Research on Granular Computing Approach in Rough Set

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Abstract

Granulation of information appears in many areas, such as machine learning, evidence theory, and data mining. Granular computing is the core research field in granulation of information. It is an effective tool for complex problem, massive data mining and fuzzy information processing. In the basis of principle of granularity, we aim to study the granular decomposing method in granules space based on rough set. Moreover, the criteria conditions for attribution necessity and attribute reduction are proposed. Finally, the corresponding equivalence is proved to traditional rough set theory. It will lay the foundation for attribute reduction under the granular representation in rough set.

Keywords: rough set; granular computing; attribute reduction; equivalence relation

1. Introduction

Rough set theory (by Prof. Pawlak in 1982 [1, 2]) is an important theory for uncertainty knowledge in a wide variety of applications such as pattern recognition, image processing, neural computing, decision support, data mining and knowledge discovery [3-8]. Recently, several extensions of the rough set model have been proposed, such as the decision theoretic rough set model [9], the rough set model based on tolerance relation [10], the Bayesian rough set model [11], the Dominance-based rough set model [12], game-theoretic rough set model [6, 7], the fuzzy rough set model and the rough fuzzy set model [8].

Granular computing (GrC), which was introduced by Prof. Lin in 1997 [13], is an intelligent calculation theory based the division of concept space. It is an effective tool for complex problem, massive data mining and fuzzy information processing. With the rapid development of data technology, it has been paid more attention gradually to artificial intelligence researchers. In general, GrC is a superset of fuzzy information granulation theory, rough set theory and interval computations, and is a subset of granular mathematics. Many researchers have argued that information granulation is very essential for solving fault diagnosis problem, and hence there is a very significant impact on the design and implementation of intelligent fault diagnosis system [14]. Description of granules means to name and label granules using certain languages, in which each label represents a concept such that an element in the granule is an instance of the named category. The granulated view summarizes available information and knowledge about the universe. It may be argued that the construction, interpretation, description, and connection of granules are of fundamental importance in the understanding, representation, organization and synthesis of data, information and knowledge [15].
On the basic of granular model in rough set [14], we aim to study the granular decomposing method in granules space based on rough set. Moreover, the decomposition method of granular space and the necessity of attributes under the granular representation and the determine conditions of attribute reductions are proposed. Furthermore, the corresponding equivalence is proved to traditional rough set theory.

2. Preliminary Knowledge

Definition 1 (relative positive region [2]): Given the domain \( U \), \( P \) and \( Q \) are the equivalence relationships in \( U \). The positive domain \( P \) of \( Q \) is denoted as \( Pos_P(Q) \), \( Pos_P(Q) = \bigcup_{a \in Q} P_a(X) \).

Definition 2 (relative core [2]): Given the domain \( U \), \( P \) and \( Q \) are the equivalence relationship in \( U \). If \( Pos_P(Q) = Pos_{P \cup R}(Q) \), \( R \) is unnecessary for \( Q \) in \( P \), otherwise necessary. The necessary relationship attributes set for \( Q \) in \( P \) is called \( Q \)-core of \( P \) and denoted as \( CORE_Q(P) \).

Definition 3 (relative reduction [2]): Given the domain \( U \), \( P \) and \( Q \) are the equivalence relationship in \( U \). If the independent subset \( S(S \subset P) \) has \( Pos_S(Q) = Pos_P(Q) \), \( S \) is called the \( Q \) reduction of \( P \).

The function \( f^{-1}(a,v) \) is described as the objects set, which is constructed by the objects whose value on attribute \( a(a \in A) \) is \( v \). Therefore, the granule of the decision table is defined as \( Gr = ((a,v), f^{-1}(a,v)) \), where \((a,v)\) is the syntax for granule \( Gr \). \( Gr \) is called the atomic granule in decision table.

Suppose \( \varphi \) and \( \psi \) are logical composition which are constructed by the atomic formulas (such as \((a,v)\)) conjunction with the logical symbols (\(\neg, \land, \lor, \rightarrow \) and \(\leftrightarrow\)). The function \( f^{-1}(\varphi) \) represents the objects set to satisfy \( \varphi \). So, \( Gr = (\varphi, f^{-1}(\varphi)) \) is called the composition granule to \( \varphi \).

