

A Hausdorff Distance Based Image Registration Algorithm

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Abstract

Hausdorff distance is a common image registration method that is based on the edge features in the image. Theoretically, using Hausdorff distance method, the rotation, scaling and translation factors of the image can be obtained by searching the four-dimensional space that includes one rotation factor, one scaling factor and two translation factors. However, each additional factor means that the dimension degree of searching space is increased one more. The searching speed will be greatly reduced with the increasing of dimension degree. This paper presents a new image registration algorithm that combines the Hausdorff distance with the angle transform. Using this method, the translation factors can be obtained by Hausdorff distance and the angle transform can be directly computed. There is three step for finishing the image registration through the new method. Firstly, discrete Canny edge detection is used on the image. Secondly, the linear features of the image is directly used to calculate the rotation angle parameter. Then Hausdorff distance is used searching in two-dimensional space for getting the translation parameters of the image. Simulation results verified the correctness and validity of the paper method.

Keywords: Hausdorff Distance, Image Registration, Image Features

1. Introduction

Image registration is the necessary premise of image fusion. There are two kinds of image registration methods. One is the methods that are based on gray level information and the other one is the methods that are based on edge features. The registration methods based on gray information are used to matching the images that are obtained from the homogeneous sensors due to the correlation between the gray information. For the heterogeneous images, there are no significant correlation between the gray information because of the giant differences in the imaging mechanism, therefore, the multi-sensor image registration method is often based on feature matching [1].

There are many kinds of image features, such as texture, gray scale, color [2]. The methods based on features are common to find the corresponding relationship among features in the images, and then get the image transformation through measuring the similarity between corresponding features [1]. When the number of features is more than a certain amount, the characteristics of these methods to build are more complex and the computing time will greatly increase. And when there arises the false feature or exists the missing features in the processing of feature extraction, the corresponding relationship is often difficult to get the correct results. Hausdorff distance method based on feature points does not need to establish the relationship between the feature points, but just calculates the similarity between the two point assemblies, which is called maximum distance. So it can effectively deal with the situation of a lot of feature points. **Error! Reference source not found.** The registration based on edge feature uses the spatial information to achieve the register offset. Hausdorff distance, which is an edge feature based registration method, has good robustness, but the matching efficiency will

decrease rapidly with the number of feature points and the transformation factors increasing.

Image transformation often includes three transformation such as Rotary, Scale and Translation (RST) transformation. Aimed at the image of the RT (Rotation-Translation) transform, a semi automatic image registration method based on Hausdorff distance is presents. The processing step of the method is as follows.

Firstly, the edge feature of the image is extracted. And then the typical edge feature is selected to calculate the rotation angle of the image directly. Then the image is rotated reversely. Finally, Hausdorff distance is used to search the shift factors of the edge feature that has been rotated.

The method reduces the search factor dimension degree and greatly improves the register speed. Simulation aimed at the images without scaling transformation, verifies the effectiveness of the proposed method. The scaling factor register research will be the next job.

2. Feature Extraction

Feature extraction not only is the precondition of image registration, but also is the key step of image registration algorithm. Features is usually extracted and expressed with the two-value image.

Extraction of geometric features [4]**Error! Reference source not found.**[6] is mainly divided into two categories. One is the methods that are based on gray feature, another kind is the methods that are based on the edge characteristics. The gray based methods, which use the histogram method or the operator of the point feature extraction, are simple but often cannot meet the requirements.

Canny is an edge feature based extraction method. There are Canny continuous edge extraction method and Canny discrete edge extraction method. Canny continuous edge extraction method is in favor of the extraction of lager outline targets, and Canny discrete edge extraction method is advantageous to extracting the small targets and details.

Although a lots of feature extraction algorithms are presented, but in many aspects such as accuracy, practicality and versatility, there is a great gap from the experimental stage to the requirements of large scale practical application due to the complexity of image feature understanding.

In this paper, the feature extraction method mainly adopts Canny algorithms, and then the characteristics and properties are used to research the matching algorithm.

2.1. Continuous Canny Method

Canny criterion is presented by John F. Canny in 1986, who assumed that the image and filters are continuous. In the continuous domain, there are three rules of Canny criterion such as SNR, positioning principle and unilateral response criteria, respectively.

(1) SNR

$$SNR = \frac{\left| \int_{-w}^w G(-x) f(x) dx \right|}{n_0 \left[\int_{-w}^w f^2(x) dx \right]^{\frac{1}{2}}}$$

Where $f(x)$ is the impulse response of the filter with the boundary $[-w, w]$, $G(x)$ is the representative of the boundary edge, and n_0 is the Mean Square value of Gauss noise.

(2) Location Criterion

$$\frac{\left| \int_{-w}^w G'(-x) f'(x) dx \right|}{n_0 \left[\int_{-w}^w f'^2(x) dx \right]^{\frac{1}{2}}}$$

Where $G'(-x)$ and $f'(x)$ are the first-order derivatives of $G(-x)$ and $f(x)$, respectively.

