An Improved Region Based Active Contour Model for Medical Image Segmentation

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\textbf{Abstract}

Level set methods have been widely used in image processing specially in image segmentation. This paper presents a new region based active contour model in a variational level set formulation for segmentation of real world images in the presence of intensity in-homogeneity and noise. In this paper, we derive a local intensity clustering property in the image domain with better distance regularization function. The level set methods sometimes develop irregularities during its evolution state, which may cause numerical complexity and destroy the stability of evolution. This distance regularization function is able to maintain the desired shape of level set function smoothly and eliminates the need of re-initialization of LSF. The local clustering criterion function is defined for image intensities in neighborhood of each point. Now, this local clustering criterion of point is then integrated with respect to the neighborhood of entire points for global clustering criterion of image segmentation. In which bias function is also evaluated to intensity inhomogeneity correction. Implementation of our method shows that, it is more robust to initialization, and more accurate than conventional model.

\textbf{Keywords:} Image segmentation, intensity inhomogeneity, level set, MRI.

\textbf{1. Introduction}

Image segmentation is a fundamental task in many image processing and computer vision applications. Image segmentation in general has been studied extensively in the past decades. A well-established class of methods is active contour models, which have been widely used in image segmentation with promising results. The models can achieve sub-pixel accuracy and provide closed and smooth contours/surfaces. But, in the presence of noise and intensity inhomogeneity, it is still a difficult problem in majority of applications. Intensity inhomogeneity occurs in real-world images such as microscopy, computer tomography (CT), ultrasound, and magnetic resonance imaging (MRI). The intensity inhomogeneity in real world images often occurs as a variation of intensities. As a result, variation in intensities of the same tissue region varies with the locations in the image. This variation also occurs in CT images, MRI due to the beam hardening effect, radio frequency coil, as well as in ultrasound images caused by non-uniform beam attenuation within the body. In particular, intensity inhomogeneity is a significant challenge to classical segmentation techniques. Widely used technique of medical image segmentation is level set method or active contour model. This well-established method of segmentation is used for promising results of segmentation with accuracy and provide closed and smooth contour of object boundary. This method of segmentation is categorized into two broad classes. First class is edge-based method of segmentation [1–4] uses gradient to guide contour evolution. Due to the use of gradient, segmentation is sensitive to noise and weak edges [4]. While second class is region based method [5–8] utilize the region descriptor such as intensity, texture, color etc. to identify region of interest, to guide curve evolution [12]. Region based methods of segmentation have better
result in the presence of noise and edge leakage of object boundary because this is less sensitive to initial contour location. In this paper, we will focus on region-based active contour model with variational level set formulation for image segmentation.

In this paper, we propose an improved region-based level set method for medical image segmentation in the presence of intensity inhomogeneity and multiplicative Gaussian noise. We derive a local intensity clustering property for the intensities in a neighborhood of each point. This local clustering criterion is integrated over the neighborhood center to define an energy functional, which is converted to a level set formulation with better potential function of distance regularization. This regularization function is able to maintain the desired shape of curve and eliminates the need of re-initialization of level set function. The structure of the paper is organized as follows: Section 2 presents literature survey. In Section 3, describes variational level set framework. Section 4 describes Implementation and experimental results and Section 5 concludes the paper.

2. Literature Survey

2.1. Mumford–Shah Functional Model

Let $\Omega$ be an image domain, $I: \Omega \rightarrow \mathbb{R}$ be gray level intensities of image. The segmentation of image $I$ is obtained by zero level set contour $C$, which separates the image domain $\Omega$ into disjoint regions. This can be formulated as a problem of minimizing the following Mumford-Shah functional [15]

$$ f_{MS}(u, C) = \int_{\Omega} (I - u)^2 \, dx + \mu \int_{\Omega} |\nabla u|^2 \, dx + \nu |C| $$

(1)

Where $|C|$ is a length of contour $C$. The first term is data term, in which piecewise smooth function $u$ that approximates the image $I$. And the second term is smooth term, which forces $u$ to be smooth within each of regions separated by the zero level set contour $C$. the third term is introduced to regularize the contour $C$. Because of different dimension of $u$ and $C$, Solution of equation (1) is difficult. And it may have multiple local minima.

