

Research on Granular Computing Approach in Rough Set

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Abstract

Granulation of information appears in many areas, such as machine learning, evidence theory, and data mining. Granular computing is the core research field in granulation of information. It is an effective tool for complex problem, massive data mining and fuzzy information processing. In the basis of principle of granularity, we aim to study the granular decomposing method in granules space based on rough set. Moreover, the criteria conditions for attribution necessity and attribute reduction are proposed. Finally, the corresponding equivalence is proved to traditional rough set theory. It will lay the foundation for attribute reduction under the granular representation in rough set.

Keywords: *rough set; granular computing; attribute reduction; equivalence relation*

1. Introduction

Rough set theory (by Prof. Pawlak in 1982 [1, 2]) is an important theory for uncertainty knowledge in a wide variety of applications such as pattern recognition, image processing, neural computing, decision support, data mining and knowledge discovery [3-8]. Recently, several extensions of the rough set model have been proposed, such as the decision theoretic rough set model [9], the rough set model based on tolerance relation [10], the Bayesian rough set model [11], the Dominance-based rough set model [12], game-theoretic rough set model [6, 7], the fuzzy rough set model and the rough fuzzy set model [8].

Granular computing (*GrC*), which was introduced by Prof. Lin in 1997 [13], is an intelligent calculation theory based the division of concept space. It is an effective tool for complex problem, massive data mining and fuzzy information processing. With the rapid development of data technology, it has been paid more attention gradually to artificial intelligence researchers. In general, *GrC* is a superset of fuzzy information granulation theory, rough set theory and interval computations, and is a subset of granular mathematics. Many researchers have argued that information granulation is very essential for solving fault diagnosis problem, and hence there is a very significant impact on the design and implementation of intelligent fault diagnosis system [14]. Description of granules means to name and label granules using certain languages, in which each label represents a concept such that an element in the granule is an instance of the named category. The granulated view summarizes available information and knowledge about the universe. It may be argued that the construction, interpretation, description, and connection of granules are of fundamental importance in the understanding, representation, organization and synthesis of data, information and knowledge [15].

On the basic of granular model in rough set [14], we aim to study the granular decomposing method in granules space based on rough set. Moreover, the decomposition method of granular space and the necessity of attributes under the granular representation and the determine conditions of attribute reductions are proposed. Furthermore, the corresponding equivalence is proved to traditional rough set theory.

2. Preliminary Knowledge

Definition 1 (relative positive region [2]): Given the domain U , P and Q are the equivalence relationships in U . The positive domain P of Q is denoted as $Pos_P(Q)$, $Pos_P(Q) = \bigcup_{x \in U/Q} P_{-}(X)$.

Definition 2 (relative core [2]): Given the domain U , P and Q are the equivalence relationship in U . If $Pos_P(Q) = Pos_{P-r}(Q)$, r is unnecessary for Q in P , otherwise necessary. The necessary relationship attributes set for Q in P is called Q -core of P and denoted as $CORE_Q(P)$.

Definition 3 (relative reduction [2]): Given the domain U , P and Q are the equivalence relationship in U . If the independent subset $S (S \subset P)$ has $Pos_S(Q) = Pos_P(Q)$, S is called the Q reduction of P .

The function $f^{-1}(a, v)$ is described as the objects set, which is constructed by the objects whose value on attribute $a (a \in A)$ is v . Therefore, the granule of the decision table is defined as $Gr = ((a, v), f^{-1}(a, v))$, where (a, v) is the syntax for granule Gr . Gr is called the atomic granule in decision table.

Suppose φ and ψ are logical composition which are constructed by the atomic formulas (such as (a, v)) conjunction with the logical symbols ($\neg, \wedge, \vee, \rightarrow$ and \leftrightarrow). The function $f^{-1}(\varphi)$ represents the objects set to satisfy φ . So, $Gr = (\varphi, f^{-1}(\varphi))$ is called the composition granule to φ .

Definition 4 (granule set): Suppose $gs(Gr)$ is the mapping function to describe granule to object set, for any granule $Gr = (\varphi, f^{-1}(\varphi))$, $gs(Gr) = f^{-1}(\varphi)$.

