A Novel Image Interpolation Algorithm Based on Directional Extension of AP-DCT Interpolation Kernel

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Abstract

In order to decrease the smoothing effect of traditional linear interpolation methods and avoid the complexity of edge-directed adaptive algorithms, a novel image interpolation algorithm based on the directional extension of linear interpolation kernel is proposed. At first, All Phase Non-subsampled Contourlet Transform is used to divide the image of low resolution into different frequencies and directional sub-band images, and then the All Phase Discrete Cosine Transform Interpolation Kernel is extended to 2-D with distinct directional priorities and used to enlarge respective sub-band images. The interpolated sub-band images are finally synthesized as the upsampled image. Simulation results demonstrate that the proposed algorithm improves both the subjective and objective quality of the interpolated images over conventional linear interpolation. The computation complexity of the proposed algorithm is moderate and it can be implemented with parallel structure so as to save more computation time.

Keywords: image interpolation, directional extension, All Phase Discrete Cosine Transform (APDCT), All Phase Non-subsampled Contourlet transform (APNSCT)

1. Introduction

Image interpolation is one of the old but still challenging technologies widely used in many image operations, which addresses the ill-conditioned problem of generating a high-resolution image from its low-resolution biased version. Traditional linear interpolation approaches are based on space-invariant models and can simply extend 1-D interpolation kernel directly to 2-D without taking into account the directional features and edge details of image, such as bi-linear, cubic spline [1], cubic convolution [2] *etc.,* which consequently produce annoying artifacts such as blurred edges in interpolated images. Adaptive interpolation techniques [3, 4],on the other hand, adjust the interpolation coefficients by edging detection or statistical estimation to better match the local structures around the edges, which involve high computation load. Most of the transform domain interpolation approaches transform the problem of spatial domain resolution enhancement to the problem of high-band extrapolation [5]. Wavelet-based approaches benefit from the numerical stability of the transform and various post-processing step after resolution enhancement, but the involvement of the anti-aliasing filter (low pass filter in wavelet) and the mixture of contrast and resolution enhancement makes a fair comparison between this type of interpolation algorithms and other methods almost impossible [5]. Moreover, multidimensional wavelet transform (WT), is proven to be an inefficient representations for multidimensional signals

with line or plane singularities. Contourlet Transform (CT) is a "true" 2-D representation, which uses contour segment like basis structure to approximate images [6]. There exists an iterative CT based image interpolation algorithm [7], which actually uses CT to iteratively de-noise the images recovered by inverse WT.

In order to take the advantages of linear interpolation kernel while preserving the directional details of an image, the 1-D interpolation kernel is directionally extended to 2-D in four typical fixed directions. The proposed interpolation algorithm differs from the above interpolation approaches, in which an adapted version of the Non-subsampled Contourlet Transform (NSCT) named All Phase Non-subsampled Contourlet Transform (APNSCT) is used for sub-band division and All Phase Discrete Cosine Transform (APDCT) interpolation kernel is directionally extended to interpolate sub-band images containing respective directional details, preserving the corresponding directional details of the original image. This hybrid algorithm is a compromise between linear and adaptive methods and is proven to be effective and with moderate computational complexity.

The rest of this paper is organized as follows. Section 2 presents the directional extension of APDCT interpolation Filter. Section 3 introduces the APNSCT. Section 4 studies the general structure of the proposed algorithm. Simulation results are given in Section 5 and the conclusion is made in Section 6.

2. Directional Extension of the APDCT Interpolation Kernel

The major contribution of this paper is to directionally extend the APDCT interpolation kernel to 2-D by using the APDCT interpolation in the main direction while using linear interpolation with short basis in other directions. An All Phase DCT (APDCT) interpolation kernel was designed to reduce the Gibbs effect caused by cutting off operation in discrete signal processing. In [8] it was extended directly to 2-D and demonstrated effectiveness in image resolution enhancement over 2-D bi-linear and cubic interpolation kernels.