2.2. Chan-Vese’s Model

To overcome the difficulties in solving Eq. (1), Chan and Vese [7] presented an active contour model based on a simplified Mumford–Shah functional. They proposed to minimize the following energy functional as follows [7]

$$ f_{CV}(\varphi, c1, c2) = \int_{\Omega} |I(x) - c1|^2 \, dx + \int_{\Omega} |I(x) - c2|^2 \left(1 - H(\varphi(x))\right) \, dx + \theta \int_{\Omega} |\nabla H(\varphi(x))| \, dx $$

(2)

Where $H$ is a Heaviside function to smooth the curve. $c1$ and $c2$ are two constants that approximate the image intensity in outside and inside the zero level set. The constants $c1$ and $c2$ can be far different from the original data, if the intensities in either outside(C) or inside(C) are not homogeneous that is images with intensity inhomogeneity. Consequently, the CV model generally fails to segment images in the presence of inhomogeneity that is often occurs in real world images.

2.3. Local Intensity Clustering Method

Chunming Li et al. [9] proposed new variational formulation for geometric active contours that forces the level set function to be close to a signed distance function, and therefore completely eliminates the need of the costly re-initialization procedure. Another
problem of Intensity inhomogeneities occur in real-world images and may cause considerable difficulties in image segmentation. In order to overcome the difficulties caused by intensity inhomogeneities, Local intensity clustering model have been proposed. Li et al. [9] improve their works in LBF model and proposed local intensity clustering (LIC) model [9]. They produced local clustering criterion to represent the intensity distribution in neighbourhood of each pixel. Level set formulation of LIC model [9] is described as follows

\[ F_{LIC}(\varphi, c, b) = \int e_1(x) H(\varphi(x)) \, dx + \int e_2(x)(1 - H(\varphi(x))) \, dx + \mu \int_\Omega \frac{1}{2} (|\nabla \varphi(x)| - 1)^2 \, dx + \theta \int_\Omega \delta(\varphi(x)) |\nabla \varphi(x)| \, dx \]  

(3)

The energy functional can be minimized by an interleaved process of level set evolution and estimation of intensity in-homogeneity \( b(x) \) [9]. This model uses Gaussian kernel as the locally spatially weighted function to relate the center pixel and its neighboring pixel, so the LIC model [9] can be regarded as a locally weighted k-mean clustering method. However, in which used potential function have the problem of unboundedness [13] of diffusion rate. This effect may slightly distort the zero level set contours.

3. Variational Level Set Framework with Reformed Potential Function

In this paper, image I is used as a function \( I(x, y) \) defined on domain \( \Omega \). As we know, region based image segmentation is depend on region descriptor. Here we consider local clustering property as a region descriptor, in which we consider circular neighborhood centered at each point \( y \in r \) with radius \( \rho \). Circular neighborhood defined as \( \mathcal{O}_y = x : |x - y| \leq \rho \). Segmentation of these circular neighborhoods is induced by the partition of entire domain of image \( \Omega \). In this paper, circular neighborhood intensities can be classified by calculating the cluster center and bias function using standard k-mean clustering into \( N \) clusters in the presence of intensity inhomogeneity.

