International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol. 7, No. 6 (2014), pp. 325-334
http://dx.doi.org/10.14257/ijsip.2014.7.6.28

An Algorithm of De-noising of Millimeter Wave Radar Signal based on Stochastic Resonance

Han Lingyun[1, 2] and Li Yuehua[1]

[1] *Institute of Millimeter Wave and Optical Near Sensing Technology, School of Electronic Engineering and Optoelectronic Technology, Nanjing University of Science & Technology, Nanjing 210094*

[2] *College of Physics and Electronic Information, An Hui Normal University, WuHu 241000*

nlglyh2013@sina.cn, hly9913334@126.com

Abstract

Millimeter wave radar echo signals often contain noise and clutter because of rain and fog's influence on the performance of which, and its performance drop greatly. In recent years, bi-stable stochastic resonance and multi-scale wavelet decomposition theory received great attentions in the field of signal de-noising. This paper proposed a novel mechanism of stochastic resonance which is induced by multi-scale noise for weak signal detection in millimeter wave radar signal. Firstly by multi-scale wavelet decomposition, input signal which is in heavy background noise was decomposed to several signals with different frequencies. After that they were induced by contraction factors of each noise scale, and then were as the input signal of bi-stable system. Simulations of different parameters show that under suitable contraction factors, SNR of output signal can be improved greatly.

Keywords: *millimeter wave radar signal; stochastic resonance; weak signal detection; multi-scale noise tuned*

1. Introduction

Millimeter wave radar work frequency is usually selected in the range of 30-300G hertz [1]. The wavelength of millimeter wave is between centimeter wave and light wave, so its. Compared with centimeter wave radar, millimeter wave radar has smaller volume, more light quality, and higher spatial resolution. Also compared with micro wave radar, millimeter wave radar has narrower beam, stronger ability of penetrating fog, smoke, dust, and better performance in anti-jamming and anti-stealth. So millimeter wave radar is commonly used in guidance, measurement [2]. But because rain and fog has great influence on the performance of millimeter wave radar, its performance can drop greatly and millimeter wave echo signal contains noise and clutter. In some applications, such as investigation system, useful signal amplitude is relatively smaller than the extent of noise or interference, and even submerged by them. Radar echo signal is of low signal-to-noise ratio (SNR), and detection performance of conventional detection methods, like spectrum analysis method [3], correlation function analysis method [4], and the wavelet analysis method [5], are difficult to meet the actual needs. This article adopts the method based on wavelet transformation and stochastic resonance to de-noise, and experimentally numerical simulations demonstrate that the proposed method is feasible.

Stochastic resonance is firstly put forward for the explanation of the fourth generation glacier by Benzi [6], and has been extensively studied theoretically and experimentally

ISSN: 2005-4254 IJSIP

around different power system and noise due to its vast applications in many fields [7-9]. Early research was mostly focus on stochastic resonance of the periodic signal and additive noise tuning on nonlinear systems. Recently, studies have found that stochastic resonance phenomena will occur in linear system tuned by multiplicative noise and periodic signals, even just only tuned by multiplicative noise, and have gained a great deal of attention. Guo Feng *et al.* discussed the stochastic resonance phenomena under combined effects of multiplicative noise and the periodic modulation noise, the amplitude and second-order steady-state moment of linear damping oscillator tuned by noise and system parameters [10]. Also, Guo found that stochastic resonance occurred in linear system with multiplicative noise, additive noise and modulation noise. Theory of stochastic resonance has great attention in use of weak signal detection and extraction of information signal under strong noise.

As compared to traditional techniques which mainly focused on how to suppress noise; stochastic resonance achieved the goal of signal enhancement by the aid of the noise. From stochastic resonance the output signal of the nonlinear system can have greatly better signal-to-noise ratio (SNR) by adjusting the system parameters and noise addition to the system. So, this phenomenon benefits weak signal detection in background noise, and attracts various studies, which indicates that noise enhanced signal detection has better effect than noise suppressed-based techniques, especially when target signal in heavy noise background.

