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# The Design of Stabilization Control Law for Mobile Robot based on Global Vision

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***Abstract***

*As the wheeled mobile robot is widely used in various fields, requirements of control accuracy for wheeled mobile robot are also increasing. Vision sensors get more and more attention because they are information capacity, high efficiency, non-contact measurement. The servo control problem of robot visual has also become a research hot spot. Dividing from the number of vision sensors, visual servo system can be divided into monocular visual servo system, binocular visual servo system and multi-purpose visual servo system.*

***Keywords:*** *nonholonomic systems, wheeled mobile robots, visual servo robust control*

## 1. Introduction

Applications of vision sensor in the field of robotics have many years of history. When the target of the robot is in unstructured, dynamic case, make the robot under the visual sensing control through the visual sensor has obvious advantages, and make level of intelligence of the robot has greatly improved[1-3]. Sanderson and Weiss classified visual servo control system structures according to the feedback approach of the visual information. The structure can be mainly divided into two categories: location-based visual servo system and image features-based visual servo system [4-6].

This paper quotes image-based visual servo control method to WMR motion control [7-8], proposed a rate control method that based on eliminating the error of image features. First, according to the pinhole model of the camera and WMR kinematic model the text deduces the relationship between actual velocity of WMR in task space and the velocity of WMR in the image space, and then do the transformation of the actual system and designs robust speed stabilization controller[9-10].

## 2. The Kinematics Model of Mobile Robot

### 2.1WMR Kinematic Model within the Tasks Space

Here we consider a typical wheeled mobile robot, shown in Figure 1.

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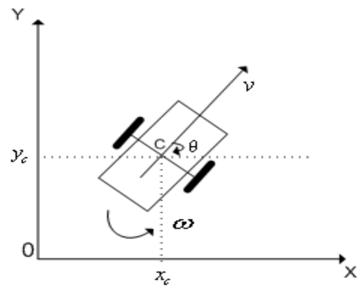


**Figure 1. Car Model of the Wheeled Mobile Robot**

The picture above shows a model of nonholonomic mobile robot, two wheel axles of the two wheels of the model coincide, the two wheels are driven by separate DC motors, caster only play a supportive role. In order to describe the movement of the robot, we establish two Cartesian coordinate system, use a point of a two-dimensional plane as the origin of the global coordinate system X-Y, and as the local coordinate system xc-yc that fixed in the nonholonomic mobile robot itself and the horizontal axis in the positive direction is always consistent with the direction the robot faces. In figure 1, C is the centroid of the robot, but also the origin of the local coordinate system. The coordinate of C in the global coordinate system (x,y) is the position of the robot, the angle θ between the positive direction of X and the positive direction of xc is the direction the robot faces, also known as direction angle.

Assume that wheels are in the case that there is only pure rolling without sliding, we can derive from the nonholonomic constraint relations of the speed and the position and orientation that[5]:

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{1}$$

In the formula, xc(t), yc(t), represent the centroid position of the robot car in the task space ,θ represents the angle between car traveling direction and the $x$-axis, $v$ represents the traveling speed of robot trolley in the task space and $\omega$ represents the rotation speed of the car in the task space.

## 2.2 Kinematic Model of WMR within the Image Space

In the global visual case, cameras are installed on the ceiling, image plane parallels to tasks plane but the two planes are in two different coordinate systems [11-12].

Servo error that is image-based visual servo system can be defined directly in the image space, that is the information characteristics the visual sensor observed can be directly used for feedback, it do not need to estimate the pose, concrete block diagram is shown in Figure 2.

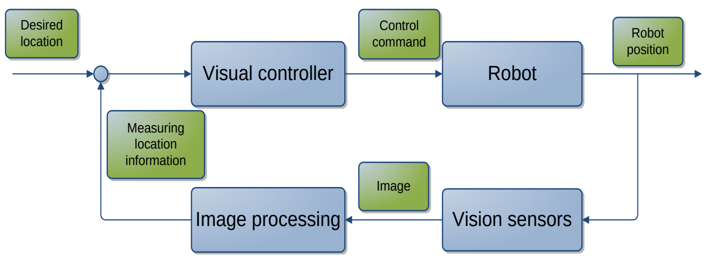


**Figure 2. Block Diagram of Visual Servo System that is Image-based**

 

Assume that the kinematics model of the robot in the image space is as follows:

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{y}}_c \\ \dot{\bar{\theta}} \end{bmatrix} = \begin{bmatrix} \cos\bar{\theta} & 0 \\ \sin\bar{\theta} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{v} \\ \bar{\omega} \end{bmatrix} \tag{2}$$

In the formula, $\bar{x}_c(t)$, $\bar{y}_c(t)$, represent the centroid position of the robot car in the image space, $\bar{\theta}$ represents the angle between car traveling direction and the $x$-axis, $\bar{v}$ represents the traveling speed of robot trolley in the image space and $\bar{\omega}$ represents the rotation speed of the car in the image space.