3.1 Energy Formulation

Image domain can be partitioned into two regions, object region and background region. These regions can be represented as the region inside and the outside the zero level set. In this method, LSF take two signs positive and negative to separate image domain. The image domain is partitioned into two disjoint region \( \Omega_1 \) and \( \Omega_2 \). Local intensity clustering property indicates that the intensities in the neighborhood \( \mathcal{O}_y \) can be separated into \( N \) clusters, with center \( c_i \), \( i = 1, 2, \ldots, N \). This allows us to apply the standard K-mean clustering. It is a iterative process to minimize the clustering criterion [9, 12]. It can be written as

\[ F_y = \sum_{i=1}^{N} \int_{\mathcal{O}_y} |I(x) - c_i|^2 m_i(x) \, dx \]  

(4)

Where \( c_i \) the cluster in center of the i-th cluster is, \( m_i \) is the membership function of the region, i.e. \( m_i(x) = 1 \) for \( x \in \Omega_i \) and \( m_i(x) = 0 \) for \( x \notin \Omega_i \) [9]. \( G(y-x) \) is a Gaussian kernel used as a window function expressed as [9]

\[ G(d) = \begin{cases} \frac{1}{a} e^{-|d|^2/2\sigma^2}, & \text{for } |d| \leq \rho \\ 0, & \text{otherwise} \end{cases} \]  

(5)
Where \( a \) is a constant, \( d \) is a distance between \( x \) and \( y \) points. \( \sigma \) is a standard deviation or scale parameter of Gaussian function, and \( \rho \) is a radius of the neighboring pixels. Note that the radius of the neighborhood should be selected appropriately according to the degree of the intensity in-homogeneity.

Bias field is used with cluster center. The clustering criterion function \( R \) can be rewritten as [9]

\[
\xi_y = \sum_{i=1}^{N} \int_{\Omega_i} G(y-x)l(x) - b(y)c_i \| dx \quad (6)
\]

The minimization of energy function for all \( y \) in image domain is done by integral of energy function \( \xi_y \) with respect to \( y \).

\[
\xi = \int (\sum_{i=1}^{N} \int_{\Omega_i} G(y-x)l(x) - b(y)c_i \|^2 u_i(\varphi(x)) \| dx ) \| dy \quad (7)
\]

Now regions can be represented with Heaviside function membership function. The membership functions for regions \( \Omega_2 \) and \( \Omega_2 \) are \( u_1 = H(\varphi) \), and \( u_2 = 1 - H(\varphi) \) respectively. Energy function \( \xi \) for data function is used to derive level set formulation as follows [9].

\[
g_e = \int (\sum_{l=1}^{N} \int_{\Omega_l} G(y-x)l(x) - b(y)c_i \|^2 \| dy u_i(\varphi(x)) \| dx + \theta L(\varphi) + \mu R_p(\varphi) \quad (8)
\]

Where \( L(\varphi) = \int |\nabla H(\varphi)| \| d x, \) is a length term that computes the length of contour. It is used to smooth the zero level contour. And \( R_p(\varphi) = \int p(\| \nabla \varphi \|) \| d x \) is a regularization term [9] with reformed potential function [13] that is described in Section 3.

The Heaviside function \( H \) is replaced by a smooth function that approximates \( H \), called the smoothed Heaviside function \( H_e \), which is defined by

\[
H_e(x) = \frac{1}{2} \left[ 1 + \frac{2}{\pi} \arctan \left( \frac{x}{\varepsilon} \right) \right] \quad (9)
\]

The Dirac delta function \( \delta \), is defined by

\[
\delta_e(x) = H_e'(x) = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + x^2} \quad (10)
\]

\( \delta_e \) is the derivative of \( H_e \). The parameter \( \varepsilon \) is set to 1.5[13]

### 3.2. Reformed Potential Function for Regularization Term

The distance regularization term expressed as previous paper [23] in the gradient flow of energy formulation \( \mu R_p'(\varphi)[9][13] \)

\[
\frac{\partial \varphi}{\partial t} = \mu \text{div} \left( d_p(\| \nabla \varphi \|) \| \nabla \varphi \| \right) \quad (11)
\]