The noise plays an important role in the weak signal detection by use of stochastic resonance. During the past decades, the constructive effects of noise have been found in numerous nonlinear systems, changing the view of that noise is always a menace, and leading to a great interest for potential applications of these phenomena. The existing research mostly focus on linear noise, such as additive noise or a linear function of color noise, but there is few of review on of stochastic resonance phenomena induced by non-linear function of color noise. In fact, in the actual non-linear physical system, driving noise is often in the form of the non-linear function of color noise. X. Gu discussed when colored noise is a quadratic function of the pump noise, the single mode laser system responses to the amplitude modulation signal [11]. Yong Xu et al. found that when multiplicative noise is of binary quadratic function, the amplitude of the steady state responses of linear system along with the change of noise intensity is also generalized stochastic resonance phenomena [12].

2. Principle of Bi-stable Stochastic Resonance

Micro particle in medium got random collision from medium molecule due to the molecular thermal movement, resulting in a Brown movement. Langevin considered that Brownian particle with mass m in the mediums under the external forces including gradient potential field force $-U'(x)$, the damping force $-\eta\dot{x}$, random force $\xi(t)$, and the external signal power $F(t)$. Due to Newton's second law, Langevin equation can be:

$$m\ddot{x} + \eta\dot{x} = -U'(x) + F(t) + \xi(t) \tag{1}$$

In the over-damped cases, the acceleration term $m\ddot{x}$ can be ignored, and selecting the appropriate units makes $\eta = \mathbf{1}$, so the Langevin equation can be simplified as:

$$\dot{x} = -U'(x) + F(t) + \xi(t) \tag{2}$$

It is known that Brownian motion is characterized well by a bi-stable system. In this paper, we deal with the case that bi-stable system induced by periodic signals, its potential function is $U(x)=-\frac{a}{2}x^2+\frac{b}{4}x^4$, and external power signal is $F(t)=A\cos(2\pi f_0 t)$, substituted to Langevin equation, we have:

$$\frac{dx}{dt}=ax-bx^3+A\cos(2\pi f_0 t)+\xi(t) \tag{3}$$

where A and f_0 are amplitude and frequency of the periodic signal, respectively; a and b are barrier parameters with positive real values.

Let $\xi(t)=\sqrt{2D}n(t)$ with $\langle\xi(t)\xi(t-\tau)\rangle=2D\delta(\tau)$, in which D is the noise intensity and $n(t)$ represents a Gaussian white noise with zero mean and unit variance.

The probability distribution function $\rho(x,t)$ of random variable x is satisfied Fokker Planck equation (FPK):

$$\frac{\partial\rho(x,t)}{\partial x}=-\frac{\partial}{\partial x}[(ax-bx^3+A\sin(2\pi f_0 t)\rho(x,t)]+D\frac{\partial^2\rho(x,t)}{\partial x^2} \tag{4}$$

where $-\frac{\partial}{\partial x}[A\sin(2\pi f_0 t)\rho(x,t)]$ is a non-autonomous item, so there is no steady state solutions for this equation, and also can not find any exactly expression.

The power spectral density $S(f)$ of the system response can be used to well understand the stochastic resonance, which in summery contains two parts, $S_1(f)$ and $S_2(f)$, corresponding to contribution of the driving periodic signal and the noise, respectively, as bellow:

$$S(f)=S_1(f)+S_2(f) \tag{5}$$

$$S_1(f)=\frac{2\mu^4A^2e^{-\mu^2/2D}/(\pi D^2)}{(2\mu^2e^{-\mu^2/2D}/\pi^2)+(2\pi f_0)^2}\delta(f_0-f) \tag{6}$$

$$S_2(f)=[1-\frac{\mu^3A^2e^{-\mu^2/2D}/(\pi^2D^2)}{(2\mu^2e^{-\mu^2/2D}/\pi^2)+(2\pi f_0)^2}][\frac{4\sqrt{2}\mu^2e^{-\mu^2/4D}/\pi}{(2\mu^2e^{-\mu^2/2D}/\pi^2)+(2\pi f)^2}] \tag{7}$$