Definition 4 (granule set): Suppose \( gs(Gr) \) is the mapping function to describe granule to object set, for any granule \( Gr = (\varphi, f^{-1}(\varphi)) \), \( gs(Gr) = f^{-1}(\varphi) \).

3. Decomposition of Decision Table Granules

Using granules to deal the decision table directly, a primary condition is to obtain sufficient granules to describe decision table. It is the decomposition of granular space of decision table.

Supposed \( GrS \) is the granules set of decision table \( S \) with decomposition. \( \forall Gr \in GrS \), the syntax of \( Gr \) is described by all attributes in condition attributes \( C(C \subseteq R) \), \( (Gr = (\varphi, f^{-1}(\varphi)), \varphi = (a_1,v_1) \land (a_2,v_2) \land \cdots (a_m,v_m), m = |C| \), and satisfies \( \exists x \in \forall Gr \Rightarrow x \in gs(Gr) (\forall x \in U) \). The granules sets satisfied the above conditions are called a granular space on the decision table. The algorithm for seeking granular space \( GrS \) of decision table is given as below. To construct the granule according to the equivalence class that is determined by the condition attribute set in \( U \), for each equivalence class in \( U \), using all attributes in the equivalence class of object value directly.
Research on traditional rough set, the information table consisted by sample objects are adapted to analyze and process. In granular computing, it will be adapted. The granules syntax table is given as following.

Given the granular space $GrS$, which is decomposed by the decision table $S$, it can construct the granular syntax table $GrT$ for decision table $S$. In $GrT$, every column is described as the granular syntax.

### Table 1. Decision Table Information System

<table>
<thead>
<tr>
<th>$\bar{R}$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
<th>$x_6$</th>
<th>$x_7$</th>
<th>$x_8$</th>
<th>$x_9$</th>
<th>$x_{10}$</th>
<th>$x_{11}$</th>
<th>$x_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$a_3$</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>$a_4$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$d$</td>
<td>$N$</td>
<td>$N$</td>
<td>$P$</td>
<td>$P$</td>
<td>$N$</td>
<td>$N$</td>
<td>$N$</td>
<td>$P$</td>
<td>$P$</td>
<td>$N$</td>
<td>$N$</td>
<td></td>
</tr>
</tbody>
</table>

**Example 1:** Table 1 is a decision table, wherein $\{a_1,a_2,a_3,a_4\}$ is the condition attribute set, and $\{d\}$ is the decision attribute set, each column in the table represents a record of decision tables. The granular space $GrS$ of the decision table can be achieved. Based on $GrS$, the granular syntax table $GrT$ of decision table can be achieved (Table 2).

### Table 2. Granular Syntax Table $GrT$

<table>
<thead>
<tr>
<th>$C$</th>
<th>$Gr_1$</th>
<th>$Gr_2$</th>
<th>$Gr_3$</th>
<th>$Gr_4$</th>
<th>$Gr_5$</th>
<th>$Gr_6$</th>
<th>$Gr_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>$a_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$a_3$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$a_4$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

After decomposition of decision table and obtaining the granular syntax table, the handle process of the sample objects is simplified to handle the granules.

### 4. Attribute Reduction on Granular Space

#### 4.1. Certainty Process with Decision Table on Granular Space

In traditional rough set, the necessity of attributes and the judgment of attribute reduction are based on the changes of positive domain of system. The changes in the positive domain is due to remove some attributes, it can be divided into the two problems. One is to remove some important attributes. It results a number of the certain equivalence classes are combined, because they have different decision attribute values. The second problem is to remove some important attributes, it causes the original certain equivalence classes and original uncertain equivalence classes are combined, so the conflict is appeared.

In the granular computing, we need find the positive domain granular space ($GrP$) firstly, that is granular space of objects of domain. Secondly, we need find a granular space of non-domain ($GrN$), which is composed by objects of negative domain.