3. Decomposition of Decision Table Granules

Using granules to deal the decision table directly, a primary condition is to obtain sufficient granules to describe decision table. It is the decomposition of granular space of decision table.

Supposed GrS is the granules set of decision table S with decomposition. $\forall Gr \in GrS$, the syntax of Gr is described by all attributes in condition attributes $C (C \subseteq R)$, ($Gr = (\varphi, f^{-1}(\varphi))$, $\varphi = (a_1, v_1) \wedge (a_2, v_2) \wedge \dots \wedge (a_m, v_m)$, $m = |C|$), and satisfies $\exists Gr \Rightarrow x \in gs(Gr)$ ($\forall x \in U$). The granules sets satisfied the above conditions are called a granular space on the decision table. The algorithm for seeking granular space GrS of decision table is given as below. To construct the granule according to the equivalence class that is determined by the condition attribute set in U , for each equivalence class in U , using all attributes in the equivalence class of object value directly.

Research on traditional rough set, the information table consisted by sample objects are adapted to analyze and process. In granular computing, it will be adapted. The granules syntax table is given as following.

Given the granular space GrS , which is decomposed by the decision table S , it can construct the granular syntax table GrT for decision table S . In GrT , every column is described as the granular syntax.

Table 1. Decision Table Information System

R	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}
a_1	4	1	1	4	3	3	4	4	2	3	2	4
a_2	1	1	1	1	4	3	2	2	3	4	3	1
a_3	3	2	2	3	1	1	2	2	1	1	1	3
a_4	0	1	0	0	0	0	1	1	0	0	0	0
d	N	N	P	P	P	N	N	N	P	P	N	N

Example 1: Table 1 is a decision table, wherein $\{a_1, a_2, a_3, a_4\}$ is the condition attribute set, and $\{d\}$ is the decision attribute set, each column in the table represents a record of decision tables. The granular space GrS of the decision table can be achieved. Based on GrS , the granular syntax table GrT of decision table can be achieved (Table 2).

Table 2. Granular Syntax Table GrT

C	Gr_1	Gr_2	Gr_3	Gr_4	Gr_5	Gr_6	Gr_7
a_1	4	1	1	3	3	4	2
a_2	1	1	1	4	3	2	3
a_3	3	2	2	1	1	2	1
a_4	0	1	0	0	0	1	0

After decomposition of decision table and obtaining the granular syntax table, the handle process of the sample objects is simplified to handle the granules.

4. Attribute Reduction on Granular Space

4.1. Certainty Process with Decision Table on Granular Space

In traditional rough set, the necessity of attributes and the judgment of attribute reduction are based on the changes of positive domain of system. The changes in the positive domain is due to remove some attributes, it can be divided into the two problems. One is to remove some important attributes. It results a number of the certain equivalence classes are combined, because they have different decision attribute values. The second problem is to remove some important attributes, it causes the original certain equivalence classes and original uncertain equivalence classes are combined, so the conflict is appeared.

In the granular computing, we need find the positive domain granular space (GrP) firstly, that is granular space of objects of domain. Secondly, we need find a granular space of non-domain (GrN), which is composed by objects of negative domain.

Theorem 1: Suppose (GrS) is a granular space that decomposed in the decision table S . Then the necessary and sufficient condition of completely certainty of the decision tables S is:

For all $\forall x_1$, there is $\forall Gr \in GrS$, and $x_2((x_1 \in Gr \wedge x_2 \in Gr) \Rightarrow d(x_1) = d(x_2))$, wherein d represents the decision attributes of the decision table S .

Proof: By definition 4, the result of theorem 1 is obvious.

According to theorem 1, the positive domain granular space GrP of domain U and non-domain granular space GrN can be obtained.

Algorithm 1: The Positive Domain Granular Space GrP and Non-domain Granular Space GrN of Domain U

Input: the granular space GrS of decision table S .

Output: the positive domain granular space GrP and non-domain granular space GrN of decision table S .

Steps:

(1) Set $GrP = \Phi$, $GrN = \Phi$.