It can be seen that $S(f)$ contains two parts, $S_1(f)$ and $S_2(f)$, corresponding to contribution of the periodic signal and the noise, respectively. $S_1(f)$ is the power spectral density response to the driving signal, and has the delta function form at the characteristic frequency. $S_2(f)$ is expressed as the response of noise and has a Lorentz distribution form, as can be seen in Figure1.

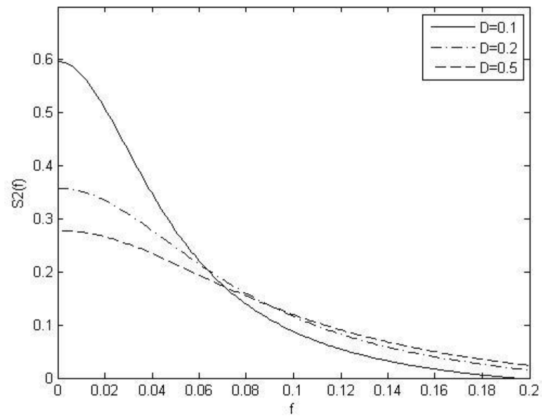


Figure 1. Wave of Power Spectral Density $S_2(f)$

Obviously, the Lorentz distribution is characterized by concentrating most of the noise energy in the low-frequency region, which means that from the nonlinear bi-stable system white noise energy that distributes uniformly in the whole spectrum will mostly be accumulated into low frequencies. From another point of view, the energy concentration leads to the stochastic resonance phenomenon for the low-frequency driving component.

Signal-to-noise ratio (SNR) is the most commonly used measurement method for stochastic resonance, which is defined as the signal frequency spectrum and the ratio of background noise spectrum value:

$$SNR = \frac{2[\lim_{\Delta\Omega \to 0} \int_{\Omega-\Delta\omega}^{\Omega+\Delta\omega} S(\omega) d\omega]}{S_N(\Omega)} \tag{8}$$

where $S(\omega)$ is the signal power, and $S_N(\Omega)$ is energy of noise in signal frequency. By (5) (6) (7), SNR for the system can be written as:

$$SNR = \frac{\sqrt{2}\mu^2 A^2 e^{-\mu^2/2D}}{4D^2} \tag{9}$$

3. Theory of Multi-scale Wavelet Decomposition

One of the mostly usage of stochastic resonance is in weak signal detection embedded in strong noise. By changing the intensity of the noise, weak signals will be clearly highlighted from the background noise.

We use $\phi_{j,k}(t) = \mathbf{2}^{j/\mathbf{2}}\phi(\mathbf{2}^j t - k)$ as the "baby scaling function" $\phi_{j,k}(t) = \mathbf{2}^{j/\mathbf{2}}\phi(\mathbf{2}^j t - k)$, and $\psi_{j,k}(t) = \mathbf{2}^{j/\mathbf{2}}\psi(\mathbf{2}^j t - k)$ as the "baby wavelet function", where $-\infty < j < +\infty$ and $-\infty < k < +\infty$. Any signal $f(t)$ in Hilbert space $L^2(R)$, $f(t) \in L^2(R)$ can be decomposed as [13]:

□□□
□□

$$f(t)=\sum_{k=-\infty}^{\infty}< f(t),\phi_{j_0,k}(t)>\phi_{j_0,k}(t)+\sum_{k=-\infty}^{\infty}\sum_{j=j_0}^{\infty}< f(t),\psi_{j,k}(t)>\psi_{j,k}(t) \quad (10)$$

□□□□□□□ □□□□□□□□□□□□□□□

where the notation for the space $L^2(R)$ is defined as: $< f,g >:=\int_{-\infty}^{\infty} f(x)\overline{g(x)}dx$, $f,g\in L^2(R)$.