**Theorem 1:** Suppose ($GrS$) is a granular space that decomposed in the decision table $S$. Then the necessary and sufficient condition of completely certainty of the decision tables $S$ is:
For all $\forall x$, there is $\forall Gr \in GrS$, and $x \in (x \in Gr \land x \in Gr) \Rightarrow d(x) = d(x)$, wherein $d$ represents the decision attributes of the decision table $S$.

Proof: By definition 4, the result of theorem 1 is obvious.

According to theorem 1, the positive domain granular space $GrP$ of domain $U$ and non-domain granular space $GrN$ can be obtained.

**Algorithm 1: The Positive Domain Granular Space $GrP$ and Non-domain Granular Space $GrN$ of Domain $U$**

**Input:** the granular space $GrS$ of decision table $S$.

**Output:** the positive domain granular space $GrP$ and non-domain granular space $GrN$ of decision table $S$.

**Steps:**

1. Set $GrP = \Phi$, $GrN = \Phi$.

2. Orderly traversing all granules in $GrS$, $\forall Gr \in GrS$, do:

   If there are two elements $x$ and $y$ in $Gr$ with unequal value in the decision attribute set ($d(x) \neq d(y)$, $d$ is decision attribute of decision table $S$), then $GrN = GrN + \{Gr\}$, else $GrP = GrP + \{Gr\}$.

3. Return $GrN, GrP$.

According to algorithm 1, the domain granular space syntax table $GrPT$ and a non-domain granular space syntax table $GrNT$ can be corresponding constructed. For each granule in positive domain granular space, because all the objects in it have the same decision-making value, so a new row can be added in the positive domain granular space syntax tables, which means that the decision-making properties of each granule contains objects. For each granule in non-domain granular space, because the decision attribute value of the object is not unique, decision-making attributes are not included in the non-domain granules syntax tables.

**4.2. Attribute Relative Necessity under the Granular Described**

Definition 2 gives the attribute relative necessary judge conditions in the traditional tough set theory. The below is the equivalent condition of attributes relative necessary in the granular described.

**Proposition 1 (the necessary condition 1 of attribute reduction):** If the attribute $a(a \in C)$ is a relative reduction in the decision table $S$, then in the domain granular syntax table and non-domain granular syntax table, after removing the rows with attribute $a$, the new granular syntax table can’t produce conflicts (there is not such granules, whose values of syntax are equal in the condition attribute set, while in the decision attribute set has different).

Proof: It is easy to prove.

**Proposition 2 (the necessary condition 2 of attribute relative reduction):** If the attribute $a(a \in C)$ is relative reduction in the decision table $S$, then remove the attribute that the rows contain $a$ in the positive domain granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$, there are not the granule $Gr_i$ and $Gr_j$, while satisfy $Gr_i \in GrP \land Gr_j \in GrN$, and they have the equal syntax in the condition attribute set.
Proof: It is easy to prove.

**Theorem 2:** Set $POS_c(D)$ to be a positive domain of decision table $S$, and $GrP$ is a positive
domain granular space in decision table $S$, then $POS_c(D) = \cup \{ gs(Gr) \mid Gr \in GrP \} .$

**Theorem 3:** The necessary and sufficient conditions of $a$ can be removed in decision
table $S$ are the prerequisites 1 and 2 are all true.

Proof: The necessity: it can be proved according to Proposition 1 and Proposition 2.

The sufficiency (reduction to absurdity):

Suppose $a$ in the decision table $S$ can’t remove, that is $ Pos_{C-\{a\}}(D) \neq Pos_c(D) $ ,
because $ Pos_{C-\{a\}}(D) \subseteq Pos_c(D) $, there should be an object $ x $, which
makes $ x \in Pos_c(D) \land x \notin Pos_{C-\{a\}}(D) $. Moreover, because of $ x \in Pos_c(D) $, so $ \exists Gr \in GrP $, which
makes the $ x \in gs(Gr) $. Set $ Gr_1 $ is a granule, which is removed attribute $ a $ in the $ Gr $, so
obviously $ x \in gs(Gr_1) $. Because $ x \notin Pos_{C-\{a\}} (D) $, so $ \exists y \in gs(Gr_2) $. $ x $ and $ y $ have the same values
in the attribute $ C-\{a\} $, but their decision attributes are different. Because of $ y \in U $, there
would be a granule $ Gr^* $ on the granular space of decision table, which makes $ y \in gs(Gr^*) $.  
For $ Gr^* $, it can be divided into two cases.