2.1 The 1-D APDCT Interpolation Kernel

In [8] the relationship between continuous and discrete signal built up by Discrete Cosine Transform (DCT) was utilized for signal reconstruction. According to [8], the continuous signal value of an arbitrary point can be reconstructed from the discrete samples around through DCT and IDCT transform as is shown in equation (1) and (2):

$$\hat{x}(t) = \sum_{n=0}^{N-1} H(t,n)x(n) \tag{1}$$

$$H(t,n) = \sum_{k=0}^{N-1} \beta(t,k)\alpha(k,n) \tag{2}$$

$H(t,n)$ is the DCT interpolation kernel, where $\alpha(l,n)$ and $\beta(t,l)$ are the DCT and IDCT transform matrixes respectively. This FIR filter results in Gibbs phenomenon in the interpolated images inevitably, which is manifested by decaying oscillations around high contrast edges [9]. The All Phase (AP) filter design takes the advantages of averaging operation to compensate for the artifacts caused by cutting off, which is much easier to be implemented than tapering the rectangular window that is the most common way of reducing Gibbs phenomenon. According to the definition of the All Phase Datum Space [10], there are $N-1$ data fields containing the data point to be interpolated as demonstrated in Figure 1 when $N=4$. The reconstructed value of the APDCT is defined to be the averaging of the DCT interpolated value of the $N-1$ fields as in equation 3.

$$\hat{x}(n+\tau) = \frac{1}{N-1}\sum_{i=0}^{N-2}\left[\sum_{j=0}^{N-1}H(i+\tau,n)x(n-i+j)\right] \qquad (3)$$

where τ ($0 \le \tau < 1$) is the delay between interpolated point and $x(n)$. Let $K = i - j$, then

$$\hat{x}(n+\tau) = \frac{1}{N-1}\sum_{k=-N+1}^{N-2}x(n-k)h(k) \qquad (4)$$

where

$$h(k) = \begin{cases} \dfrac{1}{N-1}\sum_{i=k}^{N-2}H(i+\tau,i-k), \\ \qquad\qquad k = 0,1,...N-2 \\ \dfrac{1}{N-1}\sum_{i=0}^{N-1+k}H(i+\tau,i-k), \\ \qquad\qquad k = -N+1,-N+2,...-1 \end{cases} \qquad (5)$$

$$\vdash\!\!-\tau\!-\!\!\dashv$$

$$x(n-2) \quad x(n-1) \quad x(n) \quad x(n+1) \quad x(n+2) \quad x(n+3)$$
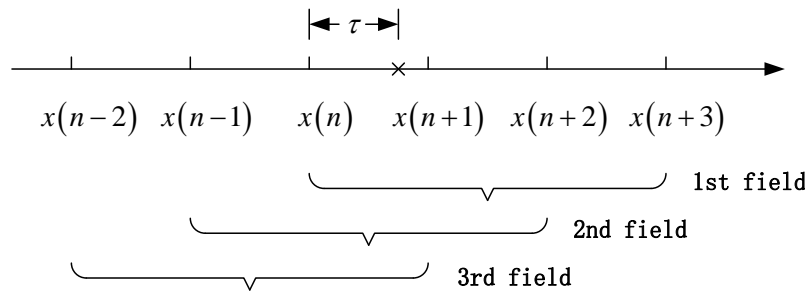
1st field

2nd field

3rd field

Figure 1. Demonstration of the APDCT Interpolation

It should be noted that the length of the APDCT interpolation kernel $h(k)$ is $2(N-1)$ rather than N. Although it is proven that a directly extended APDCT separable 2-D interpolation filter outperforms cubic interpolation of the length N in image up-sampling [8] and video format conversion [11], it actually uses more samples for reconstruction. Using more samples for interpolation not only increases the computation and storage requirement, but also the chance of cross-edge interpolation.

So we considered to extend the interpolation kernel directionally to some fixed directions to fit the geometry regularity of edge details in image.