Note that $P_{j_0}(t)=\sum_{k=-\infty}^{\infty}< f(t),\phi_{j_0}(t)>\phi_{j_0}(t)$ and $D_j(t)=\sum_{k=-\infty}^{\infty}< f(t),\psi_j(t)>\psi_j(t)$, substituted to (10), we have

$$f(t)=P_{j_0}(t)+\sum_{j=j_0}^{\infty}D_j(t) \quad (11)$$

where j denotes the scale parameter.

Equation (11) shows that the signal $f(t)$ can be decomposed in different frequency, and each component $D_j x(t)$ is the function of time. And if $f(t)$ is the band limited signal, (11) has the limited sum number of j, as follows:

$$f(t)=P_{j_0}(t)+\sum_{j=j_0}^{j_1}D_j(t)=P_{j_0}(t)+D_{j_0}(t)+D_{j_0+1}(t)+\cdots+D_{j_1}(t) \quad (12)$$

For Gauss white noise, because of its very wide frequency range, after wavelet multi-scale decomposition, it is decomposed into different frequency bandwidth.

As we know, for white noise of frequency spectrum energy under uniform distribution, its spectrum structure changed, and its frequency spectrum energy concentrated in low frequency region through the nonlinear bi-stable system. Therefore, for the signal contained noise, after the multi-scale wavelet transform, different scales of the noise has different influence on the stochastic resonance. Based on the above discussion, we can set up a multi-scale wavelet transform stochastic resonance system used to detect weak signals under strong noise, which is shown if Figure 2.

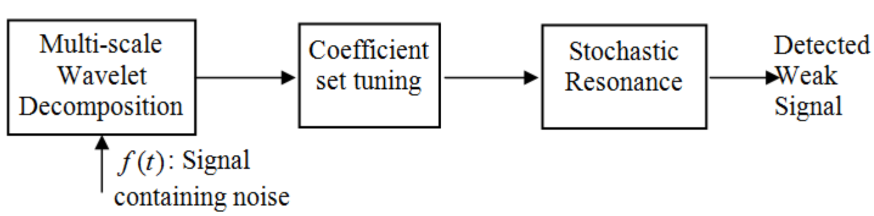


Figure 2. Numerical Simulation Scheme via Multi-scale Wavelet Decomposition based Stochastic Resonance

The strategy of this system is presented as follows. Firstly, weak signal under strong noise background $x(t)$ is decomposed into J scales wavelet transform, as formulated in the following equations:

$$x(t) = P_{j_0} x(t) + \sum_{j=j_0}^{j_1} D_j x(t) = P_{j_0} x(t) + D_{j_0} x(t) + D_{j_0+1} x(t) + \cdots + D_{j_1} x(t) \tag{13}$$

Secondly, each component is multiplied by a coefficient, which has the effect to adjust the amplitude of different components, and then affect the value of the signal-to-noise radio at the output of the nonlinear bi-stable system.

The set of wavelet coefficient are obtained as $C = \{K_{dj\mathbf{0}}, K_{j\mathbf{0}}, K_{j\mathbf{0}+\mathbf{1}}, \cdots, K_{j\mathbf{1}}\}$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□. After adjusting the coefficient set C, the new reconstructed signal is then sent to the nonlinear bi-stable system as expressed by Langevin equation. Since the signal at different scales also contributes to the stochastic resonance effect as analyzed above, there would be the best condition of coefficient set to reach the balance among the driving force, the noise, and the nonlinear system for stochastic resonance. In the stochastic resonance model of with multi-scale wavelet decomposition, the key point is to find a suitable coefficient set for a given set of stochastic resonance parameter $\{D, J, f_{\mathbf{0}}\}$. This model will be verified in the following numerical simulation and practical application.