1. $ Gr^* \in GrP $, $ Gr^* $ is a granule in the positive domain granular space of decision table.
   $ x \in gs(Gr) $, $ y \in gs(Gr^*) $. $ x $ and $ y $ have the same values on the attribute $ C-\{a\} $, but their
decision attributes are different, so the $ Gr $ and $ Gr^* $ are equal values on the condition attribute
set, but the values on the decision attribute set are not equal, it is contradicted with proposition 1.

2. $ Gr^* \in GrN $, $ Gr^* $ is a granule in the non-domain granular space of decision table.
   $ x \in gs(Gr) $, $ y \in gs(Gr^*) $. $ x $ and $ y $ have the same values on the attribute set $ C-\{a\} $, but their
values of decision attribute are different, so $ Gr $ and $ Gr^* $ are equal values on the condition
attribute set, it is contradicted with proposition 2.

In a summary, theorem 3 is proved.

After the judgment of the relative necessity under the granular described of decision table,
we can obtain the relative core of decision table.

**Algorithm 2: The Relative Core of Decision Table $S$.**

Input: The domain granular grammar table $ GrPT $ and non-domain particles grammar table $ GrNT $ of decision table $ S $.
Output: The relative core $ CORE_c(D) $ of $ S $.

Steps:

1. Set $ CORE_c(D) = \Phi $.

2. Set $ i = 1 $.

3. While ($ i \leq |C| $)
1) Remove the $i$th row (the $i$th condition attributes) in the domain positive granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$.

2) Compare any two columns in $GrPT$, if these are some conflicts (the values are same on condition attribute and they have different values of the decision attribute), set $CORE_c(D) = CORE_c(D) + \{a\}$ and turn to step 4), else turn to the step 5).

3) Compare any two columns between $GrPT$ (does not contain the rows of decision attribute) and $GrNT$, if there are any two rows are equal, then $CORE_c(D) = CORE_c(D) + \{a\}$.

4) Restore the $i$th rows in the positive domain granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$.

5) $i = i + 1$.

(4) Return $CORE_c(D)$.

4.3. Attribute Reduction under the Granular Described

**Definition 5 (projection granule and preimage granule):** Given subset of attributes $B \subseteq C$, choose any $Gr \in GrS$ on the original granular space. There is a corresponding column on the $GrPT_b$ or $GrNT_b$, and use this column as the granule to structure the granular syntax ($Gr_b$). $Gr_b$ is called the projection granule of $Gr$ on the attribute set $B$. $Gr$ is called the preimage granule of $Gr_b$.

From the above definitions, given subset of attribute $B \subseteq C$, there is a corresponding relationship between the projection granule and preimage granule.

**Definition 6 (conflict granule, conflict column):** Set $GrP$ is a domain granular space and $GrN$ is a non-domain granular space. For any $Gr \in GrP$ in attribute subset $B \subseteq C$, if the conditions are satisfied as follow:

1) If there exists the granule $Gr_i \in GrP$, which makes the values of syntax between $Gr$ and $Gr_i$ are equal in the attribute set $B$, but the values are not equal in the decision attribute set.

2) If there exists the granule $Gr_i \in GrN$, which makes the value of syntax between $Gr$ and $Gr_i$ are equal in the attribute set $B$.

$Gr$ is called the conflict granule about $B$ in the $GrP$, and the granule does not meet any of the above conditions $Gr(Gr \in GrP)$ is called non-conflict granule about $B$ in the $GrP$. The $Gr$’s corresponding columns in the $GrPT_b$ or $GrNT_b$ are called conflict columns of $GrPT_b$ or $GrNT_b$.