2.2 Directional Extension of the APDCT Interpolation Kernel

Separable 2-D interpolation kernels are directly extended from their 1-D polynomial interpolators. In practical implementation, the same interpolation function is applied in both directions. Figure 2 (a)-(d) gives the 2-D interpolation kernels and contours of bi-linear and the APDCT respectively. The bi-linear interpolation kernel is easier to be implemented because it has shorter basis. While the 2-D APDCT interpolation kernel is closer to the ideal interpolation function named Sinc function. But the symmetric operations in both the bi-linear and the 2-D APDCT interpolation are not optimal as typical scenes and images are not symmetric and separable.
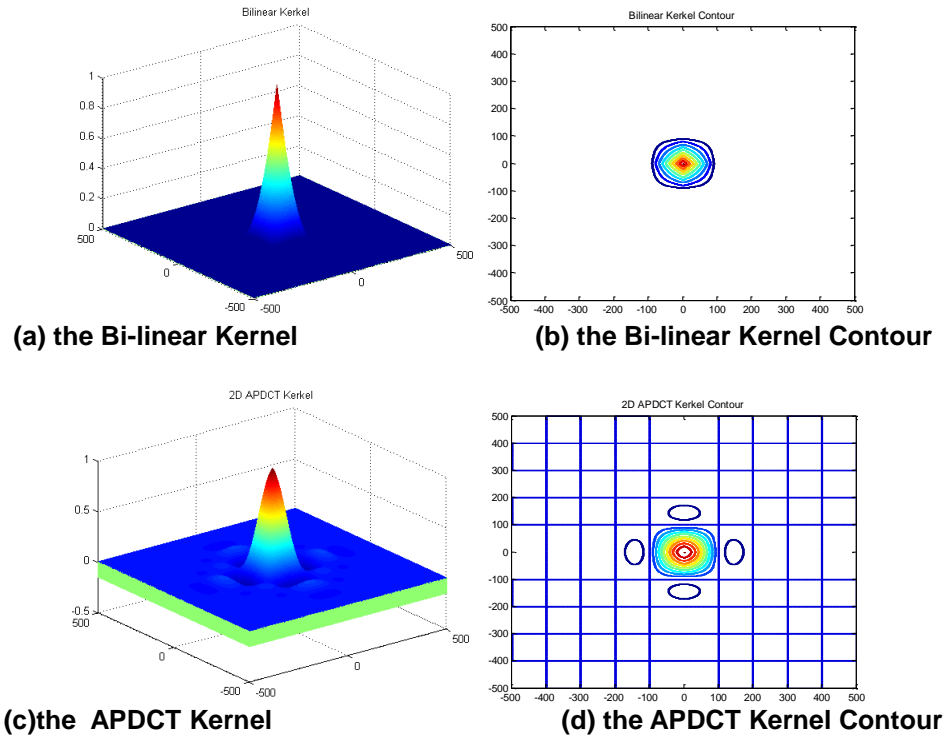
(a) the Bi-linear Kernel