Here diffusion rate \( d_p(\| \nabla \varphi \|) \) can be positive or negative in the term of potential \( p \). The diffusion \( D = d_p(\| \nabla \varphi \|) \) is called a forward and backward diffusion. When, diffusion rate \( D \) is positive then the diffusion is forward which decreases \| \nabla \varphi \| . And when diffusion rate \( D \) is negative then the diffusion is backward which increases \| \nabla \varphi \|. FAB diffusion increases or decreases the sign distance function \( \| \nabla \varphi \| \) to get one of the minimum points for potential function. The minimum point is 1, which is used to maintain the desired shape of LSF [13]. The main drawback of this potential function is the unboundedness of diffusion rate. In which, when \( \| \nabla \varphi \| \) is at zero then diffusion rate \( D \) leads to negative infinity by \( \mu d_p(\| \nabla \varphi \|) = \mu (1 - \frac{1}{\| \nabla \varphi \|}) \). Backward diffusion with large diffusion rate increases \( \| \nabla \varphi \| \).
drastically and as a result oscillation in \( \varphi \), that appears peaks and valleys in the LSF evolution. This may reason of irregularities [13] in zero level contours. In this paper, we will use another kind of potential function to avoid this undesirable side effect of slightly distortion in zero level set contours.

A new potential function is able to maintain signed property \(|\nabla \varphi| = 1 \) only near of zero level set. In which, the potential function \( p(|\nabla \varphi|) \) must have minimum points at \(|\nabla \varphi| = 0 \) and \(|\nabla \varphi| = 1 \). This is called double well potential function. This potential function avoids the drawback of previous potential function [13]. A preferable potential function \( p \) for the distance regularization term \( R_p \) is double-well potential. Here provide a specific construction of the double-well potential \( P_2(s) \) as [13]

\[
P(s) = \begin{cases} 
\frac{1}{(2\pi)^2}(1 - \cos(2\pi s)), & \text{if } s \leq 1 \\
\frac{1}{2}(s - 1)^2, & \text{if } s \geq 1
\end{cases}
\]

(12)

The potential \( P_2(s) \) has two minimum points at \( s=0 \) and \( s=1 \) to eliminate the previous problem of unboundedness, here signed function defined as \( s = |\nabla \varphi| \). Potential function \( P_2(s) \) have the first derivatives given by[13]

\[
P'_2(s) = \begin{cases} 
\frac{1}{2\pi}\sin(2\pi s), & \text{if } s \leq 1 \\
s - 1, & \text{if } s \geq 1
\end{cases}
\]

(13)

The diffusion rate for the potential function \( d_p(s) = P'_2(s)/s \) satisfies [13] the boundedness as follows

The sign of function \( d_p(s) \) for potential function that indicates the property of the FAB diffusion in the following three cases: [13]

1. For \( s > 1 \), the diffusion rate \( \mu D \) is positive, and the diffusion is forward, which decreases \( s \) [13].
2. For \( \left( \frac{1}{2} \right) < s > 1 \), the diffusion rate \( \mu D \) is negative, and the diffusion becomes backward, which increases \( s \) [13].
3. For \( < \left( \frac{1}{2} \right) \), the diffusion rate \( \mu D \) is positive, and the diffusion is forward, which further decrease \( s \) down to zero [13].

The boundedness [13] of the corresponding diffusion rate \( D \) with the property of diffusion for 3rd case is the main difference between diffusions with previous potential and new potential [13].

