# An Improved Delay-Limited Source and Channel Coding of Quasy-Stationary Sources over Block Fading Channels: Design and Scaling Laws

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#### Abstract

In this paper, delay-limited transmission of Quasi-stationary sources over block fading channels is considered. Considering distortion outage probability as the performance measure, two source and channel coding schemes with power adaptive transmission are presented. The delay limited transmission of a quasi-stationary source over a block fading channel is from the perspective of source and channel coding design and it utilizes performance scaling laws. Because In a Quasi-stationary sources time utilization takes a major role. For that the first one is optimized for fixed rate transmission, and hence enjoys simplicity of implementation. The second one is a high performance scheme, which also benefits from optimized rate adaptation with respect to source and channel states. In high SNR regime, the performance scaling laws in terms of outage distortion exponent and asymptotic outage distortion gain are derived, where two schemes with fixed transmission power and adaptive or optimized fixed rates are considered as benchmarks for comparisons. Various analytical and numerical results are provided which demonstrate a superior performance for source and channel optimized rate and power adaptive scheme.

**Keywords:** Delay limited transmission, quasi-stationary source, Channel coding, fading channel

#### 1. Introduction

Low-complexity cooperative diversity protocols that combat fading induced by multipath propagation in wireless networks. The underlying techniques exploit space diversity available through cooperating terminals' relaying signals for one another. We outline several strategies employed by the cooperating radios, including fixed relaying schemes such as amplify-and-forward and decode-and-forward, selection relaying schemes that adapt based upon channel measurements between the cooperating terminals, and incremental relaying schemes that adapt based upon limited feedback from the destination terminal. We develop performance characterizations in terms of outage events and associated outage probabilities, which measure robustness of the transmissions to fading, focusing on the high signal-to-noise ratio (SNR) regime. Cooperative diversity is a cooperative multiple antenna technique for improving or maximizing total network channel capacities for any given set of bandwidths which exploits user diversity by decoding the combined signal of the relayed signal and the direct signal in wireless multi hop networks. A conventional single hop system uses direct transmission where a receiver decodes the information only based on the direct signal while regarding the relayed signal as interference, whereas the cooperative diversity considers the other signal as contribution. That is, cooperative diversity decodes the information from the combination of two signals. Hence, it can be seen that cooperative diversity is an antenna diversity that

ISSN: 2005-4254 IJSIP Copyright © 2015 SERSC uses distributed antennas belonging to each node in a wireless network. User cooperation considers an additional fact that each user relays the other user's signal while cooperative diversity can be also achieved by multi hop relay networking systems. The aim is to find the optimized power allocation strategy and fixed rate such that the distortion outage probability for communication of a quasi-stationary source over a wireless fading channel is minimized. With a fixed rate (*R* does not change from one block to another), the encoders do not need to be rate adaptive which simplifies the design and implementation of transceivers.

## 2. Model System

We consider the transmission of a quasi-stationary source over a block fading channel. Specifically, the source is finite state quasi-stationary Gaussian with zero mean and variance  $\sigma_2$  s in a given block, where  $s \in S : S = \{1, 2, ..., Ns\}[18]$ . The source state s from the set S is a discrete random variable with the probability mass function (pmf) P(s). The source coding rate in a block in state s, is denoted by Rs bits per source sample. Hence, according to the distortion-rate function of a Gaussian source [18-19], the instantaneous distortion in a block in state s is given by  $D = \sigma 2s 2-2Rs$ . We consider a point to point wireless block fading channel for transmitting the source information to the destination. Let X, Y and Z, respectively indicate channel input, output and additive noise, where Z is an i.i.d Gaussian noise  $\sim N(0, 1)$ . Therefore, we have  $Y = \sqrt{\alpha}X + Z$ , where is the multiplicative fading. The channel gain  $\alpha$  is constant across one block and independently varies from one block to another according to the continuous probability density function  $f(\alpha)$ . For a Rayleigh fading channel,  $\sqrt{\alpha}$  is a Rayleigh distributed random variable and consequently, the channel gain  $\alpha$  is an exponentially distributed random variable, where we here consider  $E[\alpha] = 1$ . The block diagram of the system is depicted in Figure 1. We consider K source samples spanning one source block coded into a finite index by the source encoder. This index is transmitted in N channel uses spanning one fading block (bandwidth expansion ratio b = NK, where  $b \ge 1$ ). We assume that K and N are large enough such that, over a given state of source and channel, the rate distortion function of the quasi stationary source and the instantaneous capacity of the block fading channel may be achieved. The source coding rate Rs in bits per source sample and channel coding rate R in bits per channel use are related by Rs= bR. Note that in general Rs and R may be both designed to depend on source and channel states, i.e., Rs= Rs  $(\sigma s, \alpha)$ .