(b) the Bi-linear Kernel Contour

(c) the APDCT Kernel

(d) the APDCT Kernel Contour

Figure 2. Directly Extended Bi-linear and 2-D APDCT Interpolation Kernels

In order to combine the advantages of the two interpolation kernels, it is proposed to extend the interpolation directionally so as to preserve the details of image in certain directions. The APDCT interpolation is implemented along that direction so as to preserve edges. For the point to be interpolated to the other directions, interpolation kernel with short basis (linear interpolation kernel) is used to avoid cross-edge interpolation. The designed 2-D directional interpolation structures are demonstrated in Figure 3. For horizontal interpolation, the 10-point ($N = 6$) horizontal APDCT interpolation and 2-point vertical linear interpolation are combined to construct the horizontal interpolation kernel function, which has longer basis in the horizontal direction while shorter basis in the vertical direction as is shown in Figure 4 (a) and (b). The 2-D vertical interpolation kernel, on the other hand, has longer basis in the vertical direction while shorter basis in the horizontal direction as is shown in Figure 4 (c) and (d). The diagonal interpolation kernels are combination of APDCT interpolation along the diagonal direction and bi-linear interpolation around, which utilizes not only the original samples but also the interpolated diagonal samples and the 2-D interpolation kernel is no longer separable. Directional extension of interpolation kernel is the major novelty of this paper. But it should be noticed that the directional 2-D APDCT interpolation kernel proposed is only proper when being used for images containing the details of the respective direction.
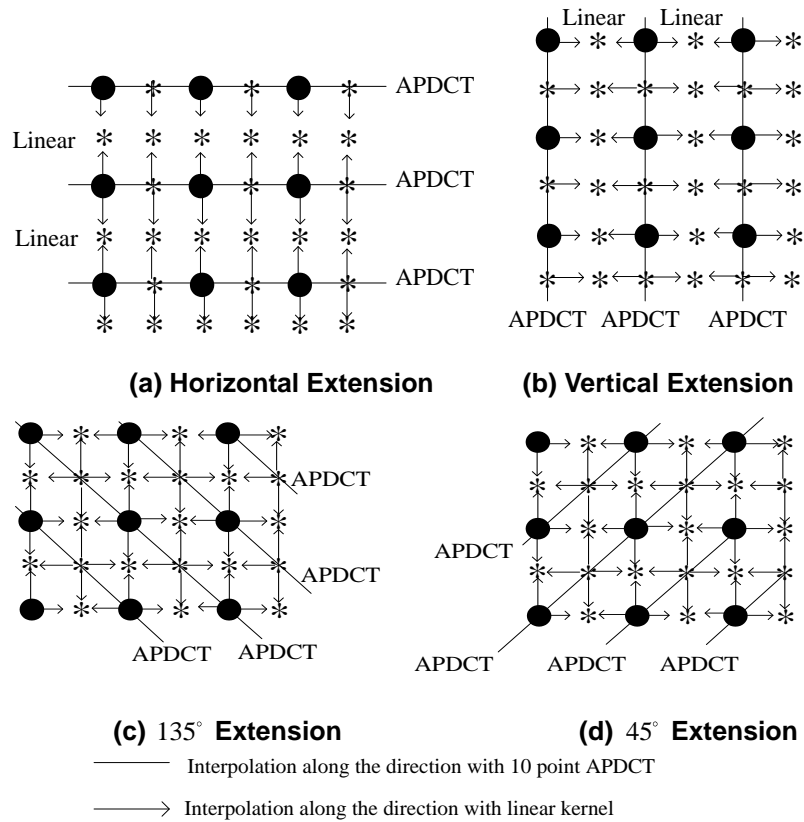
(a) Horizontal Extension