(3) Unilateral Response Criteria

$\chi_{\max}(f)$ is two-adjacent maximum distance in the noise response of f and χ_{zc} is the average distance of zero crossing point of f . Then the relationship between them is that

$$\chi_{\max}(f) = 2\chi_{zc} - kw$$

Where k is the coefficient which value is smaller than 1.

$$\chi_{zc} = \pi \left[\frac{\left| \int_{-\infty}^{+\infty} f'^2(x) dx \right|^{\frac{1}{2}}}{\left| \int_{-\infty}^{+\infty} f'(x) dx \right|} \right]$$

2.2. Discrete Canny Edge Extraction Error! Reference source not found.

The continuous optimal filter is not optimal any more in the discrete domain. Didier D. *etc.*, deduced the three principle of discrete Canny from Canny continuous criterion.

The First rule Σ : the good detecting results;

The Second criteria Λ : the influence of noise on edge location;

The Third criteria χ_{\max} :the distance between the noise value.

There are several benefits in the discrete principle. First of all, it can be used to directly obtain the optimal filter in discrete domain. Secondly, this criterion can be used to calculate the scale parameter of the classical filter. Through numerical analysis method, using the three rules can get various optimal filters which can meet one single rule or combined rules.

3. Image Registration

3.1. Hausdorff Distance Error! Reference source not found.

Given two finite assembles $A = \{a_1, a_2, \dots, a_p\}$ and $B = \{b_1, b_2, \dots, b_q\}$, then the Hausdorff distance between A and B is defined as follows.

$$H(A, B) = \max\{h(A, B), h(B, A)\}$$

Where

$$h(A, B) = \max_{a \in A} \min_{b \in B} \|a - b\|$$

$$h(B, A) = \max_{b \in B} \min_{a \in A} \|a - b\|$$

$\|\cdot\|$ is the distance norm of the point assembles A and B . This paper use the Euclidean norm.

Function $h(A, B)$ is called the directed Hausdorff distance from the point assemble A to the point assemble B . Here we define that the distance from a point to a limited assemble is the minimum value from this point to all the points of this limited assemble. Then $h(A, B)$ is the maximum value of the distances from each point of assemble A to assemble B . Obviously $h(A, B)$ and $h(B, A)$ do not equal in general. If $h(A, B) = d$, it indicates that the shortest distances from points in the assemble A to points in the assemble B are all in the range of 0 to d . The Hausdorff distance $H(A, B)$ is the maximum value among $h(A, B)$ and $h(B, A)$. Then by calculating the maximum value among $h(A, B)$ and $h(B, A)$, the matching degree can be obtained between assemble A and B .

3.2. Translation Factors based on Hausdorff Distance

From the above definition, $h(A, B)$ is the maximum value from assemble A to assemble B . That means that the Hausdorff distance from A to B is not larger than d and also means that there exists at least one point in assemble A from which the distance to assemble B is d . And those points are the most mismatching points. Obviously, Hausdorff distance represents dissimilar degree between the two assembles. In the process of matching, translation parameters between the main image and the slave image represent by using two-dimensional coordinates (x, y) . That is to say, using the Cartesian coordinates (x, y) represents the translation t . If defining the two value image A_i and B_i are the edge features of the main image A and the slave image B , respectively. Their sizes are $m_A \times n_A$ and $m_B \times n_B$, respectively. The edge characteristic value is 1, the non edge characteristic value is 0, the translation parameters (x, y) may meet

$$-m_B + 1 \leq x \leq m_A - 1$$

$$-n_B + 1 \leq y \leq n_A - 1$$

Feature points numbers of image A and image B are p and q , respectively. And the range shift (x, y) that make the smallest Hausdorff distance between the image A and the image B are the translation parameters.

3.3. Hausdorff Distance and Angle Calculation

In the real matching process, there not only needs to consider the translation (x, y) between the reference image and the slave image, but also needs to consider the rotation relationship θ between images, and sometimes also needs to consider the scale relationship s between two images. In this paper, the current research only considers the image translation and rotation. Translation and the rotation between the main image and the slave image are consist of a set of optimal parameters (x, y, θ) that obtained the highest matching.

Translational optimal parameter images can be obtained by the above method. But the premise is the image rotation angle θ should be solved firstly.

When an image is rotated angle θ , the edge feature is also be rotated angle θ . If we divide the edge into a number of small segments, each segment rotation angle also is θ . So we can calculate the gradients of the segments and then average those gradients of the main image and the slave image, respectively. Then the difference of the average gradients between two images is relationship with the rotation angle θ . Specific calculation steps are as follows.

(1) Firstly, observing same object edge features in the main image and the slave image, and selecting the minimum interference and the most close to the edge of a straight line.

(2) Retaining these features, filtering the other features and obtaining the new two-value images A_m and B_m .

(3) The feature points in images A_m and B_m is divided into several small line segments, and then calculating the gradients of every line segments.

(4) Calculating the inverse Cotangent of the gradients of the small segments, then obtaining the averages of inverse Cotangents of two images, and then calculating the difference of the two averages which is the rotation angle θ .

