The Singularity Detection of Stress Wave Signal in One-dimensional Components Based on Quantitative Information Entropy

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Abstract

According to existing methods of singularity detection in stress wave signal, raise and define the quantitative information entropy and the mean vector model of Information entropy, the definitions are based on probability theory and mathematical statistical theory and information entropy, Define the method of singularity detection based on the quantitative information entropy. This method is tested and simulated using stress wave of pile detection and compared with the modulus maxima method. The experimental results show that the quantitative information entropy method has advantages in anti-noise performance and can achieve a high accuracy and locations information compared with the modulus maxima method. In the quantitative information entropy method, multidimensional signal is processed by dimensionality reduction and can be widely applied to various research fields, such as fingerprint identification and facial identification.

Keywords: Quantitative information entropy; Mean vector of Information entropy; Stress wave; Singularity; Time Window

1. Introduction

Integrity detection using stress wave is the most important method which is used to detect damage and defects of one-dimensional components like building pile foundations, cement beam structures and mechanical rotators. The most important thing of integrity detection using stress wave is finding the singularities in stress wave signal of components, therefore, it is important to detect the singularities of stress wave signal. There are lots of theories in the study of components’ singularities [1-3],although mature methods are used, it still depends on artificial experiences and reexamination when it comes to accuracy, locations and de-noising of singularities detection [4-7],then, detecting singularities of components’ damage and defects is important to improve quality of damage detection. Therefore, this paper defines quantitative information entropy and make it as feature of singularities and builds mean vector of Information entropy, use this method to detect singularities in stress wave signal of one-dimensional components, experimental results show that our method has higher anti-noise performance and can achieve a higher accuracy and locations information as compared to traditional modulus maxima method under the same situation of resource consumption.

2. Quantitative Information Entropy and the Mean Vector Model

Information entropy can reflect the fluctuate information of signal, we can obtain quantitative information entropy through quantifying amplitudes and time of the
one-dimensional stress wave signal, then we can find singularities of signal effectively even though under the influence of complex noise.

### 2.1. Quantitative Information Entropy

Information theory expert C.E. Shannon defined information entropy firstly using the method of probability theory and mathematical statistics, information entropy means uncertainty reduction of systems or things, its traditional definition as follows [8]:

Assume that the signal source is given by discrete random variable: \( X = \{x_1, x_2, \ldots, x_n\} \), the probability distribution of \( X \) expressed as: \( p_i = p(x_i) \), and \( \sum_{i=1}^{n} p_i = 1 \) is known, then the information entropy of the signal source can be defined as:

\[
H(X) = \sum_{i=1}^{n} \left( p_i \log_2 \frac{1}{p_i} \right) \tag{1}
\]

From the definition of information entropy we can know that the information entropy of system will be lower if the system is more complex and has more information and greater uncertainty.

The definition of quantitative information entropy in this paper is based on the traditional definition of information entropy, it is obtained through quantifying the power (amplitudes) and time of signal.

By assuming that the sampled amplitudes set of signal is \( X = \{x_1, x_2, \ldots, x_n\} \), the probability distribution of \( X \) expressed as: \( p_i = p(x_i) \), and \( \sum_{i=1}^{n} p_i = 1 \), choose the max value and the min value and name them \( x_{\text{max}} \) and \( x_{\text{min}} \) respectively. According to the actual situation, divide amplitudes into \( D \) quantitative amplitude ranges averagely from \( x_{\text{min}} \) to \( x_{\text{max}} \), the length of every range is:

\[
\Delta d = \frac{x_{\text{max}} - x_{\text{min}}}{D} \tag{2}
\]

Every quantitative range of amplitudes is:

\[
(x_{\text{min}} + i\Delta d, x_{\text{max}} + (i+1)\Delta d) \quad (i = 0, 1, 2, \ldots, D-1)
\]

Suppose that the \( i \) amplitudes range has \( \text{Num}_i \) sample points and its statistical probability can be defined as:

\[
p_i = \frac{\text{Num}_i}{n} \tag{4}
\]

The quantitative information entropy of signal can be defined as:

\[
DH(X) = \sum_{i=0}^{D-1} \left( \text{Num}_i p_i \log_2 \frac{1}{p_i} \right) \tag{5}
\]

In the above formula, \( DH(X) \) is the quantitative information entropy of signal, \( D \) is the number of amplitudes ranges, \( \text{Num}_i \) is the number of sample points in the \( i \) amplitudes range, \( p_i \) is the statistical probability of sample points in the \( i \) amplitudes range.
2.2. Quantitative Information Entropy based on Time

The original low strain dynamic tress wave signal is pre-processed by wavelet packet decomposition. Wavelet packet is built based on orthogonal wavelet [9-10], and can decompose the high frequency and the low frequency of signal simultaneously [11]. Wavelet packet decomposition can decompose signal more subtly and will not lose the information of signal and can achieve better time-frequency localization effect. Signal is processed by wavelet packet decomposition into $n$ layers, can get $2^n$ wavelet signals in different frequency ranges. As shown in the Figure 1, the 4 wavelet signals are achieved after the low strain dynamic tress wave signal is processed by sym8 wavelet packet decomposition into 2 layers.