(b) Vertical Extension
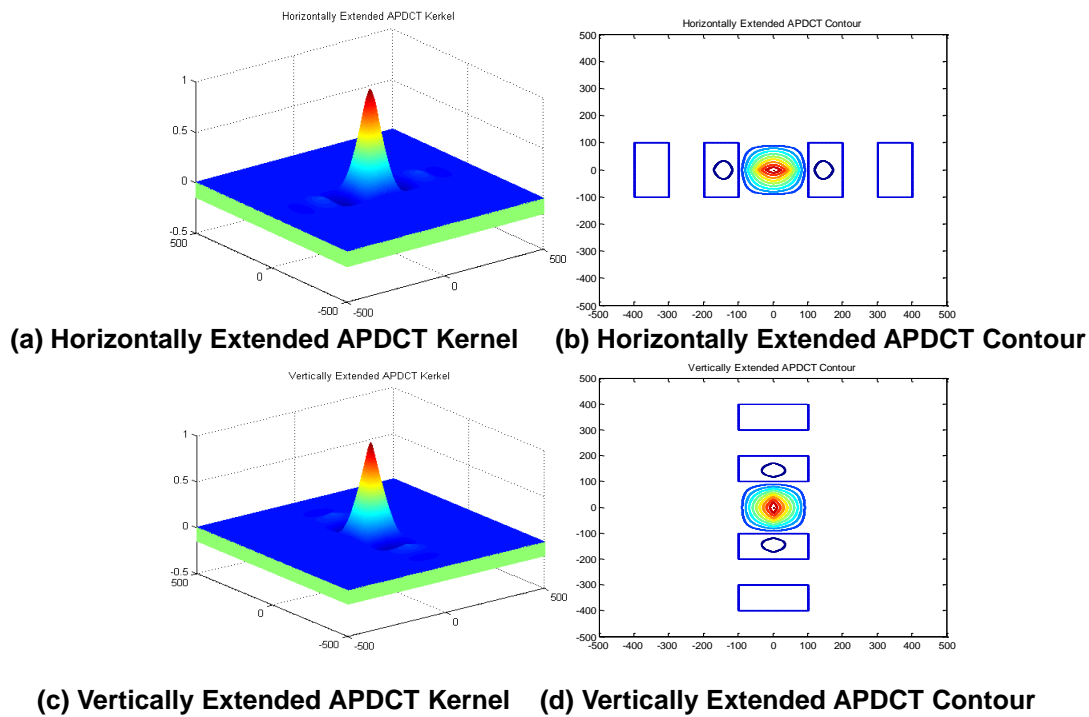

(c) $135°$ **Extension**

(d) $45°$ **Extension**

―――― Interpolation along the direction with 10 point APDCT

――――→ Interpolation along the direction with linear kernel

Figure 3. Directional Interpolation and Extension

(a) Horizontally Extended APDCT Kernel

(b) Horizontally Extended APDCT Contour

(c) Vertically Extended APDCT Kernel

(d) Vertically Extended APDCT Contour

Figure 4. the Directionally Extended 2-D APDCT Interpolation Kernel

3. General Structure of the Proposed Algorithm

The block diagram of the proposed algorithm is shown in Figure 5, which can be summarized as follows.

1. Decompose the low resolution original image into one low frequency (LF) sub-band image and four high frequency (HF) directional sub-band images (Horizontal, Vertical, $135°$ and $45°$) with the APNSCT.

2. Interpolate the LF and HF sub-band images separately with different approaches. The APDCT interpolation kernel is extended directly to 2-D and used to interpolate the LF sub-band image. Meanwhile, the 2-D directional APDCT interpolation kernels are constructed specifically for different directional sub-band images so as to preserve their directional features.

3. Synthesize the interpolated sub-images to obtain the high resolution image.
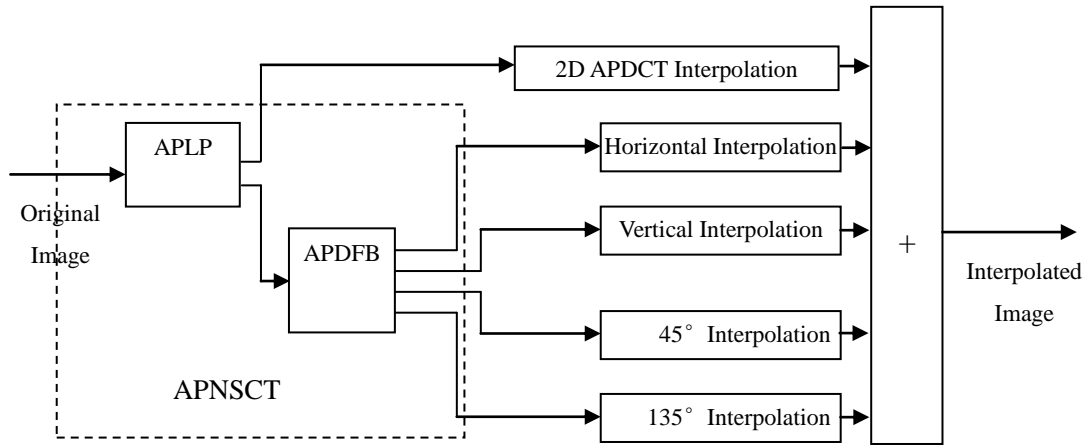

Figure 5. the Block Diagram of the Proposed Algorithm

4. APNSCT

APNSCT is an improved version of the Non-subsampled Contourlet Transform (NSCT). [12] Gives the detail of the design of APNSCT and demonstrates that the APNSCT outperforms NSCT in image denoising with better recovered image quality and higher speed.

Since APNSCT is of great importance in the proposed image interpolation algorithm, a brief introduction of its structure and implementation is given for integrity.

4.1 Basic Structure of APNSCT

NSCT is a fully shift-invariant and over complete transform adapted from Contourlet Transform targeting applications where redundancy is not a major concern. APNSCT has the similar structure with NSCT, as is shown in Figure 6. The major difference is that both the Laplacian Pyramid (LP) and the Directional Filter Bank (DFB) in APNSCT are filter banks constructed with 2-D All Phase Discrete Cosine Transform (APDCT) filters, namely All Phase LP (APLP) and All Phase DFB (APDFB) [12].