4. Numerical Experiments and Discussion

In the simulation, millimeter wave radar signal with length of 512 and Gauss white noise. Firstly, multi-scale wavelet decomposition is done using Daubechies5, and $J = \mathbf{8}$, as shown in Figure 3.

In Figure 3, time domain waveform of $P_{\mathbf{0}}(t)$, $D_{\mathbf{0}}(t)$, $D_{\mathbf{1}}(t), ..., D_{\mathbf{7}}(t)$ is shown from left to right, from top to bottom, respectively. And then adjusting scale contraction factors $C = \{K_{d\mathbf{0}}, K_{j\mathbf{0}}, K_{j\mathbf{1}}, \cdots, K_{\mathbf{7}}\}$ to change the amplitude of each scale component. Since the noise at different scales contributes to the stochastic resonance effect at different level, system output SNR can be improved greatly under suitable scale contraction factors. When $C = \{\mathbf{6}, \mathbf{4}, \mathbf{0}, \mathbf{1}, \mathbf{0.5}, \mathbf{0}, \mathbf{0}\}$, the system output SNR is improved by 27dB. Figure 4 shows the millimeter wave radar signal and the signal de-noised using the method proposed in this paper. From the figure we can see that, the weak signal submerged in strong noise can be detected using this method.

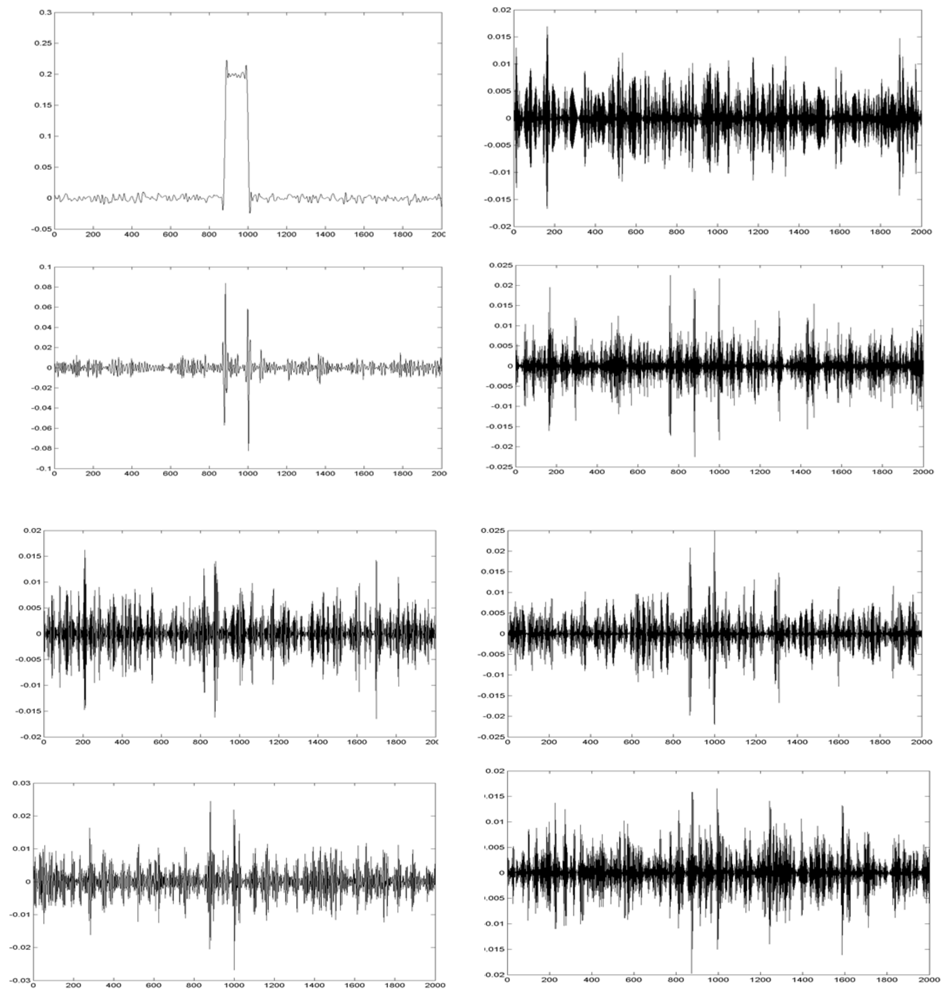
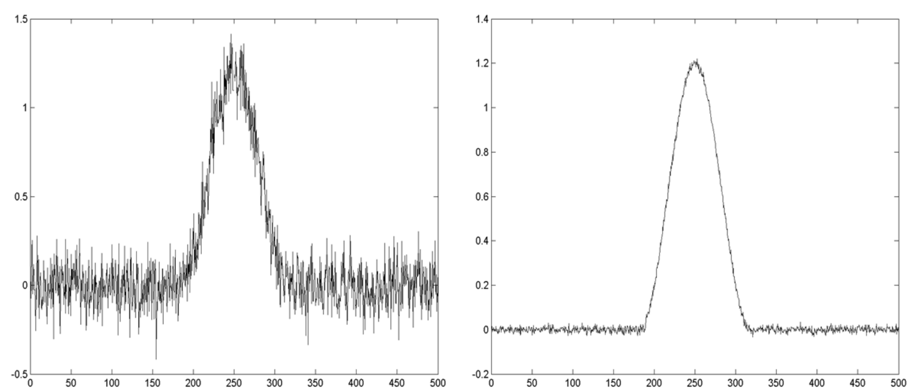