4. Simulation Result

According to the above analysis, we selected two images for the experiment. Figure 1(a) is the main image and the slave image is shown in Figure 2(a). We use discrete Canny method to extracting the edge features of the main image and the slave image. the results of the edge feature extraction are shown in Figure 1(b) and Figure 2(b), respectively. From the edge feature images, we can see that the edge is too much to auto registration. So using the presented method in this paper, we use manual image segmentation method to choose the longer straight lines for matching image research. In Figure 1(b) and Figure 2(b), the longer straight lines are shown in Figure 3 and Figure 4. The longer straight lines are manually divided into four sections and the results are shown in Figure 5(a)-(b) and Figure 6(a)-(b) respectively.



Figure 1. Main Image and its Edge Feature Image



Figure 2. Slave Image and its Edge Feature Image



Figure 3. Straight Lines Features of Main Image

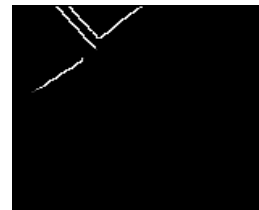


Figure 4. Straight Lines Features of Slave Image

Figure 5(a)-(d) are corresponding with Figure 6(a)-(d), respectively, which means the same objects in the main image and the slave image.

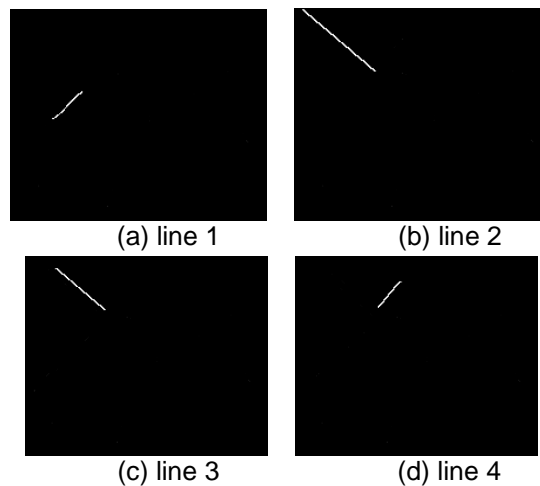


Figure 5. Divided Segment Feature of the Main Image

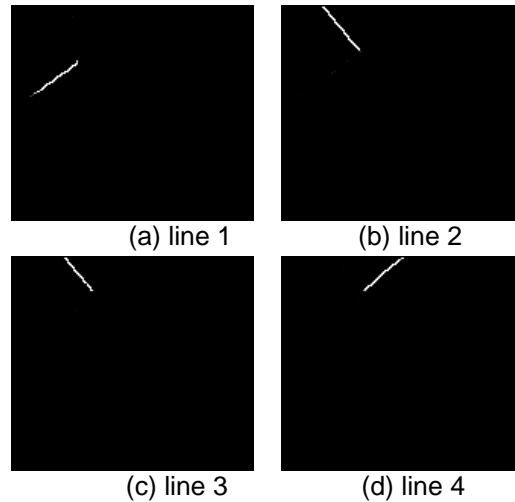


Figure 6. Divided Segment Feature of the Slave Image

Calculating the gradients of each sections, the results are shown in Table 1. According to the above angles, the image rotation angle is -8.4939 degrees, in which, the counter clockwise rotation is positive. The results show that the image rotated clockwise 8.4939 degrees. Rotating the results of Canny edge Extracting for Figure 4, then Hausdorff distance calculation, we get the optimal translation value $(8, 21)$, thus three optimal image registration value are obtained as shown in Table 2.

Table 1. The Straight Lines Angle in the Image(Unit: degree)

Line	Main image	Slave image
Line 1	-0.8491	-0.9746
Line 2	0.8729	0.7086
Line 3	0.8827	0.7086
Line 4	-0.7621	-0.8914

Table 2. The Optimal Parameters of Registration

Parameter	Value
x	8
y	21
θ	8.4939

5. Conclusion

The method of Hausdorff distance combining the angle transform for image registration is proposed. Firstly, the discrete Canny is extracted for image edge, and then straight lines in the image is remained to calculate the angle of rotation parameters, then Hausdorff distance is used to search in 2D space, and finally translation parameter of the image is obtained. The simulation results, verify the correctness and effectiveness of the method. In the simulation process, there are many details need further explanation. First, when calculating the gradients to be attention the correspondence of the start point and the end point of the straight. Secondly, after obtaining the image angle parameters, the image should be rotated and re-sampled. Here we give the re-sampled result in Figure 7. The translation

parameter of registration is the parameter between the resample image and the main image. Thirdly, the registration precision has not be considered in the method which is the next research work.

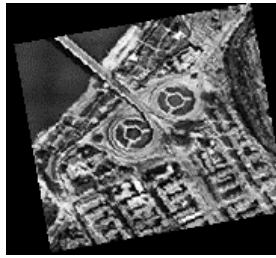


Figure 7. Re-sampled Image

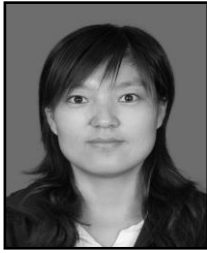
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