(2) Orderly traversing all granules in GrS , $\forall Gr \in GrS$, do:

If there are two elements x and y in Gr with unequal value in the decision attribute set ($d(x) \neq d(y)$, d is decision attribute of decision table S), then $GrN = GrN + \{Gr\}$, else $GrP = GrP + \{Gr\}$.

(3) Return GrN, GrP

According to algorithm 1, the domain granular space syntax table $GrPT$ and a non-domain granular space syntax table $GrNT$ can be corresponding constructed. For each granule in positive domain granular space, because all the objects in it have the same decision-making value, so a new row can be added in the positive domain granular space syntax tables, which means that the decision-making properties of each granule contains objects. For each granule in non-domain granular space, because the decision attribute value of the object is not unique, decision-making attributes are not included in the non-domain granules syntax tables.

4.2. Attribute Relative Necessity under the Granular Described

Definition 2 gives the attribute relative necessary judge conditions in the traditional tough set theory. The below is the equivalent condition of attributes relative necessary in the granular described.

Proposition 1 (the necessary condition 1 of attribute reduction): If the attribute $a(a \in C)$ is a relative reduction in the decision table S , then in the domain granular syntax table and non-domain granular syntax table, after removing the rows with attribute a , the new granular syntax table can't produce conflicts (there is not such granules, whose values of syntax are equal in the condition attribute set, while in the decision attribute set has different).

Proof: It is easy to prove.

Proposition 2 (the necessary condition 2 of attribute relative reduction): If the attribute $a(a \in C)$ is relative reduction in the decision table S , then remove the attribute that the rows contain a in the positive domain granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$, there are not the granule Gr_1 and Gr_2 , while satisfy $Gr_1 \in GrP \wedge Gr_2 \in GrN$, and they have the equal syntax in the condition attribute set.

Proof: It is easy to prove.

Theorem 2: Set $POS_c(D)$ to be a positive domain of decision table S , and GrP is a positive domain granular space in decision table S , then $POS_c(D) = \cup\{gs(Gr) | Gr \in GrP\}$.

Theorem 3: The necessary and sufficient conditions of a can be removed in decision table S are the prerequisites 1 and 2 are all true.

Proof: The necessity: it can be proved according to Proposition 1 and Proposition 2.

The sufficiency (reduction to absurdity):

Suppose a in the decision table S can't remove, that is $POS_{C-\{a\}}(D) \neq POS_c(D)$, because $POS_{C-\{a\}}(D) \subseteq POS_c(D)$, there should be an object x , which makes $x \in POS_c(D) \wedge x \notin POS_{C-\{a\}}(D)$. Moreover, because of $x \in POS_c(D)$, so $\exists Gr \in GrP$, which makes the $x \in gs(Gr)$. Set Gr_1 is a granule, which is removed attribute a in the Gr , so obviously $x \in gs(Gr_1)$. Because $x \notin POS_{C-\{a\}}(D)$, so $\exists y \in gs(Gr_1)$. x and y have the same values in the attribute $C - \{a\}$, but their decision attributes are different. Because of $y \in U$, there would be a granule Gr^* on the granular space of decision table, which makes $y \in gs(Gr^*)$. For Gr^* , it can be divided into two cases.

(1) $Gr^* \in GrP$, Gr^* is a granule in the positive domain granular space of decision table. $x \in gs(Gr)$, $y \in gs(Gr^*)$. x and y have the same values on the attribute $C - \{a\}$, but their decision attributes are different, so the Gr and Gr^* are equal values on the condition attribute set, but the values on the decision attribute set are not equal, it is contradicted with proposition 1.

(2) $Gr^* \in GrN$, Gr^* is a granule in the non-domain granular space of decision table. $x \in gs(Gr)$, $y \in gs(Gr^*)$. x and y have the same values on the attribute set $C - \{a\}$, but their values of decision attribute are different, so Gr and Gr^* are equal values on the condition attribute set, it is contradicted with proposition 2.

In a summary, theorem 3 is proved.

After the judgment of the relative necessity under the granular described of decision table, we can obtain the relative core of decision table.