### 2.3 Transformation from Task Space to the Image Space

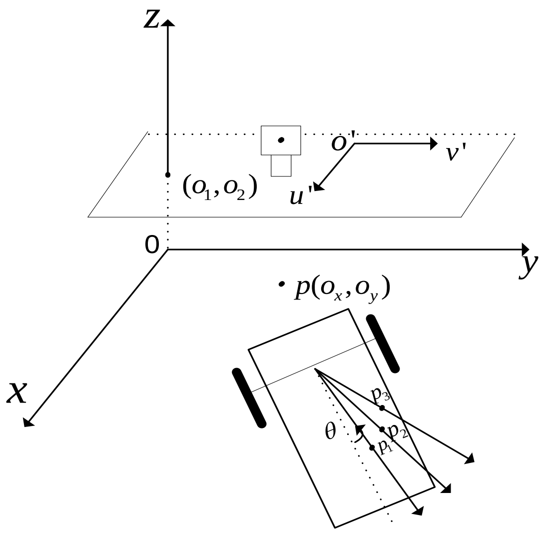


**Figure 3. Car Model of the Soccer Robot**

Mobile robot shown in the above in figure, in the figure $XYZ$ is the inertial coordinate system. p is the intersection of the optical center of the camera with the $XY$ plane, $U'OV'$ is a coordinate system of a two-dimensional image plane .

Assume that p - the optical center of the camera has a projection point in the task plane and the coordinates of projection point is $(o_x, o_y)$. In the image coordinates, the pixel coordinates of the origin point $o$ of the task space coordinates is $(o_1, o_2)$. What can obtain from the pinhole model of the camera is the pose relationship between the pose of robot car in the image space and the pose of robot car in the task space. That is:

$$\begin{bmatrix} \bar{x}_c(t) \\ \bar{y}_c(t) \end{bmatrix} = HR(\theta_0)\left[\begin{bmatrix} x_c(t) \\ y_c(t) \end{bmatrix} - \begin{bmatrix} o_x \\ o_y \end{bmatrix}\right] + \begin{bmatrix} o_1 \\ o_2 \end{bmatrix} \tag{3}$$

In the formula, the matrix H is a 2×2 constant diagonal matrix, $H = \begin{bmatrix} \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix}$, $\alpha_1 = \frac{f}{z\beta_1}$, $\alpha_2 = \frac{f}{z\beta_2}$, R is a 2×2 rotation matrix,

$$R(\theta_0) = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \tag{4}$$

$\theta_0$ is the angle between the $x$ axis in the positive of the task space and $x$ axis in the positive of the image space. Coefficient z usually called the depth; f is the focal length of the camera. $\beta_1$ Represents the physical size of pixel on the x-axis and $\beta_2$ represents the physical size of pixel on the y-axis.

In order to obtain the relationship between the line speed $\bar{v}(t)$ of robot car in the image space and the line speed $v(t)$ of robot car in the task space, do the derivation for formula (3) on both sides we can obtain that:

$$\begin{bmatrix} \dot{\bar{x}}_c \\ \dot{\bar{y}}_c \end{bmatrix} = \begin{bmatrix} v\alpha_1 \cos(\theta+\theta_0) \\ v\alpha_2 \sin(\theta+\theta_0) \end{bmatrix} \tag{5}$$

Introduce formula (2) to formula (4) and do the transformation, we can obtain that:

$$v\begin{bmatrix} \alpha_1 \cos(\theta+\theta_0) \\ \alpha_2 \sin(\theta+\theta_0) \end{bmatrix} = \bar{v}\begin{bmatrix} \cos\bar{\theta} \\ \sin\bar{\theta} \end{bmatrix} \tag{6}$$

We only consider a situation that is $\alpha_1 = \alpha_2 = \alpha$ , there we can obtain that:

$$\begin{cases} v\alpha \cos(\theta+\theta_0) = \bar{v}\cos\bar{\theta} \\ v\alpha \sin(\theta+\theta_0) = \bar{v}\sin\bar{\theta} \end{cases} \tag{7}$$

What we can know from formula (7) is:

$$v\bar{v}\alpha\left[\sin(\theta+\theta_0)\cos\bar{\theta} - \cos(\theta+\theta_0)\sin\bar{\theta}\right] = 0$$

What we can get from the arbitrariness of $v$ 、 $\bar{v}$ is:

$$\sin\left(\theta+\theta_0-\bar{\theta}\right) = 0$$

So

$$\theta+\theta_0-\bar{\theta} = k\pi \ (k \text{ is an integer}) \tag{8}$$

Thus

$$\omega = \bar{\omega} \tag{9}$$

Based on this formula we can get that:

$$\begin{cases} v\alpha \cos^2(\theta+\theta_0) = \bar{v}\cos\bar{\theta}\cos(\theta+\theta_0) \\ v\alpha \sin^2(\theta+\theta_0) = \bar{v}\sin\bar{\theta}\sin(\theta+\theta_0) \end{cases} \tag{10}$$