APNSCT is designed without rotation and resampling operations so as to preserve more image details with reduced computational complexity. In addition, the directions divided by APDFB include the horizontal and vertical as is shown in Figure 6. These directions represent more typical image directional features than the tilt wedge directions decomposed in NSCT.

In APLP, the ideal pass band of the low pass filter of level j is $[-(\pi/2^j),\pi/2^j]^2$, $j=1,2\cdots$. And the directions decomposed in each level can be achieved by using different APDFB accordingly. In Figure 6, the high pass sub-band of the first level is divided into 2 directional sub-bands, while the high pass sub-band of the second level is divided into 4 directional sub-bands. The number of APLP levels and APDFB directions can be selected according to requirement of specific applications.
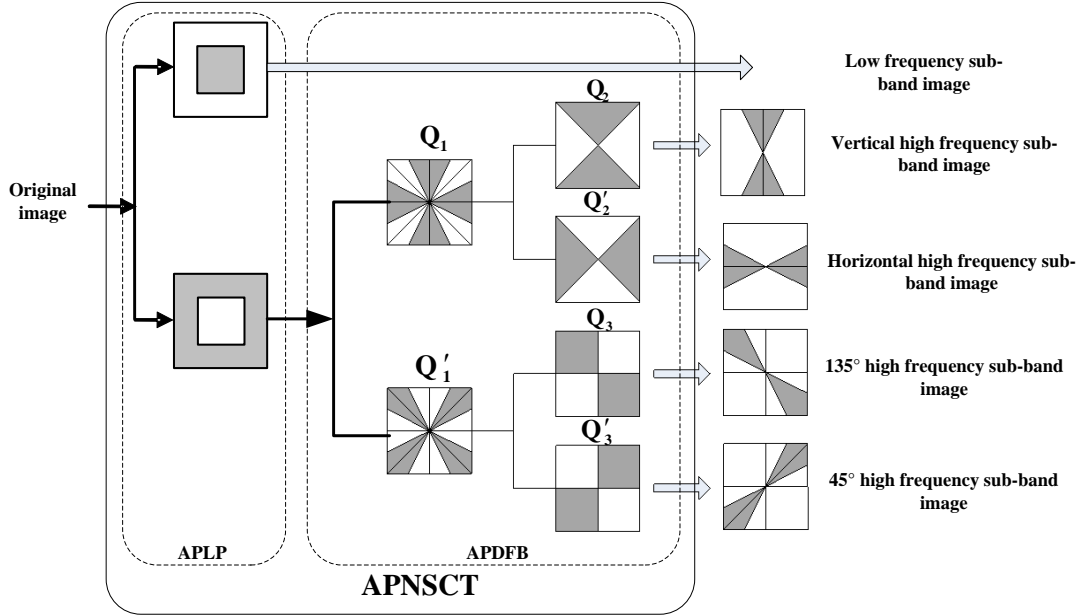

Figure 6. The Structure and Frequency Decomposition of the APNSCT

Both the APLP and APDFB are constructed based on 2-D APDCT filters. 2-D Directional filters are the filters that have the property of directional selection, which is of special importance for NSCT. The 2-D directional filters in NSCT are obtained from 1-D ones through mapping [13]. APDCT, on the other hand, is a direct 2-D filter design approach, with which most of the filters for directional decomposition are constructed by designing sequence response according to the spectrum requirement directly. This increases not only flexibility but also efficiency of DFB.

4.2 Design of APDCT Filters

APDCT can only be used to design linear-phased filters directly. So a 2-D APDCT filter $\mathbf{Q}$ of the size $(2N-1)\times(2N-1)$ is generally symmetric in the four quadrants,

$$Q(m,n)=Q(-m,n)=Q(m,-n)=Q(-m,-n)$$
$$0\le m\le N-1, \quad 0\le n\le N-1 \tag{6}$$

Then the matrix of $\mathbf{Q_{1/4}}$ is defined as a quarter of $\mathbf{Q}$ to simplify the design.