Algorithm 2: The Relative Core of Decision Table S .

Input: The domain granular grammar table $GrPT$ and non-domain particles grammar table $GrNT$ of decision table S .

Output: The relative core $CORE_c(D)$ of S .

Steps:

- (1) Set $CORE_c(D) = \Phi$.
- (2) Set $i = 1$.
- (3) While($i \leq C$)

1) Remove the i th row (the i th condition attributes) in the domain positive granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$.

2) Compare any two columns in $GrPT$, if these are some conflicts (the values are same on condition attribute and they have different values of the decision attribute), set $CORE_C(D) = CORE_C(D) + \{a\}$ and turn to step 4), else turn to the step 5).

3) Compare any two columns between $GrPT$ (does not contain the rows of decision attribute) and $GrNT$, if there are any two rows are equal, then $CORE_C(D) = CORE_C(D) + \{a_i\}$.

4) Restore the i th rows in the positive domain granular syntax table $GrPT$ and non-domain granular syntax table $GrNT$.

5) $i = i + 1$.

(4) Return $CORE_C(D)$.

4.3. Attribute Reduction under the Granular Described

Definition 5 (projection granule and preimage granule): Given subset of attributes $B \subseteq C$, choose any $Gr \in GrS$ on the original granular space. There is a corresponding column on the $GrPT_B$ or $GrNT_B$, and use this column as the granule to structure the granular syntax (Gr_B). Gr_B is called the projection granule of Gr on the attribute set B . Gr is called the preimage granule of Gr_B .

From the above definitions, given subset of attribute $B \subseteq C$, there is a corresponding relationship between the projection granule and preimage granule.

Definition 6 (conflict granule, conflict column): Set GrP is a domain granular space and GrN is a non-domain granular space. For any $Gr \in GrP$ in attribute subset $B \subseteq C$, if the conditions are satisfied as follow:

(1) If there exists the granule $Gr_1 \in GrP$, which makes the values of syntax between Gr and Gr_1 are equal in the attribute set B , but the values are not equal in the decision attribute set.

(2) If there exists the granule $Gr_1 \in GrN$, which makes the value of syntax between Gr and Gr_1 are equal in the attribute set B .

Gr is called the conflict granule about B in the GrP , and the granule does not meet any of the above conditions $Gr(Gr \in GrP)$ is called non-conflict granule about B in the GrP . The Gr 's corresponding columns in the $GrPT_B$ or $GrNT_B$ are called conflict columns of $GrPT_B$ or $GrNT_B$.

Given attribute subset $B \subseteq C$, for all conflicts granules about B in positive domain granular space GrP , their projection granules called GrU_B on the attribute subset B . For all non-conflicts granules on B in GrP , their projection granule set called $GrNU_B$ in the attribute subset B .

GrP is not decomposed into GrU_B and $GrNU_B$. The granular syntax in GrP contains all the condition attributes, but the granular syntax of GrU_B and $GrNU_B$ only contains few parts of the condition attributes (attribute set B). Moreover, the granules in the GrU_B and $GrNU_B$

has corresponding relationship with the granules in the GrP . In addition, $|GrP| = |GrU_B| + |GrNU_B|$.

Theorem 4: Given attribute subset $B \subseteq C$, the element set of granules included in $GrNU_B$ is the positive domain about decision attribute set of decision table S on B , that is $POS_B(D) = \cup(g_s(Gr) | Gr \in GrNU_B)$.

Proof: Firstly prove $\cup(g_s(Gr) | Gr \in GrNU_B) \subseteq POS_B(D)$.

According to the definition of positive domain and non-conflict granule, it is easy to prove.

Next prove $POS_B(D) \subseteq \cup(g_s(Gr) | Gr \in GrNU_B)$.

$\forall x \in POS_B(D)$, because of $POS_B(D) \subseteq POS_C(D)$, $x \in POS_C(D)$. Use the value of x on the attribute set B to construct the syntax of granule, and then use the syntax to construct Gr_1 and Gr_1 (belong $GrNU_B$ or GrU_B).