And then we can get that:

$$v = \frac{1}{\alpha}\bar{v}\cos\left(\theta+\theta_0-\bar{\theta}\right) \tag{11}$$

Similarly we can get that:

$$\begin{cases} v\alpha \cos(\theta+\theta_0)\cos\bar{\theta} = \bar{v}\cos^2\bar{\theta} \\ v\alpha \sin(\theta+\theta_0)\sin\bar{\theta} = \bar{v}\sin^2\bar{\theta} \end{cases} \tag{12}$$

It can be obtained:

$$\bar{v} = v\alpha\cos\left(\theta+\theta_0-\bar{\theta}\right) \tag{13}$$

To sum up: We get the transformation relationship of the amount of the velocity of the robot car from the image space to the task space, as the formula (8), (11), (12) show, and from the formula (11) and (12) we can know that $v$ and $\bar{v}$ are determined between each other.

## 3. Design of Stabilization Control Law

### 3.1 System Transformation

Introduce formula (12) to formula (2) we can get that:

$$\begin{pmatrix} \dot{\bar{x}}_c \\ \dot{\bar{y}}_c \\ \dot{\bar{\theta}} \end{pmatrix} = \begin{pmatrix} \cos\bar{\theta} & 0 \\ \sin\bar{\theta} & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} v\alpha\cos\left(\theta+\theta_0-\bar{\theta}\right) \\ \omega \end{pmatrix} \tag{14}$$

 

In the formula, $\theta+\theta_0-\bar{\theta}=k\pi$ （ $k$ is an integer）. That is:
$\cos\left(\theta+\theta_0-\bar{\theta}\right)=\pm 1$

First consider the case that $\cos\left(\theta+\theta_0-\bar{\theta}\right)=1$.

Then

$$\begin{pmatrix}\dot{\bar{x}}_c\\ \dot{\bar{y}}_c\\ \dot{\bar{\theta}}\end{pmatrix}=\begin{pmatrix}\cos\bar{\theta} & 0\\ \sin\bar{\theta} & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}v\alpha\\ \omega\end{pmatrix}$$

That is:

$$\begin{cases}\dot{\bar{x}}_c=v\alpha\cos\bar{\theta}\\ \dot{\bar{y}}_c=v\alpha\sin\bar{\theta}\\ \dot{\bar{\theta}}=\omega\end{cases} \tag{15}$$

Make that :

$$\begin{cases}z_1=\bar{x}_c\sin\bar{\theta}-\bar{y}_c\cos\bar{\theta}\\ z_2=\bar{x}_c\cos\bar{\theta}+\bar{y}_c\sin\bar{\theta}\end{cases} \tag{16}$$

So

$$\begin{cases}\dot{z}_1=\omega z_2\\ \dot{z}_2=v\alpha-\omega z_1\\ \dot{\bar{\theta}}=\omega\end{cases} \tag{17}$$

And then consider the situation $\cos\left(\theta+\theta_0-\bar{\theta}\right)=-1$.

Then

$$\begin{pmatrix}\dot{\bar{x}}_c\\ \dot{\bar{y}}_c\\ \dot{\bar{\theta}}\end{pmatrix}=\begin{pmatrix}\cos\bar{\theta} & 0\\ \sin\bar{\theta} & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-v\alpha\\ \omega\end{pmatrix} \tag{18}$$

If we make that:

$$\begin{cases}z_1=\bar{x}_c\sin\bar{\theta}-\bar{y}_c\cos\bar{\theta}\\ z_2=\bar{x}_c\cos\bar{\theta}+\bar{y}_c\sin\bar{\theta}\end{cases} \tag{19}$$

We can get that:

$$\begin{cases}\dot{z}_1=\omega z_2\\ \dot{z}_2=-v\alpha-\omega z_1\\ \dot{\bar{\theta}}=\omega\end{cases} \tag{20}$$

Without loss of generality, we assume $\cos\left(\theta+\theta_0-\bar{\theta}\right)=1$.

### 3.2 The Design of Control Law

Design $\omega$ to make index $\bar{\theta}$ stable to 0. Chose the following:

$$\sigma>0,\beta>0,k>0 \text{ and } k>\sigma \tag{21}$$

Design the control law is as follows:

$$\omega=-k\bar{\theta}+\beta e^{-\sigma t}$$

So $\dot{\bar{\theta}}=-k\bar{\theta}+\beta e^{-\sigma t}$ We can get that: $\bar{\theta}(t)=\frac{\beta}{k-\sigma}e^{-\sigma t}+\left(1-\frac{\beta}{k-\sigma}\right)e^{-kt}$

Thus

$$\omega=-\left[\frac{\beta\sigma}{k-\sigma}e^{-\sigma t}+k\left(1-\frac{\beta}{k-\sigma}\right)e^{-kt}\right] \tag{22}$$

Therefore $\bar{\theta}(t)$ index converges to zero.