And according to [12]

$$\mathbf{Q}_{1/4}=\mathbf{GFG}^T \tag{7}$$

$\mathbf{F}$ is the sequence response matrix, which is to be designed according to the frequency response of the filter. For APDCT, the elements in $\mathbf{G}$ are calculated from (8):

$$G(i,j) = \begin{cases} \dfrac{N-i}{N^2} & 0 \le i \le N-1, j = 0 \\ \dfrac{1}{N^2}\left[(N-i)\cos\dfrac{ij\pi}{N} - \csc\dfrac{j\pi}{N}\sin\dfrac{ij\pi}{N}\right] & \begin{array}{l} 0 \le i \le N-1 \\ 0 < j \le N-1 \end{array} \end{cases} \tag{8}$$

So APDCT filters can be very easily designed by constructing the sequence response matrix according to the frequency response required. For the example of the directional filter shaping like a four-leaf clover demonstrated in Figure 7, its sequence response matrix is designed according to the shape of its frequency response as follows.

$$\mathbf{F} = \begin{bmatrix} 0.5 & 0.75 & 1 & 1 \\ 0.75 & 0 & 0.25 & 0.75 \\ 1 & 0.25 & 0 & 0 \\ 1 & 0.75 & 0 & 0 \end{bmatrix}$$
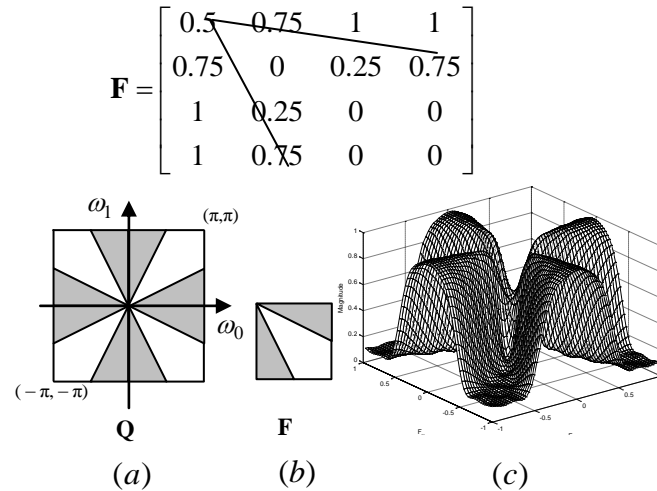

Figure 7. Directional Filter (Four-Leaf Clover Filter)

Then the matrix of this 2-D filter can be calculated from equation 7 directly. The frequency response of the designed APDCT directional filter is given in Figure 7 (c). APDCT is simple to be implemented and it can be used to design linear-phased filters that are too complicated to be designed by the other methods

4.3 APDFB

Based on the 2-D APDCT filters, the APDFB can be easily constructed to meet different directional decomposition requirements. The structure of the 2-level APDFB used in the proposed interpolation algorithm is demonstrated in Figure 6, which can divide a high frequency image into four directional sub-band images: horizontal, vertical, $45°$ and $135°$. The two directional filters $\mathbf{Q}_1$ and $\mathbf{Q}'_1$ used in level 1 are the Four-leaf Clover Filter designed in 4.2 and its complementary filter. The fan filters $\mathbf{Q}_2$ and $\mathbf{Q}'_2$ used in level 2 are designed in a similar way. But for the quadrant filters $\mathbf{Q}_3$ and $\mathbf{Q}'_3$, since they are not linear-phase filters, it is impossible to design them directly with the APDCT approach. Fortunately however, they can simply be obtained by rotating the fan filter $\mathbf{Q}_2$ and $\mathbf{Q}'_2$ by $45°$.

Figure 8 gives an example of the decomposed sub-band images of a naturally taken image named 'building'. (a) is the original image, (b) is the low frequency sub-band image obtained by a 1-level APLP, (c) is the high frequency sub-band image obtained by a 1-level APLP, (d)-(g) corresponds to the horizontal, vertical, $45°$ and $135°$ high frequency directional sub-band images respectively. It should be noted that the directions in space domain are perpendicular

to that of frequency domain. The sub-band images are named according to their space directions for simplicity.

