International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol.8, No.10 (2015), pp.147-154
http://dx.doi.org/10.14257/ijsip.2015.8.10.17

Combined Forecasting Mode of Subgrade Settlement Based on Forecasting Availability and Real-coded Quantum Evolutionary Algorithm

Gao Hui[1] and Huang Jun[2,3]

1.School of Electrical and Information Engineering, Heilongjiang Institute of Technology, Harbin, Heilongjiang, China 150050.
2. Faculty of Geosciences and Environmental Engineering, Southwest Jiaotong University, Chengdu, Sichuan, China 610031.
3. Chengdu Donglu Traffic Science and Technology Co.,Ltd. Chengdu, Sichuan, China 610031.
E-mail:Gaohui_dl@sina.com

Abstract

Due to the normal forecasting methods for subgrade settlement using observation data have different explicabilities, and the predicting results has bigger volatility and lower accuracy. The Combined forecasting model of subgrade settlement based on forecasting availability and real-coded quantum evolutionary algorithm (RQEA) is put forward in this paper. At the first, according to the basic settlement law of subgrade and characteristics of settlement curve, the growth curve with the S-type characteristics are chosen as single forecasting model; Then, to get the weights of each single forecasting model, objective function is build on the basis of standard of forecasting availability maximization, and RQEA is employed to solve the objective function, and to construct the combined forecasting model of subgrade settlement. The result of engineering practice shows that the proposed method has better prediction accuracy and stability, and can meet engineering demand.

Keywords: *Subgrade Settlement prediction, Combination forecast model, forecasting availability, Real-coded quantum evolutionary algorithm*

1. Introduction

The development of subgrade settlement has complex characteristics, such as non-linear, non-stabilization, and including numerous uncertain information, so it is very difficult to forecast the subgrade settlement accurately. At present, the methods of subgrade settlement prediction mainly include layer-wise summation method (LWSM), numerical analysis method (NAM) and modeling method based on the observed data (MMBD) [1-2].

LWSM and NAM demand the precise geotechnical parameters and model of elasto plastic constitutive relation, but the geotechnical parameters and the model of elasto plastic constitutive relation is difficult to be acquired, so their application is limited. The observed data of subgrade settlement is an integrated reaction of all kinds of factors, and contains abundant information, then MMBD is paid more attention. MMBD includes experience formula method (EFM) (such as hyperbolic method, exponential curve method, growth-curve approach and parabolic method), system analysis and control theory method (grey system method and nerve network method). In a word, each forecasting model has their own advantages and their scopes of application, but there are some shortcomings too [3-4].

ISSN: 2005-4254 IJSIP

Combined forecast makes full use of the advantages of single-phase prediction model and overcomes its shortcomings, improving the forecasting precision and reducing the forecasting risk. Combined forecasting method combines different single-phase prediction models through considering the characteristics of each single-phase prediction model, so the predicted results could make full use of the obtained information from each single-phase prediction model, it has strong adaptability and good stability [5-6]. We use machine learning and optimized calculation to establish combined forecasting model of subgrade settlement. Firstly, we lead Usher, Logistics, Gompertz of S-type single-phase prediction model into combined forecast according to the development law and sedimentation curve characteristic of subgrade settlement, and calculating the corresponding prediction results; then we take the largest forecast effectiveness of each single forecast model as the rule to build objective function, and to determine the weight of each single forecast model, and to build combined forecasting model of subgrade settlement through real coded quantum inspired evolutionary algorithm. Engineering case analysis shows that the combined forecasting model has better prediction accuracy and stability, it has important theory and engineering practical value.