Figure 3. Multi-scale Wavelet Decomposition of Millimeter Wave Radio Meter Signal with Noise



(a) (b)

Figure 4. De-noise of Millimeter Wave Radio Meter Signal (a) Millimeter Wave Radio Meter Signal (b) Signal after De-noising

On the other hand, the bi-stable system output signal to noise ratio (SNR) changes with the input noise variance D, as shown in Figure5. From the figure we can see that when $D=\mathbf{1}$, the system output SNR comes to the peak value.

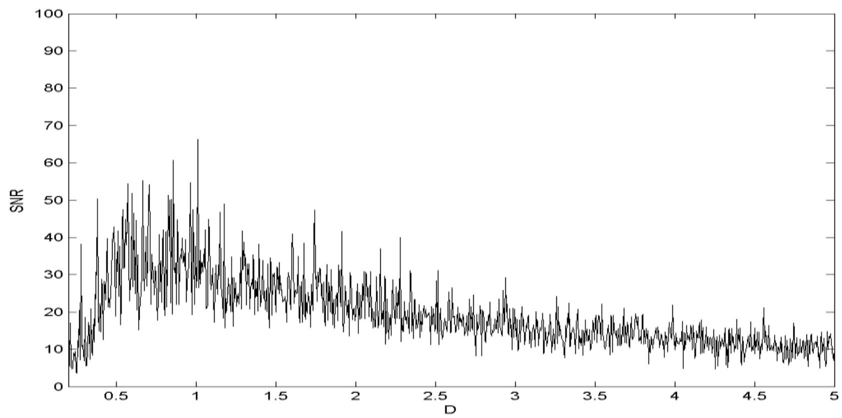


Figure 5. SNR of the System Output Versus D

To explain the validity of the algorithm proposed, we compared the SNR improvement using different methods, consisting of Wiener filter method, wavelet soft threshold de-noising method and wavelet hard threshold de-noising method, and under different SNR input signals. Here we use Daubechies5 as wavelet base function, and select layer decomposition. For each SNR of input signal, we conducted Monte-Carlo experiments of 200 times, and output SNR and RMSE (root-mean-square error) of statistical average is shown in Table1.