The following describe how $z_1$ and $z_2$ converges to zero.

Introduce formula (18) to formula (16) we can get that:

$$\begin{cases} \dot{z}_1 = -z_2\left[\frac{\beta\sigma}{k-\sigma}e^{-\sigma t} + k\left(1-\frac{\beta}{k-\sigma}\right)e^{-kt}\right] \\ \dot{z}_2 = v\alpha + z_1\left[\frac{\beta\sigma}{k-\sigma}e^{-\sigma t} + k\left(1-\frac{\beta}{k-\sigma}\right)e^{-kt}\right] \\ \dot{\bar{\theta}} = -\left[\frac{\beta\sigma}{k-\sigma}e^{-\sigma t} + k\left(1-\frac{\beta}{k-\sigma}\right)e^{-kt}\right] \end{cases} \tag{23}$$

Set

$$\begin{cases} \bar{z}_1 = \frac{z_1}{\beta e^{-\sigma t}} \\ \bar{z}_2 = z_2 \end{cases} \tag{24}$$

Then

$$\begin{cases} \dot{\bar{z}}_1 = \sigma\bar{z}_1 - \frac{\sigma}{k-\sigma}\bar{z}_2 - \bar{z}_2\frac{k}{\beta}\left(1-\frac{\beta}{k-\sigma}\right)e^{-(k-\sigma)t} \\ \dot{\bar{z}}_2 = v\alpha + \bar{z}_1\left[\beta^2\frac{\sigma}{k-\sigma}e^{-2\sigma t} + k\beta\left(1-\frac{\beta}{k-\sigma}\right)e^{-(k+\sigma)t}\right] \end{cases} \tag{25}$$

Consider the following linear time-varying system:

$$\dot{x} = \left(A_1 + A_2(t)\right)x \tag{26}$$

if $A_1$ is steady array, and $A_2(t)$ meet the conditions as follows：

1） $\lim_{t\to\infty} A_2(t) = 0$

2） $\int_0^\infty \|A_2(t)\| < \infty$

The linear time-varying system (26) shows that global exponential stable.

Next we consider the design of control law.

Case 1: $\alpha$ is known.

Theorem 1: Selection $k$ 、 $\beta$ and $\sigma$ satisfy the equation (22), $k_1$ 、 $k_2$ are constant as follow.

$$\begin{cases} k_1 > -(k-\sigma)k_2 \\ k_2 < -\sigma \end{cases} \tag{27}$$

then

(1)matrix $\begin{pmatrix} \sigma & -\frac{\sigma}{k-\sigma} \\ k_1 & k_2 \end{pmatrix}$ is steady array;

(2)The controller (28)can guarantees the system (25) be from any initial state index converge to the origin.

$$v = \frac{1}{\alpha}\left(k_1\bar{z}_1 + k_2\bar{z}_2\right) \tag{28}$$

Proof: take formula (28) into formula (25),then

$$\begin{pmatrix} \dot{\bar{z}}_1 \\ \dot{\bar{z}}_2 \end{pmatrix} = \left(A_1 + A_2(t)\right)\begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \end{pmatrix} \tag{29}$$

Among them

 

$$A_1 = \begin{pmatrix} \sigma & -\dfrac{\sigma}{k-\sigma} \\ k_1 & k_2 \end{pmatrix}$$

$$A_2(t) = \begin{pmatrix} 0 & -\dfrac{k}{\beta}\left(1-\dfrac{\beta}{k-\sigma}\right)e^{-(k-\sigma)t} \\ \beta^2 \dfrac{\sigma}{k-\sigma} e^{-2\sigma t} + k\beta\left(1-\dfrac{\beta}{k-\sigma}\right)e^{-(k+\sigma)t} & 0 \end{pmatrix}$$

By formula (28) and (29) we know: $A_2(t)$ meet the conditions of lemma (2). Is easy to prove $A_1$ is stable array.

Note: the final control law is

$$\begin{cases} v = \dfrac{1}{\alpha}\left(k_1 \bar{z}_1 + k_2 \bar{z}_2\right) \\ \omega = -k\bar{\theta} + \beta e^{-\sigma t} \end{cases} \tag{30}$$

Case 2: $\alpha$ is unknown, we make the following assumptions

$$0 < \alpha \le m_2 \tag{31}$$

This assumption is not strict, because the actual distance $f$, pixel scale factor $\beta_1$ 、 $\beta_2$ and depth $z$ are bounded.

Because of the unknown at this time, so the control law (30) can't use, therefore, theorem 2 is given below.