2. Combined Forecasting Model based on Forecasting Availability

Assuming the observation data sequence of subgrade settlement is s_t, $t = 1,2,\cdots,N$, there are m single forecasting models included in combined forecasting model, and the forecast result of the i th single forecasting model at the t th time observation is $\hat{s}_{it}$, $i = 1,2,\cdots,m$, $t = 1,2,\cdots,N$. The weight values of each single forecasting model in combined forecasting models are respectively $\omega_{1t},\omega_{2t},\cdots,\omega_{mt}$, and $\sum_{i=1}^{m}\omega_{it} = 1$, the predicted result of combined forecasting model is shown as:

$$\hat{s}_t = \sum_{i=1}^{m}\omega_{it}\hat{s}_{it} \tag{1}$$

From Eq.(1), we can know that the key to construct combined forecasting model is to determine weight value of each single forecasting model. To get weight value of each single forecasting model, Several concepts is introduce.

Definition 1: the prediction relative error of the i th single forecasting model at the t th time observation in combined forecasting model is:

$$e_{it} = \begin{cases} (s_t - \hat{s}_{it})/s_t, & |(s_t - \hat{s}_{it})/s_t| \le 1 \\ 1, & |(s_t - \hat{s}_{it})/s_t| > 1 \end{cases} \tag{2}$$

where $i = 1,2,\cdots,m$, $t = 1,2,\cdots,N$, and then the matrix $E_R = [e_{it}(t)]_{m\times N}$ is called as relative error matrix of combined forecasting model. Obviously, $0 \le |e_{it}| \le 1$, the i th row of matrix E is the prediction relative error sequence of the i th single forecasting model at the t th time observation.

Definition 2: the prediction accuracy of the i th single forecasting model at the t th time observation in combined forecasting model is:

$$A_{it} = 1 - |e_{it}| \tag{3}$$

where $i = 1,2,\cdots,m$, $t = 1,2,\cdots,N$, and then the matrix $A = [A_{it}]_{m\times N}$ is called as prediction accuracy matrix of combined forecasting model. Obviously, $0 \le A_{it} \le 1$, when $A_{it} = 0$, it indicates the predict result of the i th forecasting model at the t th time observation is invalid. At the same time, we can see that the value of A_{it} is more approximate 1, the prediction accuracy is better.

From eq.(3) , The prediction accuracy of combined forecasting model at the t th time observation is represented as:

$$\begin{aligned} A_{ct} &= 1-|e_{ct}| \\ &= 1-|(s_t-\hat{s}_t)/s_t| = 1-\left|\sum_{i=1}^{m}\omega_{it}(s_t-\hat{s}_{it})/s_t\right| \end{aligned} \tag{4}$$

Definition 3: the prediction availability of the i th single forecasting model at the t th time observation in combined forecasting model is:

$$AV_i = \frac{1}{N}\sum_{t=1}^{N} A_{it} \tag{5}$$

where $i = 1,2,\cdots,m$.

From eq.(4) , The prediction availability of combined forecasting model is shown as:

$$AV_c = \frac{1}{N}\sum_{t=1}^{N} A_{ct} \tag{6}$$

We can find from eq.(6) that the value of the prediction availability AV_c is larger, the combined forecasting model is more valid, namely, the prediction value of combined forecasting model is more precise. To get the weights of each single forecasting model in combined forecasting model, and construct combined forecasting model, based on the standard of forecasting availability maximization , we can build the objective function shown as:

$$\begin{aligned} &\max AV_c = \frac{1}{N}\sum_{t=1}^{N} A_{ct} \\ &st.\begin{cases} A_{ct} = 1-|e_{ct}| \\ \sum_{i=1}^{m}\omega_{it} = 1 \end{cases} \end{aligned} \tag{7}$$

3. Real-coded Quantum Evolutionary Algorithm (RQEA)

3.1. Basic Ideas of RQEA

In Ref.[7,8], a real-coded chromosome, whose allele is composed of one component x_i of variable vector X and probability amplitudes $(\alpha_i,\beta_i)^T$ of one qubit, $i = 1,2,\cdots,n$ in RQEA, is represented as:

$$q = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ \alpha_1 & \alpha_2 & \cdots & \alpha_n \\ \beta_1 & \beta_2 & \cdots & \beta_n \end{bmatrix} \tag{8}$$

where n is the length of chromosome, and lies on the dimensions of variable vector X .

