N((\mathbf{X}\beta)_{i,j}, \lambda^{-1})$$

$$\beta\, |\alpha \sim N(0, \alpha^{-1} I_{C\times C}),$$

$$\varepsilon \sim N(0, \lambda^{-1}),$$

$$\alpha \sim Ga(a_0, b_0),$$

$$\lambda \sim Ga\,(c_0, d_0). \tag{8}$$

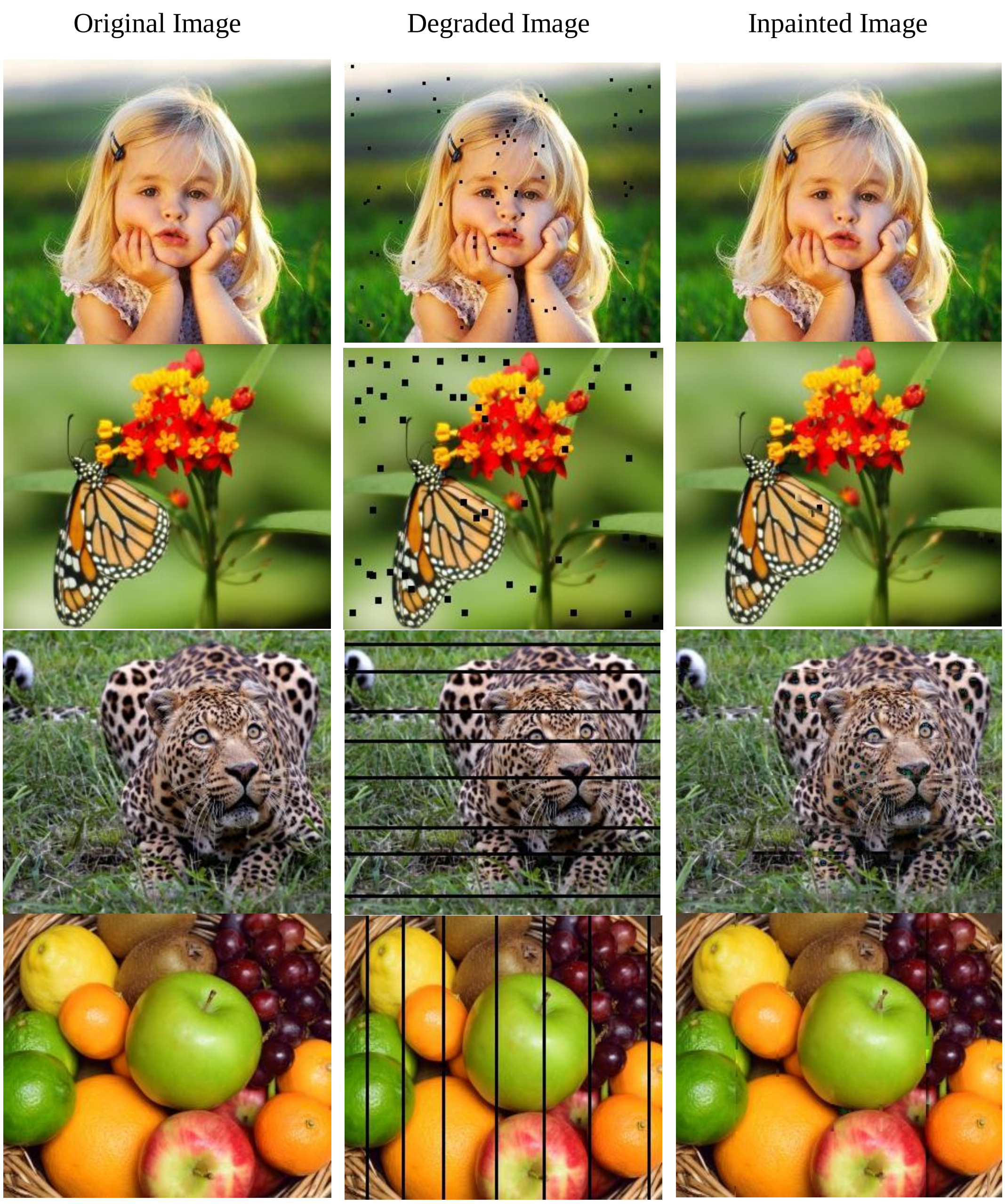

Figure 3. Original, Degraded and Inpainted Image

where $N(\cdot\cdot,\cdot\cdot)$ and $Ga\ (\cdot\cdot,\cdot\cdot)$ represents the normal distribution and gamma distribution respectively. α and λ are known as precision of parameters and noise precision respectively. Now the joint distribution is given by the expression

$$p\ (y, \beta, \alpha) = p\ (y|\ \beta)\ p\ (\beta|\ \alpha)\ p\ (\alpha) \tag{9}$$

This model can be represented using a graphical model as shown in Figure 2 [17-19].