(1) If Gr_1 belongs to $GrNU_B$, according to the granular computation rules, $x \in g_s(Gr_1)$, then $POS_B(D) \subseteq \cup(g_s(Gr) | Gr \in GrNU_B)$.

(2) If Gr_1 belongs to GrU_B , then there exists $y \in g_s(Gr_1)$ satisfies $d(x) \neq d(y)$, that values of x and y in the condition attribute set are equal, but the values in the decision attribute set are different. According the definition of positive domain ($x \notin POS_B(D)$), it is conflict with the known conditions. Gr_1 can only belong to $GrNU_B$.

Lemma 1: The necessary and sufficient condition of attribute a is reduction of decision table S is: there is no conflict granule about $C - \{a\}$ in the positive domain granular space GrP .

Proof: According to theorem 2, $POS_C(D) = \cup(g_s(Gr) | Gr \in GrP)$; according to theorem 4, $POS_{C-\{a\}}(D) = \cup(g_s(Gr) | Gr \in GrNU_{C-\{a\}})$.

Sufficiency:

$\forall Gr \in GrP$, set Gr_1 is a projection of Gr in the attribute set $C - \{a\}$. Because there is no conflict granule about $C - \{a\}$ in the positive domain granular space GrP , so $Gr_1 \in GrNU_{C-\{a\}}$ and $Pos_C(D) = Pos_{C-\{a\}}(D)$, that a can be removed for decision table S .

Necessity:

Because the attribute a can be removed for decision table S , $Pos_C(D) = Pos_{C-\{a\}}(D)$. Because the decision set does not change, and according to the definition of the positive domain, $U / IND(C) = U / IND(C - \{a\})$.

Reduction to absurdity: Suppose there is conflict granule Gr_1 about $C - \{a\}$ in the completely positive domain granular space GrP , according to the definition of the conflict granule, the following two cases may occur.

(1) $\exists Gr_2 \in GrP$, which makes syntax of Gr_1 and Gr_2 has the same values on the attribute set $C - \{a\}$, but the values are not equal on the decision attribute set. Because of $Gr_1 \in GrP$ and $Gr_2 \in GrP$, so $g_s(Gr_1) \subseteq Pos_C(D)$ and $g_s(Gr_2) \subseteq Pos_C(D)$. And because the elements of Gr_1

and Gr_2 has the same values with the elements on the attribute set $C - \{a\}$, but the values are not equal on the decision attribute set. Obviously, $gs(Gr_1) \not\subseteq Pos_{C-\{a\}}(D)$ and $gs(Gr_2) \not\subseteq Pos_{C-\{a\}}(D)$, it is contradicted with $Pos_C(D) = Pos_{C-\{a\}}(D)$.

(2) $\exists Gr_2 \in GrN$, which makes syntax of Gr_1 and Gr_2 has the same values on the attribute set $C - \{a\}$. Because of $Gr_1 \in GrP$, $\forall x \in gs(Gr_1) \Rightarrow x \in Pos_C(D)$, $\exists y \in Gr_2 \Rightarrow d(x) \neq d(y)$. The values of x and y in the attribute subset $C - \{a\}$ are equal, but the values in the decision attribute set are different. Therefore $x \notin Pos_{C-\{a\}}(D)$, this is contradicted with $Pos_C(D) = Pos_{C-\{a\}}(D)$.

Theorem 5 (necessary and sufficient conditions for attribute reduction): Given any attributes subset $B \subseteq C$ (included core attributes set), the necessary and sufficient conditions, which B is the D reduction to C , is there is no conflict granule about B in the positive domain granule space GrP .

Proof: The proof is similar with lemma 1, so it unnecessary to repeatedly prove.

So far, the judgment conditions of attributes reduction under granular representation of decision table and the algorithm of solving attribute cores are obtained. The different heuristic methods can be constructed to sort the non-core attributes, and achieve attribute reduction algorithm under the different granular models.

Set $CGr(B)$ to represents the number of conflict granules about attribute set B in the domain granular space GrP , $CGr(B) = |GrU_B|$.