This paper focuses and uses the method of quantity in time and creates the time-window with fixed width, calculate and obtain the quantitative information entropy of every time-window by steps. As shown in the Figure 1, we can obtain features of signal use the method of quantity in time.

![Figure 1. The Method of Quantity in Time](image)

Assume that the width of time-window is $W$, the step length is $B$. The width of time-window $W$ must cover at least 1 singularity and less than 2 singularity; the step length $B$ should satisfy: $\frac{W}{8} < B < \frac{W}{4}$. those sets can ensure that we will not miss any singularities when obtain features by steps and will not process one same singularity repeatedly, besides, those sets can save time and improve the resolution of singularities.

2.3. Mean Vector Model of Quantitative Information Entropy

Assume that the tress wave is processed by wavelet packet into $k$ layers, we can get $m = 2^k$ wavelet signals, the number of time-windows steps is $n$, obtain quantitative information entropy as feature and the entropy vector of every wavelet signal is:

$$H_m = [DH_{m1}, DH_{m2}, DH_{m3}, \ldots, DH_{mn}]_{(m = 1,2,\ldots,2^k)} \quad (6)$$

The matrix of low strain dynamic tress wave signal quantitative information entropy built as the follow:

$$H = [DH_{11}, DH_{12}, \ldots, DH_{1n}]

[DH_{21}, DH_{22}, \ldots, DH_{2n}]

[\ldots \ldots \ldots \ldots]

[DH_{m1}, DH_{m2}, \ldots, DH_{mn}] \quad (7)$$
In this formula, the data of every row is the quantitative information entropy vector of one wavelet signal, \( m \) wavelet signals create a quantitative information entropy matrix with \( m \) rows and \( n \) columns. Singularities cannot be detected precisely from the complex feature matrix, we can calculate and obtain the quantitative information entropy mean of time-windows in wavelet signals which reflect the same low strain dynamic tress wave signal part, the quantitative information entropy mean can be expressed as follows:

\[
\overline{D\bar{H}}_i = \frac{1}{m} (DH_{i1} + DH_{i2} + \ldots + DH_{im}) \quad (i = 1, 2, \ldots, n)
\]  

(8)

The mean vector of quantitative information entropy is:

\[
\bar{H} = \left\{ \overline{D\bar{H}}_1, \overline{D\bar{H}}_2, \overline{D\bar{H}}_3, \ldots, \overline{D\bar{H}}_m \right\}
\]  

(9)

Detect Singularities by the mean vector of quantitative information entropy. If the quantitative information entropy mean of a point is the convergent minimum value from right and left ranges with same specific length, the low strain dynamic tress wave signal part reflected by this point is singularity location. For example, the \( \overline{D\bar{H}}_i \) of \( \bar{H} \) is the convergent minimum value, the range:

\[
((i-1)B, (i-1)B+W) \quad (i \in [1,n])
\]  

(10)

The range is the singularity location of low strain dynamic tress wave signal, the \( B \) is the steps length, the \( W \) is the width of time-window.

3. Experimental Simulation and Analysis

In our study we have chosen pile detection low strain dynamic stress wave signal with already known singularities location to perform simulation experiments, these experiments used method based on quantitative information entropy where we also compared with other methods mentioned in [12-14]. We used well known factors to compare with other method such as analysis of accuracy, anti-noise performance, locations and time consumptions.

3.1. Experimental Comparison and Analysis

As shown in the Figure 2, the signals are the original low strain dynamic tress wave signal \( S \) and the 8 wavelet signals (\( s0, s1, s2, s3, s4, s5, s6, s7 \)) which are from the signal \( S \) is processed by wavelet packet decomposition into 3 layers, the abscissa is time (ms), the ordinate is the amplitude of signal wave velocity, the whole signal time is 10.23ms. The singularities locations are: 2.15ms, 3.70ms, 4.55ms, 5.75ms, 9.10ms.

![Figure 2. The Original Low Strain Dynamic Tress Wave Signal and Wavelet Signals](image-url)
Use the quantitative information entropy method, assume the time-window width $W$ is 1 ms, the sample points number of every time-window is 100 according to the time average, the last time-window just includes part of signal, set the rest part of the time-window as 0 to complete the last time-window; set the step length $B$ as 0.2 ms (20 simple points); Set the number of amplitude ranges as 10, achieve the quantitative information entropy of every time-window and the mean vector of quantitative information entropy, the mean curve of quantitative information entropy shown as the follow Figure 3, the abscissa is the ordinal number of time-window, the ordinate is the mean of quantitative information entropy, by using this method we can detect 4 singularities locations named ①、②、③、④ respectively, they represent the 4 ranges: 2.00-3.00ms、2.80-3.80ms、5.60-6.60ms、8.20-9.20ms, they are all exact singularities locations.