Table 1. SNR and RMSE under Different De-noising Method

	SNR	*RMSE*
Wiener filter method	*20.3802*	*0.2441*
The wavelet soft threshold de-noising method	*22.3824*	*0.2235*
The wavelet hard threshold de-noising method	*23.3572*	*0.2132*
Method proposed in this paper	*25.7825*	*0.2042*

From Table 1 we can see that, algorithm proposed in this paper can be used to effectively remove noise in the millimeter wave radar signal. And compared to classical de-noising method, the output SNR is improved, in favor of subsequent of characteristic detection, identification and other signal processing.

5. Conclusion

In this paper, an algorithm for de-noising of millimeter wave radar echo signal based on bi-stable stochastic resonance and multi-scale wavelet decomposition is investigated. It is verified that the system output SNR can be further improved by adjusting the contraction factor of each noise scale. This research also has practical application value in weak signal detection, and has profound physical meaning in the investigation of stochastic resonance mechanism.

Acknowledgements

The authors would like to thank the anonymous reviewer and editors for their helpful comments and suggestions. This work is supported by National Natural Science Foundation of China under Grants 61371038, National Ministry Foundation of China under Grants 9140A05070910BQ02 and 51305050102.

References

[1] [1] M.I.Skolnik,Introduction to Radar Systems, 2nd ed.,McGraw-Hill,New York, **(1980)**.

[2] [2] K.J.Button,J.C.Wiltse,Infrared and Millimeter Waves,Vol.4,Academic Press,New York, **(1981)**.

[3] [3] Gardner. Measurement of Special Correlation. IEEE Transaction on ASSP, 34 (5):1111-1123 **(1986)**.

[4] [4] Li Min Zhu, Han Ding, Xiang Yang Zhu. Extraction of Periodic Signal without External Reference by Time Domain Average Scanning. IEEE Transactions on Industrial Electronics, 55(2):918-927 **(2008)**

[5] [5] Smith C B, Agaian S, Akopoan D. A Wavelet De-noising Approach Using Polynomial Threshold Operators. IEEE Signal Processing Letters, 15:906-909 **(2008)**

[6] [6] Benzi, R., S. Alfonso and A. Vulpiani. The Mechanism of Stochastic Resonance. J. Phys. A, 14: 453-457 **(1981)**.

[7] [7] Benzi, R., G. Parisi and A. Vulpiani. Theory of Stochastic Resonance in Climatic Change. Siam J. Applied Math, 43: 565-578 **(1983)**.

[8] [8] S. K. Nguang and P. Shi, Fuzzy Hinfinity Output Feedback Control of Nonlinear Systems under Sampled Measurements, Automatica, vol.39, no.12, pp.2169-2174 **(2003)**.

[9] [9] E. K. Boukas, Z. Liu and P. Shi, Delay-dependent Stability and Output Feedback Stabilization of Markov Jump Systems with Time-delay, IEE-Part D, Control Theory and Applications, vol.149, no.5, pp.379-386 **(2002)**.

[10] [10] Qingbo He, Jun Wang, Effects of Multi-scale Noise Tuning on Stochastic Resonance for Weak Signal Detection, Digital Signal Processing, Vol.22, pp.614-621 **(2012)**

[11] [11] Guo Feng, Cheng Xiao-feng, Li Shao-fu, Entropic Stochastic Resonance in a Confined Structure Driven by Dichotomous Noise and White Noises, China, Phys. B, Vol.21, No.8 **(2012)**

[12] [12] X.Gu, Stochastic Resonance Driven by Time-delayed Feedback in a Bi-stable System with Colored Noise, the European Physical Journal D, Vol.66, 67 **(2012)**

[13] [13] Yong Xu, Juan Wu, Hui-Qing Zhang, Stochastic Resonance Phenomenon in an Under-damped Bi-stable System Driven by Weak Asymmetric Dichotomous Noise, Nonlinear Dynamic, Vol.70,.531-539 **(2012)**