Theorem 2: Selection $k$ 、 $\beta$ and $\sigma$ satisfy the equation (22), $k_1$ 、 $k_2$ are constant as follow.

$$\begin{cases} k_1 > -(k-\sigma)k_2 \\ k_2 < -\dfrac{\sigma}{m_2} \end{cases} \tag{32}$$

then

(1) matrix $\begin{pmatrix} \sigma & -\dfrac{\sigma}{k-\sigma} \\ \alpha k_1 & \alpha k_2 \end{pmatrix}$ is steady array;

(2) The controller (33) can guarantee the system (25) be from any initial state index converge to the origin.

$$v = k_1 \bar{z}_1 + k_2 \bar{z}_2 \tag{33}$$

Proof: take formula (33) into formula (25), then

$$\begin{pmatrix} \dot{\bar{z}}_1 \\ \dot{\bar{z}}_2 \end{pmatrix} = \left(A_1 + A_2(t)\right)\begin{pmatrix} \bar{z}_1 \\ \bar{z}_2 \end{pmatrix} \tag{34}$$

Among them

$$A_1 = \begin{pmatrix} \sigma & -\dfrac{\sigma}{k-\sigma} \\ \alpha k_1 & \alpha k_2 \end{pmatrix} \tag{35}$$

$A_2(t)$ is same as theorem 1 in the form.

By formula (21) and (32) we known $A_2(t)$ is meet the conditions of lemma (2). Next we prove $A_1$ is stable array.

The Characteristic polynomial of matrix $A_1$ is as follow:

$$\det(\lambda I - A_1) = \begin{vmatrix} \lambda-\sigma & \dfrac{\sigma}{k-\sigma} \\ -\alpha k_1 & \lambda-\alpha k_2 \end{vmatrix} = (\lambda-\sigma)(\lambda-\alpha k_2)+\alpha k_1 \frac{\sigma}{k-\sigma}$$

$$= \lambda^2 - (\sigma+\alpha k_2)\lambda + \alpha k_2 \sigma + \alpha k_1 \frac{\sigma}{k-\sigma}$$

According to the principle of stability, if we want to make $A_1$ stabilize array, then the characteristic value need have negative real part. That is:

$$\begin{cases} \sigma+\alpha k_2 < 0 \\ k_2\sigma + k_1 \dfrac{\sigma}{k-\sigma} > 0 \end{cases} \tag{36}$$

By formula (35) and (36) we can get as follow：

$$\begin{cases} \sigma+\alpha k_2 < \sigma + k_2 m_2 < 0 \\ k_2\sigma + k_1 \dfrac{\sigma}{k-\sigma} > 0 \end{cases} \tag{37}$$

Then we can get $A_1$ is steady array. Note: the final control law is

$$\begin{cases} v = k_1\bar{z}_1 + k_2\bar{z}_2 \\ \omega = -k\bar{\theta} + \beta e^{-\sigma t} \end{cases}$$

Case 3: $\theta_0$ is known, but $\alpha_1$ , $\alpha_2$ are unknown, We make the following assumptions:

(1) If $\alpha_1 = \alpha_2 = \alpha$ , we can get that:

$$\begin{pmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\alpha\cos(\theta-\theta_0) \\ -v\alpha\sin(\theta-\theta_0) \\ \omega \end{pmatrix} \tag{38}$$

Suppose to $\theta$ replace $\theta-\theta_0$ ,then we can get that:

$$\begin{pmatrix} \dot{x}_m \\ \dot{y}_m \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} v\alpha\cos\theta \\ -v\alpha\sin\theta \\ \omega \end{pmatrix} \tag{39}$$

If we make $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} x_m \\ y_m \end{pmatrix}$, Then formula（40） will be converted to the form below

$$\begin{cases} \dot{z}_1 = -\omega z_2 + v\alpha \\ \dot{z}_2 = \omega z_1 \\ \dot{\theta} = \omega \end{cases}$$

We set $\omega = -k\theta$ , then $\dot{\theta} = \theta(0)e^{-kt}$ （$\theta(0)$ is the initial value of $\theta$）. Assuming that

$$\begin{cases} y_1 = z_1 \\ y_2 = \dfrac{z_2}{\theta} \end{cases}$$

Then

$$\begin{cases} \dot{y}_1 = k\theta^2 y_2 + v\alpha \\ \dot{y}_2 = -ky_1 + ky_2 \end{cases} \tag{40}$$

We can get the final control law is

$$\begin{cases} \omega = -k\theta \\ v = k_1 y_1 + k_2 y_2 \end{cases} \tag{41}$$

 

Among them $k_1$ and $k_2$ are constant: From the above two formulas we can get formula ( 42 )

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} k\theta^2 y_2 + \alpha k_1 y_1 + \alpha k_2 y_2 \\ -ky_1 + ky_2 \end{pmatrix} = \left(A_0 + A_1(t)\right)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{42}$$

In the formula , $A_0 = \begin{pmatrix} \alpha k_1 & \alpha k_2 \\ -k & k \end{pmatrix}$, $A_1(t) = \begin{pmatrix} 0 & k\theta^2 \\ 0 & 0 \end{pmatrix}$.

To illustrate what kind of $k_1$ and $k_2$ can make the system (42) asymptotic stability, the paper first introduces the following propositions.