The single-gene mutation is adopted to update population at each iteration in RQEA. Assume that RQEA maintains a population $p(t) = \{p_1^t,\cdots,p_j^t,\cdots,p_N^t\}$ at the t-th iteration, where N is the population size. Select the i-th gene $(x_{ji}^t,\alpha_{ji}^t,\beta_{ji}^t)^T$ of p_j^t, and update the value of $x_{j,i}^t$ using Gaussian mutation, which is expressed as:

$$x_{j,i}^{t+1,k} = x_{ji}^t + (x_{i,\max}-x_{i,\min})N(0,(\sigma_{j,i}^k)^2) \tag{9}$$

where $k \in \{\alpha, \beta\}$, $(\sigma_{j,i}^{k})^2$ denotes Gaussian distribution variance, and its value is designed as:

$$(\sigma_{j,i}^{k})^2 = \begin{cases} \left|\alpha_{j,i}^{t}\right|^2, & k = \alpha \\ \left|\alpha_{j,i}^{t}\right|^2 \big/ 5, & k = \beta \end{cases} \tag{10}$$

To avid generation the infeasible solution, the value of $x_{j,i}^{t+1,k}$ is clipped according to Eq.(12). Until the value of $x_{j,i}^{t+1,k}$ lies in the feasible solution space, Eq.(11) has to be performed repeatedly.

$$\begin{cases} x_{j,i}^{t+1,k} = 2x_{i,\max} - x_{j,i}^{t+1,k},\ x_{j,i}^{t+1,k} > x_{i,\max} \\ x_{j,i}^{t+1,k} = 2x_{i,\min} - x_{j,i}^{t+1,k},\ x_{j,i}^{t+1,k} < x_{i,\min} \end{cases} \tag{11}$$

If the feasible solution derived from Eq.(9)~(11) $(x_{j,1}^{t}, \cdots, x_{j,i}^{t+1,k}, \cdots x_{j,n}^{t})^T$ is superior to the feasible solution $(x_{j,1}^{t}, \cdots, x_{j,i}^{t}, \cdots x_{j,n}^{t})^T$, then the valid evolution is carried out, and $x_{j,i}^{t} = x_{j,i}^{t+1,k}$, $\alpha_{j,i}^{t+1} = \alpha_{j,i}^{t}$, $\beta_{j,i}^{t+1} = \beta_{j,i}^{t}$. Otherwise ,the invalid evolution is done ,the feasible solution $(x_{j,1}^{t}, \cdots, x_{j,i}^{t}, \cdots x_{j,n}^{t})^T$ is retained ,and $(\alpha_{ji}^{t}, \beta_{ji}^{t})^T$ is updated by quantum rotation gates as:

$$\begin{bmatrix} \alpha_{j,i}^{t+1} \\ \beta_{j,i}^{t+1} \end{bmatrix} = \begin{bmatrix} \cos(\Delta\theta_{j,i}^{t}) & -\sin(\Delta\theta_{j,i}^{t}) \\ \sin(\Delta\theta_{j,i}^{t}) & \cos(\Delta\theta_{j,i}^{t}) \end{bmatrix} \begin{bmatrix} \alpha_{j,i}^{t+1} \\ \beta_{j,i}^{t+1} \end{bmatrix} \tag{12}$$

where $\Delta\theta_{j,i}^{t}$ is the rotation angle, and the value of $\Delta\theta_{j,i}^{t}$ is design as:

$$\Delta\theta_{j,i}^{t} = \mathrm{sgn}(\alpha_{j,i}^{t}\beta_{j,i}^{t})\theta_0 \exp\left(-\frac{\left|\beta_{j,i}^{t}\right|}{\left|\alpha_{j,i}^{t}\right| + \gamma}\right) \tag{13}$$

where $\mathrm{sgn}(\cdot)$ is the sign function and determines the direction of $\Delta\theta_{j,i}^{t}$, θ_0 is the initial rotation angle; γ is evolutionary scale. θ_0, γ and $(\alpha_{j,i}^{t}\beta_{j,i}^{t})^T$ decide the size of $\Delta\theta_{j,i}^{t}$ together , control further the convergence rate.