![Figure 3. The Mean Curve of Quantitative Information Entropy](image)

Choose modulus maxima method as contrast, set the wavelet transform scale from 200 to 20 and the interval is 20, track the modulus maxima from the largest to the tiniest scale taking the advantage of the effective information of the modulus maxima map and draw the modulus maxima lines. As shown in the Figure 4, the abscissa is the time, the ordinate is the scale, the points ①、②、③、④、⑤、⑥、⑦ and ⑧ represent the 8 singularity locations, the points ③、④、⑤ and ⑥ are exact singularities.

![Figure 4. The Modulus Maxima Lines](image)

The comparative results of the 2 singularity detection methods shown in Table 1:
Table 1. The Contrast Results

<table>
<thead>
<tr>
<th>Detection method</th>
<th>Number of singularities</th>
<th>Number of simulation singularities</th>
<th>Number of right singularities</th>
<th>Accuracy of number</th>
<th>Accuracy of locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantitative information entropy method</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>80%</td>
</tr>
<tr>
<td>Modulus maxima method</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>50%</td>
<td>80%</td>
</tr>
</tbody>
</table>

As shown in the Table 1, singularity detection of the quantitative information entropy method can achieve a higher accuracy and locations information compared with the modulus maxima method.

3.2. Comparison and Analysis of Singularity Detection Performance

Select 200 groups of stress wave signals as experiment objects, every stress wave signal has 4 known singularities locations, add white Gaussian noise to every stress wave signal, the SNR are 30, 40 and 50 respectively, singularity detection using the quantitative information entropy method and the modulus maxima method, contrast and analyze anti-noise performance and time consumption(s). Compare the mean result of each group of experiments. The results as shown in the Table 2 and Table 3:

Table 2. Accuracy of Singularity Locations and Time Consumption

<table>
<thead>
<tr>
<th>SNR</th>
<th>Detection method</th>
<th>Number of locations</th>
<th>Number of detection locations</th>
<th>Accuracy of locations</th>
<th>Time consumption(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No noise</td>
<td>Quantitative information entropy method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>0.3327</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>0.3383</td>
</tr>
<tr>
<td>30</td>
<td>Quantitative information entropy method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>0.3436</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>4</td>
<td>3.9</td>
<td>97.5%</td>
<td>0.3404</td>
</tr>
<tr>
<td>40</td>
<td>Quantitative information entropy method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>0.3331</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>4</td>
<td>3.9</td>
<td>97.5%</td>
<td>0.3326</td>
</tr>
<tr>
<td>50</td>
<td>Quantitative information entropy method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
<td>0.3423</td>
</tr>
</tbody>
</table>
As shown in the Table 2 and Table 3, the quantitative information entropy method can achieve a higher accuracy of singularities number and locations, and will not need more time consumption; the quantitative information entropy method has better anti-noise performance, it has an excellent detection performance under the situation of complex noise.

4. Conclusions

In this paper, the quantitative information entropy and the mean vector model of Information entropy have been developed, the pile foundation component stress wave signals have been chosen as experiment objects, the experimental results show that the quantitative information entropy method has more advantage on the accuracy of singularities number and locations and anti-noise performance compared with the modulus maxima method.

The quantitative information entropy method has the equal level of the whole processing time consumption compared with the modulus maxima method while add signal process time when move the time-window by steps. The quantitative information entropy method achieves dimensionality reduction of multidimensional signal, and has higher application value in research fields, such as fingerprint identification and facial identification, especially in processing mass features of information.

<table>
<thead>
<tr>
<th>SNR</th>
<th>Detection method</th>
<th>Number of detection</th>
<th>Number of right singularities</th>
<th>Accuracy of number</th>
</tr>
</thead>
<tbody>
<tr>
<td>No noise</td>
<td>Quantitative information entropy method</td>
<td>4</td>
<td>4</td>
<td>100%</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>5</td>
<td>4</td>
<td>80%</td>
</tr>
<tr>
<td>30</td>
<td>Quantitative information entropy method</td>
<td>15.5</td>
<td>10.6</td>
<td>68.39%</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>74.1</td>
<td>28.4</td>
<td>38.33%</td>
</tr>
<tr>
<td>40</td>
<td>Quantitative information entropy method</td>
<td>14.4</td>
<td>9.6</td>
<td>66.67%</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>50.6</td>
<td>15.5</td>
<td>30.36%</td>
</tr>
<tr>
<td>50</td>
<td>Quantitative information entropy method</td>
<td>12.8</td>
<td>8.8</td>
<td>68.75%</td>
</tr>
<tr>
<td></td>
<td>Modulus maxima method</td>
<td>32.1</td>
<td>8.4</td>
<td>26.17%</td>
</tr>
</tbody>
</table>
Acknowledgments

The authors would like to thank the anonymous reviewers and editors for their valuable comments to improve the presentation of the paper. This work is supported by National Natural Science Foundation of China (NSFC), project number is 61371174. The authors also would like to thank the lab mates and professors for their kind help and valuable suggestions.

References


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