Lemma 1：

If there are linear time-varying systems

$$\dot{x} = \left(B_0 + B_1(t)\right)x \tag{43}$$

Among them， $\dot{x}$ is a n dimension vector， $B_0$ is $n \times n$ Hurwitz matrix,， $B_1(t)$ satisfy the following formula：

$b_{ij}(t) \to 0 (t \to \omega)\ i, j = 1,2 \ldots n$

Then the system (42) is asymptotic stability.

The following content will prove the asymptotic stability of the system (41).

Proof: the characteristic polynomial of $A_0$ is:

$$|\lambda I - A_0| = \begin{vmatrix} \lambda - \alpha k_1 & -\alpha k_2 \\ k & \lambda - k \end{vmatrix} = \lambda^2 - (k + \alpha k_1)\lambda + \alpha k k_2$$

The sufficient and necessary conditions for $A_0$ belongs to *Hurwitz* matrix is:

$$\begin{cases} k + \alpha k_1 < 0 \\ \alpha k k_2 > 0 \end{cases}$$

We set $\alpha \geq \alpha_0 > 0$ ,and $k > 0$ ,

Then we can get the sufficient and necessary conditions for $A_0$ belongs to *Hurwitz* matrix is:

$$\begin{cases} k_2 > 0 \\ k_1 < -\dfrac{k}{\alpha_0} \end{cases} \tag{44}$$

Because $\dot{\theta} = \theta(0)e^{-kt}$ , then $\theta \to 0$ , $A_1(t) \to 0\ (t \to \omega)$, According to lemma 1, system (43) is asymptotically stable.

So the control law as follow can guarantee $(x, y, \theta)$ asymptotic stability：

$$\begin{cases} \omega = -k\theta \\ v = k_1 y_1 + k_2 y_2 \end{cases}$$

(2)If $\alpha_1$、 $\alpha_2$ unknown , and $\alpha_1 \neq \alpha_2$

Assuming that

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos(\theta - \theta_0) & -\sin(\theta - \theta_0) \\ \sin(\theta - \theta_0) & \cos(\theta - \theta_0) \end{pmatrix}\begin{pmatrix} x_m \\ y_m \end{pmatrix}$$

Put the system equation in the derivative of the above formula ,we can get next equation：

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \omega\begin{pmatrix} -z_2 \\ z_1 \end{pmatrix} + v\begin{pmatrix} \alpha_1 \cos^2(\theta - \theta_0) + \alpha_2 \sin^2(\theta - \theta_0) \\ (\alpha_1 - \alpha_2)\sin(\theta - \theta_0)\cos(\theta - \theta_0) \end{pmatrix}$$

Set $\omega = -k(\theta - \theta_0)$ , then

$$\theta - \theta_0 = e^{-kt}h \tag{45}$$

 

$h$ is the initial value of $\theta(t)-\theta_0$

If $\begin{cases} y_1 = z_1 \\ y_2 = \dfrac{z_2}{-\omega} \end{cases}$,we can get

$$\begin{cases} \dot{y}_1 = -\omega z_2 + v\left(\alpha_1 \cos^2(\theta-\theta_0) + \alpha_2 \sin^2(\theta-\theta_0)\right) \\ \dot{y}_2 = -ky_1 + ky_2 + \dfrac{v}{\theta-\theta_0}(\alpha_1-\alpha_2)\sin(\theta-\theta_0)\cos(\theta-\theta_0) \end{cases} \tag{46}$$

Take control law as

$$v = k_1 y_1 + k_2 y_2 \tag{47}$$

Take the formula (46) into the formula (45) ,we can get

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} v\alpha_1 + k(\theta-\theta_0)^2 y_2 + v(\alpha_2-\alpha_1)\sin^2(\theta-\theta_0) \\ -ky_1 + ky_2 + v(\alpha_1-\alpha_2) + v(\alpha_1-\alpha_2)\left(\dfrac{\sin(\theta-\theta_0)\cos(\theta-\theta_0)}{\theta-\theta_0} - 1\right) \end{pmatrix} = \left(A_{20} + A_2(t)\right)\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Among them $A_{20} = \begin{pmatrix} \alpha_1 k_1 & \alpha_2 k_2 \\ -k + k_1(\alpha_1-\alpha_2) & k + k_2(\alpha_1-\alpha_2) \end{pmatrix}$

$$A_2(t) = \begin{pmatrix} k_1(\alpha_2-\alpha_1)\sin^2(\theta-\theta_0) & k(\theta-\theta_0)^2 + k_2(\alpha_2-\alpha_1)\sin^2(\theta-\theta_0) \\ k_1(\alpha_1-\alpha_2)\left(\dfrac{\sin(\theta-\theta_0)\cos(\theta-\theta_0)}{\theta-\theta_0} - 1\right) & k_2(\alpha_1-\alpha_2)\left(\dfrac{\sin(\theta-\theta_0)\cos(\theta-\theta_0)}{\theta-\theta_0} - 1\right) \end{pmatrix}$$