Definition 7 (importance of attribute): Set decision table S , C is condition attribute set and D is decision attribute set, and attribute set $B \subseteq C$. The attribute importance $SGF(a, B, D)$ of any property $a \in (C - B)$ is defined as fellow:

$SGF(a, B, D) = CGr(B) - CGr(B \cup \{a\}) = |GrU_B| - |GrU_{B \cup \{a\}}|$ ($B \neq \Phi$, and it is unmeaning when B equals Φ).

Obviously, $SGF(a, B, D) \geq 0$, the value of $SGF(a, B, D)$ is more larger, which means the attribute a the more important for decision-table D under the conditions of knowing B , and this definition of attribute importance belongs to algebra definition. When the number of attributes a is increased, if you cannot change the number of conflict granules on $B \cup \{a\}$ in the completely domain granular space GrP , the importance of this attribute will be 0.

5. Conclusion

Granulation of information appears in many areas, such as rough sets, evidence theory, machine learning, and data mining. Granular computing is similar to the way of the human understand things, people can summarize some concepts from several similar instances. In the basis of principle of granularity, we aim to study the granular decomposing method in granules space based on rough set. Moreover, the criteria conditions for attribution necessity and attribute reduction are proposed. Finally, the corresponding equivalence is proved to traditional rough set theory. It will lay the foundation for attribute reduction under the granular representation in rough set.

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References

- [1] Z. Pawlak, "Rough Sets: Theoretical Aspects of Reasoning about Data, System Theory", Kluwer, Dordrecht, (1991).
- [2] Z. Pawlak and A. Skowron, "Rudiments of rough sets", Information Sciences, vol. 177, (2007) pp. 3–27.
- [3] T. Feng, S. P. Zhang and J. S. Mi, "The reduction and fusion of fuzzy covering systems based on the evidence theory", International Journal of Approximate Reasoning, vol. 53, no. 1, (2012), pp. 87–103.
- [4] R. Jensen and Q. Shen, "Fuzzy-rough sets assisted attribute selection", IEEE Transactions on Fuzzy Systems vol. 15, no. 1, (2007), pp. 73–89.
- [5] G. Jeon, D. Kim and J. Jeong, "Rough Sets Attributes Reduction Based Expert System in Interlaced Video Sequences", IEEE Transactions on Consumer Electronics, vol. 52, no. 4, (2006), pp. 1348–1355.
- [6] H. X. Li, M.H. Wang, X. Z. Zhou and J. B. Zhao, "An interval set model for learning rules from incomplete information table", International Journal of Approximate Reasoning, vol. 53, no. 1, (2012), pp. 24–37.
- [7] Y. H. Qian, J. Y. Liang and C. Y. Dang, "Incomplete multigranulation rough set", IEEE Transactions on Systems, Man and Cybernetics – Part A vol. 20, (2010), pp. 420–431.
- [8] Y. H. Qian, J.Y. Liang, D. Y. Li, F. Wang and N. N. Ma, "Approximation reduction in inconsistent incomplete decision tables", Knowledge-Based Systems, vol. 23, (2010), pp. 427–433.
- [9] Y. H. Qian, J. Y. Liang, W. Wei, "Pessimistic rough decision, in: Second International Workshop on Rough Sets Theory", (2010) October 19–21, Zhoushan, PR China, pp. 440–449.
- [10] T. Y. Lin, "Granular computing [M]/Rough Sets, Fuzzy Sets, Data Mining, and Granular Computing, Springer Berlin Heidelberg, (2003), pp. 16-24.
- [11] Z. Zhang, X. Yan and W. Cheng, "Granular Computing with Application to Fault Diagnosis [J][J]", Journal of Xi'an Jiaotong University, vol. 9, (2009), p. 008.
- [12] F. Li, J. Xie and K. Me, "Granular computing theory in the application of fault diagnosis[C]/Control and Decision Conference", 2008. CCDC 2008. Chinese. IEEE, (2008), pp. 595-597.
- [13] F. E. H. Tay and L. Shen, "Fault diagnosis based on rough set theory [J]", Engineering Applications of Artificial Intelligence, vol. 16, no. 1, (2003), pp. 39-43.