### 3.3. Minimization of Energy using Gradient Descent Flow

We get the segmented region by minimization of energy function \( g_2 \) in equation (11) with respect to \( \varphi \) can be obtained by solving the gradient descent flow [9]. The gradient flow equation as

\[
\frac{\partial \varphi}{\partial t} = -\delta(\varphi)(e1 - e2) + \partial \delta(\varphi) div \left( \frac{\nabla \varphi}{|\nabla \varphi|} \right) + \mu R'_p(\varphi)
\]

(14)

Where \( \nabla \) the gradient operator, \( \text{div} (.) \) is the divergence operator, and the function \( R'_p(\varphi) \) is defined as reformed potential function \( \frac{\partial \varphi}{\partial t} = \mu \text{div}(d_p(|\nabla \varphi|)\nabla \varphi) \) that is described above subsection 3.2

In energy minimization, first term is defined as [9]

\[
e_1(x) = \int G(y-x)l(x) - b(y)c_2^2 \, dy
\]

(15)

And

\[
e_2(x) = \int G(y-x)l(x) - b(y)c_2^2 \, dy
\]

(16)

Note that \((e1 - e2)\) is not dependent to scale of local intensities due to intensity inhomogeneity. Energy minimization with respect to \( c \) defined by [9]
\[ \hat{c}_i = \frac{\int (G \ast b)u_i \, dy}{\int (G \ast b^2)u_i \, dy}, \quad i = 1 \ldots N \] (17)

The bias field \( b \) that is used to minimize the energy with respect to \( b \), denoted by \( \hat{b} \) as follows[9]

\[ \hat{b} = \frac{(l \sum_{i=1}^{N} c(u_i) + G)}{\sum_{i=1}^{N} c_i^2 u_i + G} \] (18)

With \( u_1(y) = H_1(\varphi(y)) \) and \( u_2(y) = 1 - H_2(\varphi(y)) \).

We minimize the energy formulation \( g_\epsilon \) to obtain the region of interest. This is iterative process, in each iteration energy is minimized [13].

4. Implementation and Experiment

The implementation of our proposed iterative procedure is straightforward, that is presented below:

Algorithm implementation:
- Data: The input image \( (I_j)_{j=1}^{N} \).
- Result: Level set function \( \varphi \).

Step1. Initializing the level set function \( \varphi = \varphi^0 \).

Step2. Initialize the bias function \( (b_j)_{j=1}^{N} = 1 \).

Step3. For \( t \leftarrow 1 \) to maxiter do

   Step4. Computing the mean constant \( c \) by eq. (17)

   Step5. Updating the level set function by eq. (14).

   Step6. Updating the bias function \( b \) by eq. (18)

   Step7. If \( \varphi^t = \varphi^{t-1} \). Then stop.

Step8. Otherwise, return to step 2 until the convergence criteria is met.

4.1 Numerical Implementation

The implementation of our method is iterative with less numerical complexity. In this paper, we are using the double potential function which would greatly raise the accuracy. Choosing the parameter [9] in our proposed method is simple, such as time step \( t \) and parameter \( \mu \) are fixed to 0.1 and 1.0 respectively. Most of digital images, the parameter \( \theta \) is set to 0.001 \times 255^2 with intensity values 0-255. \( G \) is a \( w \times w \) mask window operator with odd number of \( w \). The experiments are implemented in MATLAB2012a using PC with Intel (R) Core (TM) 2 Duo CPU.

Fig.1 shows the results for synthetic image in which image domain have two regions. Firstly initial contour are plotted on the image anywhere, after that curve evolution steps are presented after sufficient evolution, we obtained region of interest [12]. Fig.2 shows that, segmentation of region of interest does not depend on shape of initial contour, it may be circular, rectangular etc. In which, first column represent circular initial contour on brain MRI, second column represent rectangular initial contour, and third column represents different initial contour on MRI. We obtained region of interest in all cases with accuracy in the presence of intensity inhomogeneity and noise.
Figure 1. Contour Evolution Steps from Initial to Final Contour for Synthetic Image

Figure 2. Shows Robustness to Contour Initialization on brain MRI Lasast Row Presents the Tumor Segmentation