The characteristic polynomial of $A_{20}$ is

$$|\lambda I - A_{20}| = \begin{vmatrix} \lambda - \alpha_1 k_1 & -\alpha_2 k_2 \\ k - k_1(\alpha_1-\alpha_2) & \lambda - k - k_2(\alpha_1-\alpha_2) \end{vmatrix} = \lambda^2 - \left(k + k_1\alpha_1 + k_2\alpha_1 - k_2\alpha_2\right)\lambda + k\left(k_1+k_2\right)\alpha_2$$

The sufficient and necessary conditions for $A_0$ belongs to *Hurwitz* matrix is:

$$\begin{cases} k + k_1\alpha_1 + k_2\alpha_1 - k_2\alpha_2 < 0 \\ k\left(k_1+k_2\right)\alpha_2 > 0 \end{cases} \tag{48}$$

## 4. Results and Analysis

### 4.1. Simulate by α known

When the initial state is (1，0.5，1), we can get the state of the time trajectory and the robot motion geometric path by obtained (23) under the control law (28), as shown in figure 4, 5. In the simulation process the value of β, σ and κ can be determined first, and then the value of $\kappa_2$ and $\kappa_1$ can be determined by formula (27).

Control parameters are $\alpha = 2$, $\beta = 1$, $\sigma = 1$, $k_1 = 6$, $k_2 = -2$, $k = 3$.

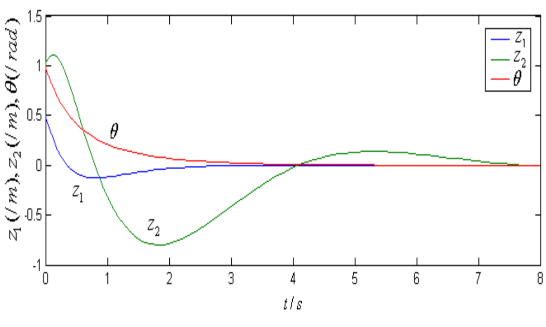


**Figure 4. The Time Curve in Each State of System (23) by $\alpha$ Known**

 

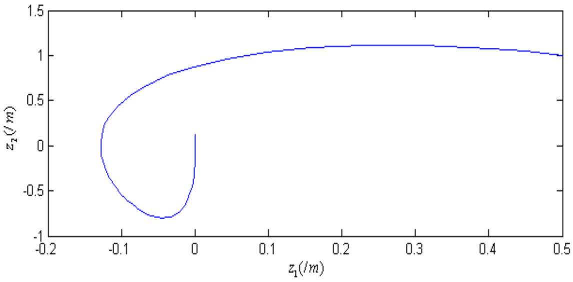


**Figure 5. Geometrical Locus of Robot Movement by $\alpha$ Known**

### 4.2 Simulate by α Unknown

When the initial state is (1, 0.5, 1), we can get the state of the time trajectory and the robot motion geometric path by obtained (23) under the control law (33), as shown in figure 6, 7. In the simulation process the value of β, σ and κ can be determined first, and then the value of $\kappa_2$ and $\kappa_1$ can be determined by formula (32).

Control parameters are $m_2 = 4$, $\beta = 1$, $\sigma = 1$, $k_1 = 3$, $k_2 = -1$, $k = 3$.

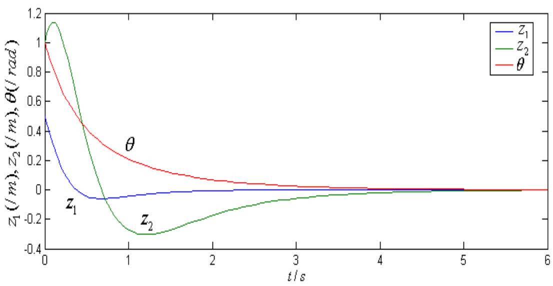


**Figure 6. The Time Curve in Each State of System (23) by α Unknown**

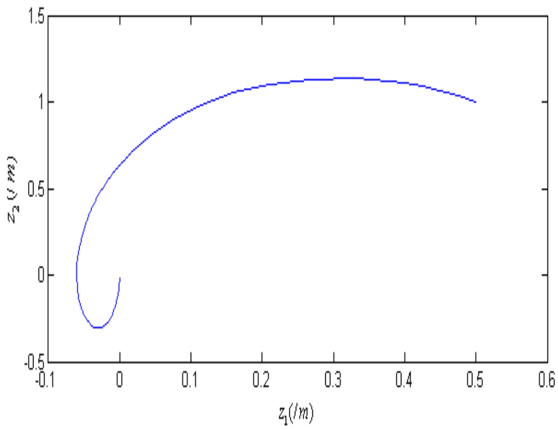


**Figure 7. Geometrical Locus of Robot Movement by α is Unknown**

### 4.3 Simulate by α1and $\alpha_2$ are Unknown

Case1: $\alpha_1 = \alpha_2 = \alpha$, and α is unknown

When the initial state is (1, 0.5, 1), we can get the state of the time trajectory and the robot motion geometric path by obtained (40) under the control law (41), as shown in figure 8, 9. In the simulation process the value of β, σ and κ can be determined first, and then the value of κ2 and κ1 can be determined by formula (44).