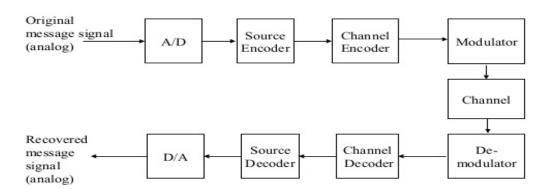


Figure 1. Block Diagram of the System

The instantaneous capacity of the fading Gaussian channel [3] over one block (in bits per channel use) is defined as:

$$C(\alpha, \gamma) = \frac{1}{2} \log_2(1 + \alpha \gamma) \tag{1}$$

The outage distortion exponent is defined as:

$$\Delta_{OD} = \lim_{\bar{P} \to \infty} -\frac{\ln P_{\text{Dout}}}{\ln P}.$$
 (2)

Let P1 and P2 be the average powers transmitted to asymptotically achieve a specific distortion outage probability by two different schemes. We define the asymptotic outage distortion gain as follows:

$$G_{OD} = 10 \log_{10} \bar{P}_2 - 10 \log_{10} \bar{P}_1.$$
 (3)

# 3. Channel Optimized Power Adaptation with Fixed Rate Source & Channel Coding

In this section, the aim is to find the optimized Power allocation strategy and fixed rate such that the distortion outage probability for communication of a quasi-stationary source over a wireless fading channel is minimized. With a fixed rate (*R* does not change from one block to another), the encoders do not need to be rate adaptive which simplifies the design and implementation of transceivers. Noting (3) the distortion outage probability is computed as follows:

$$P_{\text{Dout}} = \Pr(R > C(\alpha, \gamma)) \Pr(\sigma_s^2 > D_m) + \left(1 - \Pr(R > C(\alpha, \gamma))\right) \Pr(\sigma_s^2 2^{-2bR} > D_m).$$
(4)

The power allocation in this setting may be interpreted as water-filling with respect to the channel state and the source and channel statistics. The next two Propositions quantify the performance of the proposed COPA-MDO scheme in terms.

**Problem 1:** The problem of delay-limited channel optimized power adaptation for communication of a quasi-stationary source with minimum distortion outage probability (COPA-MDO) is formulated as the solution to Problem 1 is obtained in two steps. For a given fixed rate and no tin. Problem 2 below formulates this (sub) problem and provides the optimum power adaptation strategy as a function of R. Next, solving Problem 3, as described below, provides the optimum solution for R, and hence problem 1 is solved.

**Problem 2**: With COPA-MDO scheme and with a given fixed channel coding rate R, the power adaptation problem is formulated.

**Proposition 1:** The solution to Problem 2 for optimized power adaptation over a block fading channel is given by:

$$\gamma^*(\alpha, R) = \begin{cases} \frac{2^{2R} - 1}{\alpha} & \text{if } \alpha \ge \frac{2^{2R} - 1}{q_1^*(R)} \\ 0 & \text{otherwise,} \end{cases}$$
(5)

**Proposition 2:** Solution to Problem 3 for block fading:

$$C(\alpha, \gamma) = \begin{cases} R & \text{if } \frac{2^{2R} - 1}{\alpha} \le q_1^*(R) \\ 0 & \text{otherwise} \end{cases}$$
(6)

where  $C(\alpha, \gamma)$  is given in (1). Let Dm be a nonnegative constant and represent the maximum allowable distortion. The distortion outage probability evaluated at Dm is defined as. Whereas without channel outage, distortion is given by  $\sigma$  2 s 2 -2bR. Thus, the distortion at a given state (s,  $\alpha$ ) is equal to:

$$D(\sigma_s, \alpha, \gamma) = \begin{cases} \sigma_s^2 2^{-2bR} & \text{if } R \le C(\alpha, \gamma) \\ \sigma_s^2 & \text{if } R > C(\alpha, \gamma), \end{cases}$$

$$(7)$$

In case of a channel outage, in each state of the source and the channel  $(s, \alpha)$ , the instantaneous distortion is equal to the variance of the source and the decoder reconstructs the mean of the source.