4.2. Performance Evaluation:

4.2.1. RMSE (Root Mean Square Error): As a level set method, our method provides a contour as the segmentation result. Therefore, we use the following contour-based metric for precise evaluation of the segmentation result. Let C be a contour as a segmentation result, and S be the true object boundary, which is also given as a contour. For each point $P_i$, $i=1,2,…,N$ on the contour, we can compute the distance from the point $P_i$ to the ground truth contour, denoted by $dist(P_i, S)$. Then, we define the deviation from the contour C to the ground truth S by [13, 9].

$$e_{mean}(C)=\frac{1}{N}\sum_{i=1}^{N}dist(P_i, S)$$  \hspace{1cm} (19)

This is referred to as the mean error of the contour. This contour-based metric can be used to evaluate a sub pixel accuracy of a segmentation result given by a contour.

4.2.2. COS (Coefficient of similarity): Coefficient of similarity is a Quantitative analysis of image segmentation. Its value should be near to 1. Where $I_{exp}$ and $I_{algo}$ are the gold standard image and segmented image respectively

$$COS=1 - \frac{|I_{exp}-I_{algo}|}{I_{exp}}$$  \hspace{1cm} (20)
4.3. Experiment Result

4.3.1. Comparison with local intensity clustering method in the presence of increasing noise: Our method works better in the presence of multiplicative Gaussian noise and intensity inhomogeneity. The images including Gaussian noise with increasing value of standard deviation. The evaluated results of the proposed method and the Local intensity clustering (LIC) method by Li et al. are presented in the second and third rows in Figure 3. The Root mean square error RMSE values are presented below table of these images.

**Table 1. Comparison of our Method and LIC Model using RMSE and COS for Figure 3**

<table>
<thead>
<tr>
<th></th>
<th>Image 1</th>
<th></th>
<th>Image 2</th>
<th></th>
<th>Image 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>COS</td>
<td>RMSE</td>
<td>COS</td>
<td>RMSE</td>
<td>COS</td>
</tr>
<tr>
<td>Our method</td>
<td>0.06</td>
<td>0.911</td>
<td>0.40</td>
<td>0.859</td>
<td>1.21</td>
<td>0.815</td>
</tr>
<tr>
<td>LIC method</td>
<td>0.09</td>
<td>0.816</td>
<td>0.58</td>
<td>0.849</td>
<td>1.52</td>
<td>0.799</td>
</tr>
</tbody>
</table>

**Figure 3. Performance of Proposed Model and LIC Model in Different MRI Image Conditions in the Presence of Different Noise, Leakage Boundary, and Intensity Inhomogeneity. Top Row Represents Initial Contours, Middle Row Represents Results of Proposed Method, and Third Row Represents Results of the LIC Model.**

4.3.2. Elimination of Re-initialization: Our method with better potential function of regularization term avoids the need of re-initialization of LSF. It maintains the desired shape of contour signed distance function near the zero level set.

4.3.3. Robustness to LSF initialization: Performance of our model with different shapes of initialization and different parameter is quantitatively evaluated in the robust manner. If we apply our method to MRI image with different initialization of contour plotting. The results of RMSE have minor difference.

**Table 2 Mean Errors and COS for 4 Different Initialization of Contour**

<table>
<thead>
<tr>
<th>Initialization of contour</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Error</td>
<td>0.23</td>
<td>0.24</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>COS</td>
<td>0.891</td>
<td>0.895</td>
<td>0.897</td>
<td>0.896</td>
</tr>
</tbody>
</table>
5. Conclusion

We have presented an improved region based level set method in a variational level set framework with much better distance Regularization potential function for medical image segmentation in the presence of intensity inhomogeneities and noise. The proposed method has an intrinsic capability of maintaining regularity of level set function. Our method is much more robust to contour initialization than the previous local intensity clustering model. It has stable performance for different scale parameters Experimental results have demonstrated superior performance of our method in terms of accuracy, efficiency, and robustness, due to better distance regularization term. This new potential function embedded in this level set evolution can eliminate the need of re-initialization of LSF. As an application, our method has been applied to MR image segmentation with promising results.

References

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