Table 1. ISNR and PSNR Results

Object	PSNR of degraded image (dB)	PSNR of inpainted image (dB)	ISNR (dB)
Girl	26.2386	46.1418	0.2452
Butterfly	21.6256	33.3834	1.0966
Tiger	17.2412	24.9861	0.5628
Fruits	17.6752	32.5319	0.2524

5. Variational Approximation and Results

Now, Variational method is applied for inpainting of the images by predicting of the missing pixels. As described in Section 2, an approximate distribution, q, is chosen for the approximation of posterior. Now using (4), α and β can be approximated as gamma distribution $q^*(\alpha) = Ga(a_R, b_R)$ and Gaussian distribution $q^*(\beta) = N(m_R, S_R)$, respectively. The results are used by factorization of the variational distribution which is given as

$$q^* (\beta, \alpha) = q^*(\beta) q^*(\alpha) \tag{10}$$

Algorithm can be initialized by initializing the parameters of $q(\alpha)$ or $q(\beta)$. In proposed method, a small window size of $R \times C$ uses to generate the regression function using the known pixels and then predicts lost pixels of that window using this regression function. The algorithm is initialized with parameter values a_0, b_0, c_0 and d_0 to 3, 0.4, 25 and 2 respectively. While, m_R and λ_R are initialized with column matrix of C zeros and identity matrix of C ones respectively. Now for new values X_{new}, predicted values Y_{new} are found using model parameters.

Mathematical results, for inpainted images of Figure 3, are given in Table 1. All images are of the size of 250 x 300 pixels. Algorithm have been applied to four different degraded images and inpainted. Peak signal to noise ratio (PSNR) and improved signal to noise ratio (ISNR) parameters use for checking the improvement of images. PSNR and ISNR are given by the expressions

$$PSNR = 20 log_{10} \frac{255}{MSE}$$

$$ISNR = 10\ log_{10} \frac{\|f-g\|^2}{\|f-h\|^2} \tag{11}$$

where MSE is mean square error, f is original image, g is degraded image and h is inpainted image. ISNR value for girl is very less as compared to that of butterfly. This is due to the amount of recovered pixel numbers. For girl image, there is less recovery and hence ISNR is low. For butterfly image, recovered pixels are more and hence ISNR is high. When image is degraded heavily, as for tiger and fruits images, then ISNR is low. This is due to the lack of sufficient neighbor pixels information while for butterfly image, there were sufficient pixels to recover the missing pixels. Hence, ISNR depends on both missing and neighbor pixels. Above experimental results make motivation for the practical use of algorithm. Proposed methodology is also applied on two practical images in Figure 4 and compared with an existing method of [3]. In [3], a Fast Marching Method (FMM) is used for image inpainting purpose which is reviewed in Section 2.

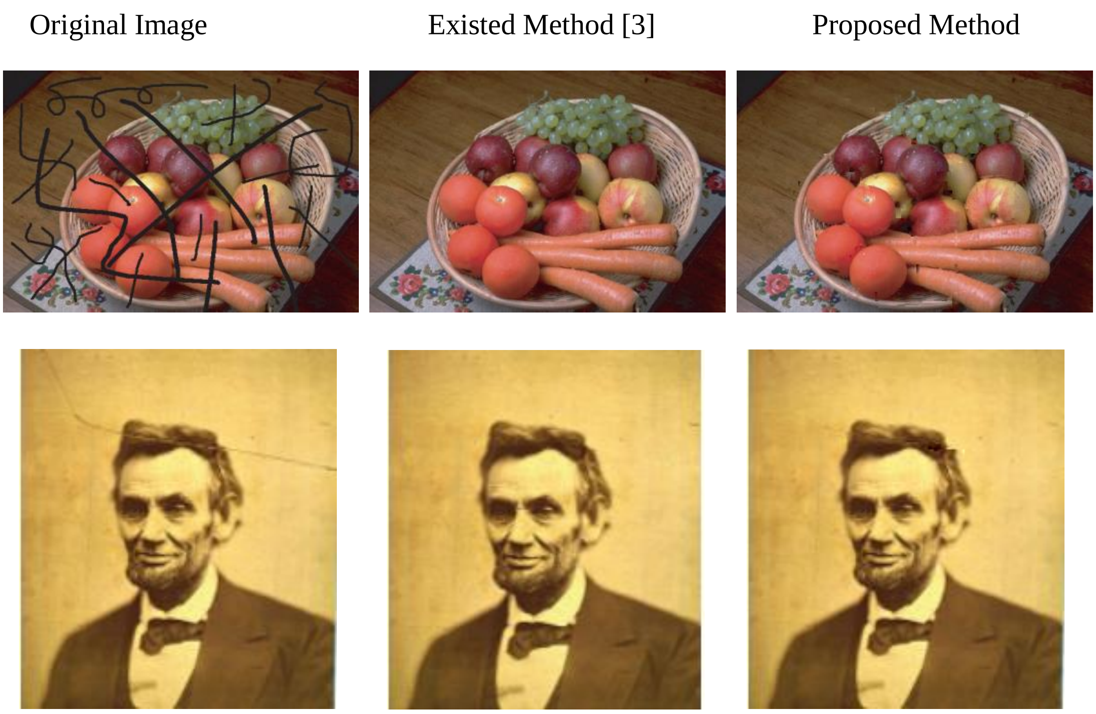

Figure 4. Comparison of Proposed Method with Existed Method

6. Conclusion

A latest approach of VB inference is applied for image inpainting. The images are restored appropriately using algorithm, termed as, VB inference based image inpainting. This is an iterative algorithm which gives guaranty for convergence. In Table 1, the PSNR and ISNR measure the performance of the restored images. Resultant images, given in Figure 3, shows the effectiveness of the algorithm. Proposed method shows the applications for practical images as shown in Figure 4.

In this paper, gamma distribution is used as prior. For further research, another type of prior can be implemented to improve the accuracy and performance of the algorithm. However, this algorithm is implemented for small window size as it is a linear approach while images are nonlinear generally.

A window of size 10x10 has been used to avoid non-linearity of image. A linear approach limits its applicability for non-linear windows and for other non-linear applications. Hence, in future, suitable modifications in algorithm can be explored for use in nonlinear window.

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