$$\min_{\gamma,R} P_{\text{Dout}} = \Pr \left( D(\sigma_s, \alpha, \gamma) > D_m \right)$$
subject to  $E[\gamma] \le P$ ,
(8)

The distortion outage probability for transmission of a quasi-stationary source using the SCOPA-MDO scheme over a Rayleigh block fading channel is given by:

$$P_{\text{Dout}} = \Pr(\sigma_s^2 > D_m) - \sum_{s: \sigma_s^2 > D_m} \exp\left(-\frac{\left(\frac{\sigma_s^2}{D_m}\right)^{\frac{1}{b}} - 1}{q_2^*}\right) P(s)$$

$$\tag{9}$$

where  $\gamma = \gamma(\sigma s, \alpha)$  is the transmission power and we have the power constraint  $E[\gamma] \le P^-$ . The outage distortion exponent is defined as [14]  $\triangle OD = \lim P^- \rightarrow \infty - 1$ .

#### 4. Performance Evaluation

In this section, we first present two constant power transmission schemes as benchmarks for comparisons. Next, we consider analytical performance comparison of different schemes followed by numerical results. To this end, we consider four quasistationary sources, with Ns = 25 where the variance of the source in the state s is given by  $\sigma$  2 s (s) = (1 + 1 6 s) 2 :  $\forall$ s  $\in$  {0, 1, ..., NS - 1}. For three of the sources, labeled as G1, G2 and G3, the probability of being in different states follows a discrete Gaussian distribution with mean 3 and variances 0.05, 0.48, 1.07, respectively. For the fourth source, U, the said distribution is considered uniform with the same mean and a variance of 1.44. We also consider an stationary source S with  $\sigma s = 3$  for a meaningful comparison. Unless otherwise mentioned, we consider the source G2 for the following results and simulations. Two constant power schemes for transmission of a quasi-stationary source over a block fading channel are considered as benchmarks for comparisons. In the first scheme, the channel coding rate is adjusted based on the channel state to minimize the distortion outage probability; hence the scheme is labeled as Channel Optimized Rate Adaptation with Constant Power (CORACP). In the second scheme with Constant Rate and Constant Power (CRCP), the aim is to find the optimized fixed rate such that the distortion outage probability is minimized. 1) Channel Optimized Rate Adaptation with Constant Power: With CORACP and constant transmission power P<sup>-</sup>, the instantaneous capacity is given by:

$$C(\alpha, \gamma) = \frac{1}{2} \log_2(1 + \alpha \gamma), \tag{10}$$

And hence to minimize PDout it is logical to consider the rate adaptation strategy of R = C. The source coding rate is then set as  $\mathbf{Rs} = \mathbf{bR}$ . The next two Propositions quantify the distortion outage performance of CORACP. Proposition 8: The distortion outage probability for transmission of a quasi-stationary source over a Rayleigh block fading channel using CORACP is given by:

$$P_{\text{Dout}} \cong \Pr\left(\sigma_s^2 > D_m\right) \exp\left(\frac{-\bar{P}}{\left(\frac{\max\left\{\sigma_s^2\right\}}{D_m}\right)^{\frac{1}{b}} - 1}\right)$$
(11)

#### 4.1. Channel Optimized Rate Adaptation with Constant Power:

With CORACP and constant transmission power P, the instantaneous capacity is given by  $C(\alpha) = 12 \log 2_1 + \alpha P$ ; and hence to minimize PDout it is logical to consider the rate adaptation strategy of  $R(\alpha) = C(\alpha)$ . The source coding rate is then set as Rs = bR. The next two Propositions quantify the distortion outage performance of CORACP. The distortion outage probability for transmission of a quasi-stationary source over a Rayleigh block fading channel using CORACP is given by:

$$\Delta_{OD} = \lim_{\bar{P} \to \infty} \frac{\ln \bar{P} - \ln \sum_{s: \sigma_s^2 > D_m} \left( \left( \frac{\sigma_s^2}{D_m} \right)^{\frac{1}{b}} - 1 \right) P(s)}{\ln \bar{P}} = 1$$
(12)