In RQEA, discrete crossover is performed at period τ_c. Select individual p_u^t and p_v^t, $u \neq v$, $u = 1,2,\cdots,N$, $v = 1,2,\cdots,N$, at random in population, let p_u^t and p_v^t as parents, exchange ever corresponding gene of them by 0.5 probability, and generate new individual c^t.

3.2. Procedures of RQEA

The procedures of RQEA can be described as follows:

Step1 Determine the parameters: N, θ_0, τ_c, and initialize a population $p^t = \{p_j^t\}$.

Step2 Evaluate fitness value of each individual and select the best individual b^t.

Step3 Update population p^t by mutation operator. For the individual p_j^t, $j = 1,2,\cdots,N$.

Step3.1 Update the variable $x_{j,i}^{t}$ of the i-th gene in turn using Eq.(9)~(11), and 'Fine search' and 'coarse search' is performed m_1 and m_2 times, respectively. Select the best individual b^t.

Step3.2 Update the probability amplitudes $(\alpha_{ji}^t, \beta_{ji}^t)^T$ of the i-th gene in turn using Eq.(12) and Eq.(13).

Until all individuals in population are updated,Step3 is performed repeatedly.

Step4 Carr out the discrete crossover repeat m_3 times for individuals selected in p^t when the discrete crossover condition is satisfied.

Step5 Loop to Step2 until a termination criterion is satisfied.

4. Combined Forecasting Model based on Predictive Availability and RQEA

4.1. Selecting Single Forecasting Model

By researching the basic settlement law of subgrade and characteristics of settlement curve,

we can know that the developing process of subgrade settlement presents 'S' type curve in linear loading, then the growth curve with the S-type characteristics are adopted as single forecasting model. In this paper, single forecasting model contains Usher model, Logistics model and Gompertz model, the expressions of each model are:

Usher model:

$$S(t) = \frac{k}{\left(1 + ae^{-bt}\right)^{\frac{1}{c}}} \tag{14}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b, c are the parameters to be estimated.

Gompertz model:

$$S(t) = ke^{-ae^{-bt}} \tag{15}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

Logistic model:

$$S(t) = \frac{k}{1 + ae^{-bt}} \tag{16}$$

where $S(t)$ is the settlement value corresponding to the time t, k, a, b are the parameters to be estimated.

4.2. Constructing Combined Forecasting Model

Construting combined forecasting model of subgrade settlement based on maximum principle of forecasting availability through Usher, Logistics and Gompertz of single forecasting models. The establishing process of combined forecasting model is that the weights of each single forecasting model is determined.

In this paper, we use RQEA to solve the following numerical optimization problem, and determine each single forecasting model weights.

$$\max m_c = \frac{1}{N}\sum_{t=1}^{N} A_{ct}$$
$$st.\begin{cases} A_{ct} = 1 - \left|\sum_{i=1}^{3} \omega_{it}\left(s_t - \hat{s}_{it}\right)/s_t\right| \\ \sum_{i=1}^{3} \omega_{it} = 1 \end{cases} \tag{17}$$

where s_t is observation data sequence of subgrade settlement, $\hat{s}_{it}$ is the predicted result of the *i*-th single forecasting model at the *t*-th time observation, ω_{it} is the weight of the *i*-th single forecasting model, $t = 1,2,\cdots,N$, $i = 1,2,3$.

5. The Application of Combined Forecasting Model

In this paper, choose Ning-Hang highway subgrade NH standard K095+520 section as observation point, select 15 group settlement observation data from 2001-08-09 to 2002-12-14 as research object[2]. Using Usher model, Logistic model, Gompertz model and combined forecasting model to model and forecast, the first 11 data are used to model and after 4 data are used to forecast.

RQEA proposed is used to Estimating each single forecasting model parameter values, and the value of estimated parameters are shown in table 1. Forecast results and relative errors of each single forecasting model and combined forecasting model are shown in table 2. Forecast results contrast of each single forecasting model and combined forecasting model is shown in figure 1, relative error contrast is shown in figure 2.