Given attribute subset $B \subseteq C$, for all conflicts granules about $B$ in positive domain granular space $GrP$, their projection granules called $GrU_b$ on the attribute subset $B$. For all non-conflicts granules on $B$ in $GrP$, their projection granule set called $GrNU_b$ in the attribute subset $B$.

$GrP$ is not decomposed into $GrU_b$ and $GrNU_b$. The granular syntax in $GrP$ contains all the condition attributes, but the granular syntax of $GrU_b$ and $GrNU_b$ only contains few parts of the condition attributes (attribute set $B$). Moreover, the granules in the $GrU_b$ and $GrNU_b$
has corresponding relationship with the granules in the \( \text{GrP} \). In addition, \(|\text{GrP}| = |\text{GrU}_B| + |\text{GrNU}_B|\).

**Theorem 4:** Given attribute subset \( B \subseteq C \), the element set of granules included in \( \text{GrNU}_B \) is the positive domain about decision attribute set of decision table \( S \) on \( B \), that is \( \text{POS}_B(D) = \cup (gs(Gr) | Gr \in \text{GrNU}_B) \).

Proof: Firstly prove \( \cup (gs(Gr) | Gr \in \text{GrNU}_B) \subseteq \text{POS}_B(D) \).

According to the definition of positive domain and non-conflict granule, it is easy to prove. Next prove \( \text{POS}_B(D) \subseteq \cup (gs(Gr) | Gr \in \text{GrNU}_B) \).

\( \forall x \in \text{POS}_B(D) \), because of \( \text{POS}_B(D) \subseteq \text{POS}_C(D) \), \( x \in \text{POS}_C(D) \). Use the value of \( x \) on the attribute set \( B \) to construct the syntax of granule, and then use the syntax to construct \( \text{Gr}_1 \) and \( \text{Gr}_2 \) (belong \( \text{GrNU}_B \) or \( \text{GrU}_B \)).

1. If \( \text{Gr}_1 \) belongs to \( \text{GrNU}_B \), according to the granular computation rules, \( x \in \text{gs}(\text{Gr}_1) \), then \( \text{POS}_B(D) \subseteq \cup (gs(Gr) | Gr \in \text{GrNU}_B) \).

2. If \( \text{Gr}_1 \) belongs to \( \text{GrU}_B \), then there exists \( y \in \text{gs}(\text{Gr}_1) \) satisfies \( d(x) \neq d(y) \), that values of \( x \) and \( y \) in the condition attribute set are equal, but the values in the decision attribute set are different. According the definition of positive domain (\( x \notin \text{POS}_B(D) \)), it is conflict with the known conditions. \( \text{Gr}_1 \) can only belong to \( \text{GrNU}_B \).

**Lemma 1:** The necessary and sufficient condition of attribute \( a \) is reduction of decision table \( S \) is: there is no conflict granule about \( C - \{a\} \) in the positive domain granular space \( \text{GrP} \).

Proof: According to theorem 2, \( \text{POS}_C(D) = \cup (gs(Gr) | Gr \in \text{GrP}) \); according to theorem 4, \( \text{POS}_{C - \{a\}}(D) = \cup (gs(Gr) | Gr \in \text{GrNU}_{C - \{a\}}) \).

Sufficiency:

\( \forall Gr \in \text{GrP} \), set \( \text{Gr}_1 \) is a projection of \( \text{Gr}_1 \) in the attribute set \( C - \{a\} \). Because there is no conflict granule about \( C - \{a\} \) in the positive domain granular space \( \text{GrP} \), so \( \text{Gr}_1 \in \text{GrNU}_{C - \{a\}} \) and \( \text{POS}_C(D) = \text{POS}_{C - \{a\}}(D) \), that \( a \) can be removed for decision table \( S \).

Necessity:

Because the attribute \( a \) can be removed for decision table \( S \), \( \text{POS}_C(D) = \text{POS}_{C - \{a\}}(D) \). Because the decision set does not change, and according to the definition of the positive domain, \( U / \text{IND}(C) = U / \text{NID}(C - \{a\}) \).