It is evident in Proposition 10 that the optimized fixed rate R with CRCP is related to the source statistics and equals 1  $2b \log 2(\sigma 2s*Dm)$ , where s\* is given in (64). Therefore, the distortion outage probability is:

$$P_{\text{Dout}} = \Pr\left(C(\alpha) < \frac{1}{2b}\log_2(\frac{\sigma_{s^*}^2}{D_m})\right)\Pr(\sigma_s^2 > D_m) + \\ \Pr\left(C(\alpha) \ge \frac{1}{2b}\log_2(\frac{\sigma_{s^*}^2}{D_m})\right)\Pr(\frac{\sigma_s^2 D_m}{\sigma_{s^*}^2} > D_m)$$

$$\tag{13}$$

the CRCP scheme achieves the distortion outage probability of:

$$\Delta_{OD} = \lim_{\bar{P} \to \infty} -\frac{-\ln(\bar{P})}{\ln \bar{P}} = 1. \tag{14}$$

For communication of a stationary source over a Rayleigh block fading channel, the CRCP scheme achieves a distortion outage probability of:

$$P_{\text{Dout}} = 1 - \exp\left(-\frac{\left(\frac{\sigma^2}{D_m}\right)^{\frac{1}{b}} - 1}{\bar{P}}\right) \tag{15}$$

and an outage distortion exponent of  $\triangle OD$  of the order O(1).

### 4.2. Analytical Performance Comparison:

In the sequel, we quantify the respective asymptotic outage distortion gain  $G_{OD}$  of SCOPA-MDO, COPA-MDO, CORACP and CRCP for transmission of a quasi-stationary source over a block fading channel.

**Proposition3**: In transmission of a quasi-stationary source over a Rayleigh block fading channel, the asymptotic outage distortion gain obtained by scheme 1 with respect to scheme 2 (see Table 1) is given by:

$$G_{OD}^{1} = 10 \log_{10} \left( \left( \frac{\max\{\sigma_{s}^{2}\}}{D_{m}} \right)^{\frac{1}{b}} - 1 \right) - 1$$

$$10 \log_{10} \sum_{s:\sigma_{s}^{2} > D_{m}} \left( \left( \frac{\sigma_{s}^{2}}{D_{m}} \right)^{\frac{1}{b}} - 1 \right) P(s),$$
(16)

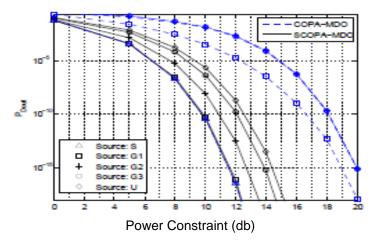


Figure 2. Distortion Outage Probability versus Power Constraint (db); b = 1 and Dm = 8Db. P

CRCP schemes give  $\Delta OD = 1$ . The distortion outage exponent indicates the speed at which the distortion outage (dB) reduces as the average power (limit) (dB) increases. Therefore, as evident, this speed is noticeably high with SCOPA-MDO and very low with CORACP and CRCP. Furthermore, the  $\Delta OD$  obtained by SCOPA-MDO and COPA-MDO depends on the average power limit P, maximum allowable distortion Dm and bandwidth expansion ratio b. It is observed that with SCOPA-MDO and COPA-MDO,  $\Delta OD$  improves as b, P or Dm increase.

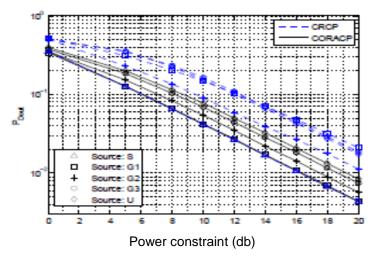


Figure 4. Distortion Outage Probability versus Provided by SCOPA-MDO and COPA-MDO for five different sources; b = 1 and Dm = 8dB

The COPAMDO scheme achieves asymptotic outage distortion gains of about 8.4dB and 6.4dB with respect to CORACP; and CORACP achieves gains of 5dB and 4.6dB with respect to CRCP.

#### 5. Conclusion

In this paper, we meet the aim of minimizing the distortion outage probability; two transmission strategies namely channel-optimized power adaptation with fixed rate (COPA-MDO) and source and channel optimized power and rate adaptation (SCOPA-MDO) were introduced. The SCOPA-MDO scheme provides a superior performance, while the COPA-MDO scheme enjoys the simplicity of single rate transmission. In high SNR regime, different scaling laws involving outage distortion exponent and asymptotic outage distortion were derived. With a fixed rate (*R* does not change from one block to another), the encoders do not need to be rate adaptive which simplifies the design and implementation of transceivers. Hence the analyses of the presented schemes in the case of stationary sources indicate the same outage distortion performance and it is improved by with or without rate adaptation scheme.

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