From table 2, we can see that the relative error maximum of fitting value of combined forecasting model is 4.0187%, it is better than one of Usher model which is 6.4282%, Logistics model which is 6.0012% and Gompertz model which is 6.7119%, the elative error maximum of predicting value of combined forecasting model is 0.6340%, it is lower than the one of Usher model is 0.7554%, Logistic model is 1.0744% and Gompertz model is 1.9443%. So the fitting and forecasting precision of combined forecasting model is more superior to each single forecasting model.

Table 1. Four Kinds of Forecast Methods Comparison of Forecast Results

Model	Parameter estimation			
	a	*b*	*c*	*k*
Usher	5.5590	-3.9067	5.6292	-5.2187
Logistic	6.1583	-4.7323	6.2051	-5.5293
Gompertz	6.3039	3.9047	6.3465	3.2549

In figure 1, the settlement development curves from each forecasting model are describes respectively. We could know that combined forecasting model can reflects the characteristics and law of settlement development curve in both fitting period and forecasting period, but each single forecasting model has good performance at some time.

Table 2. Four Kinds of Forecast Methods Comparison of Forecast Results

No.	Measured values (cm)	Usher Model		Logistics model		Gompertz model		RQEA-SVM model	
		Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）	Predicted values (cm)	Relative error（%）
1	5.35	5.5590	-3.9067	5.6292	-5.2187	5.6878	-6.3139	0.5565	-3.8598

2	5.88	6.1583	-4.7323	6.2051	-5.5293	6.2218	-5.8124	6.1163	-4.0187
3	6.56	6.3039	3.9047	6.3465	3.2549	6.3518	3.1736	6.3905	2.5830
4	7.48	6.9992	6.4282	7.0282	6.0012	6.9780	6.7119	7.1213	4.7947
5	7.82	7.6306	2.4221	7.6540	2.1229	7.5595	3.3312	7.6660	1.9689
6	8.05	7.9754	0.9271	7.9968	0.6614	7.8854	2.0452	7.7969	0.9080
7	8.38	8.1905	2.2608	8.2105	2.0227	8.0927	3.4282	8.2184	1.9281
8	8.59	8.6772	-1.0156	8.6915	-1.1821	8.5763	0.1599	8.6337	-0.5093
9	8.84	9.0932	-2.8643	9.0973	-2.9103	9.0107	-1.9309	9.0103	-1.9264
10	9.01	9.2409	-2.5629	9.2394	-2.5464	9.1711	-1.7880	9.1653	-1.7241
11	9.18	9.4438	-2.8735	9.4324	-2.7499	9.3979	-2.3735	9.3635	-1.9991
12	**9.81**	**9.7359**	**0.7554**	**9.7046**	**1.0744**	**9.7407**	**0.7069**	**9.7478**	**0.6340**
13	**10.04**	**9.9770**	**0.6274**	**9.9225**	**1.1699**	**10.0423**	**-0.0234**	**9.9954**	**0.4437**
14	**10.06**	**10.0198**	**0.3999**	**9.9604**	**0.9899**	**10.0981**	**-0.3784**	**10.0345**	**0.2527**
15	**10.11**	**10.1745**	**-0.6382**	**10.0951**	**0.1472**	**10.3066**	**-1.9443**	**10.1715**	**-0.6088**