Control parameters are $\alpha = 3$, $k = 2$, $k_1 = -2$, $k_2 = 1$.

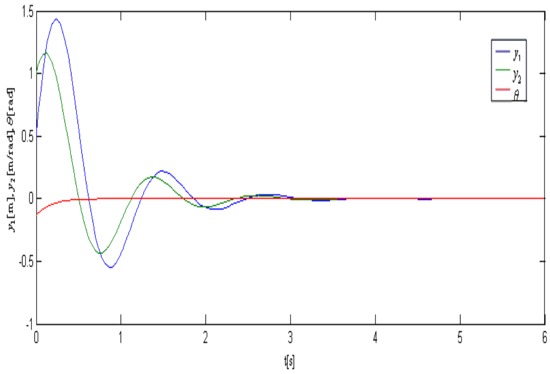


**Figure 8. The Time Curve in Each State of System (40) by $\alpha_1 = \alpha_2 = \alpha$**

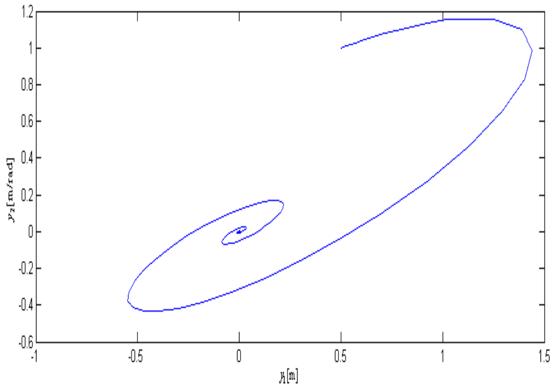


**Figure 9. Geometrical Locus of Robot ovement by $\alpha_1 = \alpha_2 = \alpha$**

Case2: $\alpha_1 \neq \alpha_2$ ,$\alpha_1$ and $\alpha_2$ are unknown

When the initial state is (1, 0.5, 1), we can get the state of the time trajectory and the robot motion geometric path by obtained (46) under the control law (47), as shown in figure 10, 11. In the simulation process the value of β, σ and κ can be determined first, then the value of $\kappa_2$ and $\kappa_1$ can be determined by formula (48).

Control parameters are $\alpha_1 = 2$, $\alpha_2 = 1$, $k_1 = -3$, $k_2 = 4$, $k = 2$.

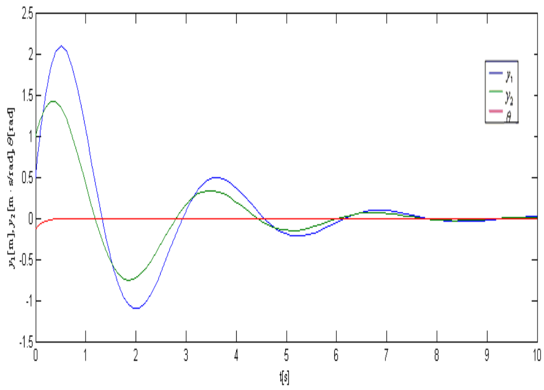


**Figure 10. The Time Curve in Each State of System (46) by $\alpha_1 \neq \alpha_2$**

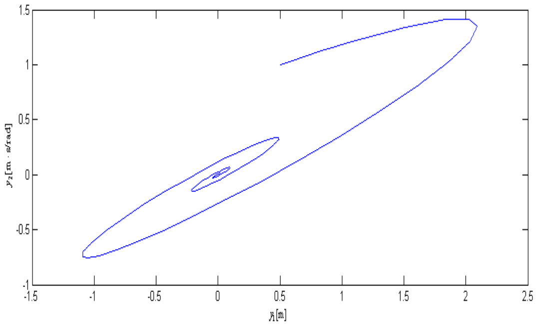

**Figure 11. Geometrical Locus of Robot Movement by $\alpha_1 \neq \alpha_2$**

By simulation α is known and unknown it can be seen that the control law designed in this paper can make the system achieve exponential convergence quickly.

Now we found in the simulation process, the control under the condition of α unknown quantity is bigger than in the case of α known, this shows that when the α unknown the system need more energy.

## 5. Conclusions

For stabilization problem of wheeled mobile robot under global visual conditions, this paper introduces a visual servo control method based on image motion control of WMR. The simulation results show that the method can solve the problem of robot car calm, it has advantages of fast convergence and good robustness. This paper just in case of $\alpha_1 = \alpha_2$ presents a robust control law design. For general situation of $\alpha_1 \neq \alpha_2$, can use the design idea of this paper to consideration.

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