Reduction to absurdity: Suppose there is conflict granule \( \text{Gr}_1 \) about \( C - \{a\} \) in the completely positive domain granular space \( \text{GrP} \), according to the definition of the conflict granule, the following two cases may occur.

1. \( \exists \text{Gr}_2 \in \text{GrP} \), which makes syntax of \( \text{Gr}_1 \) and \( \text{Gr}_2 \) has the same values on the attribute set \( C - \{a\} \), but the values are not equal on the decision attribute set. Because of \( \text{Gr}_1 \in \text{GrP} \) and \( \text{Gr}_2 \in \text{GrP} \), so \( \text{gs}(\text{Gr}_1) \subseteq \text{POS}_C(D) \) and \( \text{gs}(\text{Gr}_2) \subseteq \text{POS}_C(D) \). And because the elements of \( \text{Gr}_1 \)
and $Gr_2$ has the same values with the elements on the attribute set $C-\{a\}$, but the values are not equal on the decision attribute set. Obviously, $gs(Gr_1) \subset Pos_{C-\{a\}}(D)$ and $gs(Gr_1) \subset Pos_{C-\{a\}}(D)$, it is contradicted with $Pos_C(D) = Pos_{C-\{a\}}(D)$.

(2) $\exists Gr_2 \in GrN$, which makes syntax of $Gr_1$ and $Gr_2$ has the same values on the attribute set $C-\{a\}$. Because of $Gr_1 \in GrP$, $\forall x \in gs(Gr_1) \Rightarrow x \in Pos_{C-\{a\}}(D)$, $\exists y \in Gr_2 \Rightarrow d(x) \neq d(y)$. The values of $x$ and $y$ in the attribute subset $C-\{a\}$ are equal, but the values in the decision attribute set are different. Therefore $x \notin Pos_{C-\{a\}}(D)$, this is contradicted with $Pos_C(D) = Pos_{C-\{a\}}(D)$.

**Theorem 5 (necessary and sufficient conditions for attribute reduction):** Given any attributes subset $B \subseteq C$ (included core attributes set), the necessary and sufficient conditions, which $B$ is the $D$ reduction to $C$, is there is no conflict granule about $B$ in the positive domain granularity space $GrP$.

Proof: The proof is similar with lemma 1, so it unnecessary to repeatedly prove.

So far, the judgment conditions of attributes reduction under granular representation of decision table and the algorithm of solving attribute cores are obtained. The different heuristic methods can be constructed to sort the non-core attributes, and achieve attribute reduction algorithm under the different granular models.

Set $CGr(B)$ to represents the number of conflict granules about attribute set $B$ in the domain granular space $GrP$, $CGr(B) = |GrU_B|$.

**Definition 7 (importance of attribute):** Set decision table $S$, $C$ is condition attribute set and $D$ is decision attribute set, and attribute set $B \subseteq C$. The attribute importance $SGF(a, B, D)$ of any property $a \in (C-B)$ is defined as fellow:

$$SGF(a, B, D) = CGr(B) - CGr(B \cup \{a\}) = |GrU_B| - |GrU_{B-\{a\}|} \ (B \neq \Phi \text{, and it is unmeaning when } B \text{ equals } \Phi)$$

Obviously, $SGF(a, B, D) \geq 0$, the value of $SGF(a, B, D)$ is more larger, which means the attribute $a$ the more important for decision-table $D$ under the conditions of knowing $B$, and this definition of attribute importance belongs to algebra definition. When the number of attributes $a$ is increased, if you cannot change the number of conflict granules on $B \cup \{a\}$ in the completely domain granularity space $GrP$, the importance of this attribute will be 0.

5. Conclusion

Granulation of information appears in many areas, such as rough sets, evidence theory, machine learning, and data mining. Granular computing is similar to the way of the human understand things, people can summarize some concepts from several similar instances. In the basis of principle of granularity, we aim to study the granular decomposing method in granules space based on rough set. Moreover, the criteria conditions for attribution necessity and attribute reduction are proposed. Finally, the corresponding equivalence is proved to traditional rough set theory. It will lay the foundation for attribute reduction under the granular representation in rough set.
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