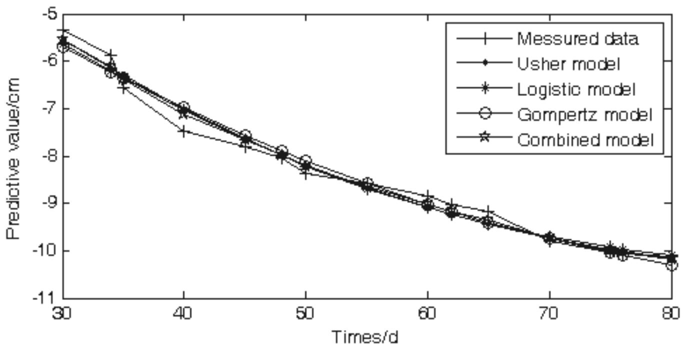

Figure 1. The Predictive Effect of Four Kinds of Forecast Methods

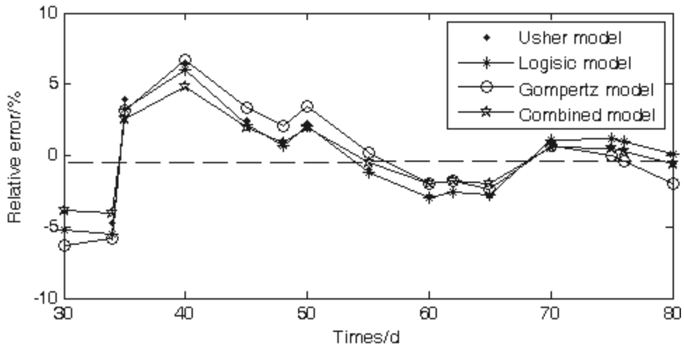

Figure 2. The Relative Error of Four Kinds of Forecast Methods

In figure 2, the error curves of each forecasting model are illustrates respectively. We could see that the relative error of single forecasting model appears larger fluctuations at the beginning of the model period, and the relative error is also larger during the forecast period, but the relative error of predicted results of combined forecasting model is relatively stable, it reduces the risk of forecasting.

6. Conclusion

This paper has proposed the combined forecasting model for subgrade settlement based on forecasting availability and RQEA. Its core is that, firstly, based on the research of the basic settlement law of subgrade and characteristics of settlement curve, we know that the growth curve with the S-type characteristics can be better to present the developing process of subgrade settlement, so Usher、Logistics and Gompertz curve is chosen as single forecasting model, then we build objective function based on the standard of forecasting availability maximization in orde to determine the weights of each single forecasting model, and employ RQEA to solve the objective function to construct the combined forecasting model for subgrade settlement. Engineering examples show that The proposed combined forecasting model could improve the predicting accuracy, low the predicting risk, and has some practical value.

Acknowledgements

This work was supported by Science and Technology Supporting Plan Project of Sichuan Province of China (No. 2013GZX0167, No. 2015FZ0056), and Applied Fundamental Research Plan Project of Sichuan Province of China (No. 2013JY0179).

References

[1] VAPNIK V. An overview of statistical learning theory [J]. IEEE Trans. On Neural networks, vol. 10, no. 5, **(1999)**, pp. 988-999.
[2] K. Tsuda, G. Ratsch and S. Mika, "Learning to predict the leave-one-out error of kemel based classifiers [M]", Lecture Notes in Computer Science, **(2001)**, 2130, pp. 331-338.
[3] O. Chapelle, V. Vapnik and O. Bousquet, "Choosing multiple parameters for support vector machines [J]", Machine Learning, vol. 46, nos. 1-3, **(2002)**, pp. 131-159.
[4] S. Keersthis, "Efficient tuning of SVM hyper-parameters using radius/margin bound and iterative algorithms [J]", IEEE Trans on Neural networks, vol. 13, no. 5, **(2002)**, pp. 1225-1229.
[5] X. Wang, C H. Yang and B. Qin, "Parameters selection of support vector regression based on hybrid optimization algorithm and its application [J]", J of Control Theory and Application, vol. 4, **(2005)**, pp. 371-376.
[6] X F. Yuan and Y N. Wang, "Selection of SVM parameters using chaos optimization algorithms [J]", Control and Decision, vol. 21, no. 1, **(2006)**, pp. 111-113.
[7] X G. Shao, H Z. Yang and G. Cheng, "Parameters selection and application of support vector machines based on particle swarm optimization algorithm [J]", Control Theory & Applications, vol. 23, no. 5, **(2006),** pp. 740-748.
[8] X Z Wang and S Y. Yu, "Improved evolution strategies for high-dimensional optimization [J]", Control Theory & Applications, vol. 23, no. 1, **(2006)**, pp. 148-151.