The low-pass sub-band image is smooth and lacks of details, so it is comparatively easy to be reconstructed. The high frequency directional sub-band images contain mainly details of respective direction. And if the conventional interpolation methods are applied to reconstruct them, the edges will become blurred. So it is reasonable to use the directional extended APDCT interpolation filters for different directional sub-band images so as to preserve more edge information.
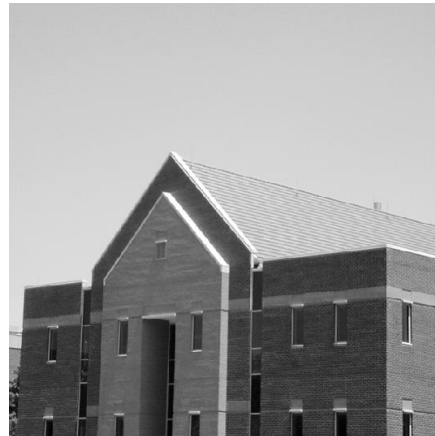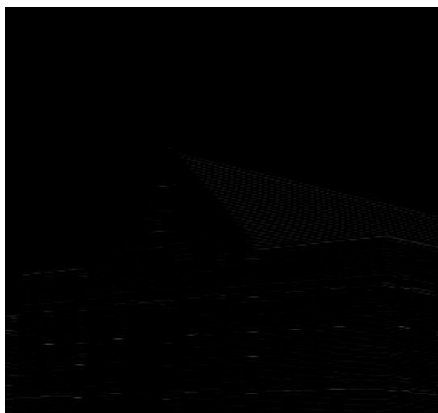

(a)   Original

(b)   Low frequency Sub-band   **(c) High Frequency Sub-band**

(d) Horizontal Sub-band   **(e) Vertical Sub-band**

(f) $45°$ **Sub-band**   **(g)** $135°$ **Sub-band**
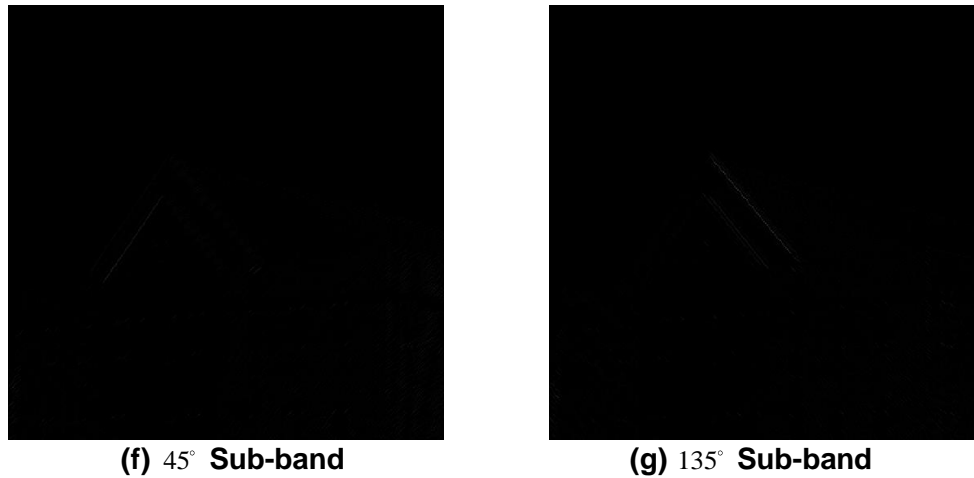
Figure 8. The Sub-band Images of a Natural Image Decomposed by APNSCT

5. Simulation Results

Five photographic images: Cameraman, Lena, Barbara, Mandrill and one synthesized image are used as benchmark images, in which Cameraman has abundant diagonal information, Lena has more low frequency components, Barbara and Mandrill are rich in high frequency details and the synthesized image contains character and standard shapes. The proposed interpolation is compared with three linear interpolation methods: cubic-spline, cubic-convolution and the directly extended 2-D APDCT. The low-resolution image is obtained by directly down sampling the original image by a factor of 2 along each dimension (aliasing may be introduced). Peak Signal Noise Ratio (PSNR) is used to evaluate the subjective quality of the interpolated images. It can be concluded from Table 1 that the proposed algorithm generates images with the higher PSNR than the other algorithms for all the benchmark images except for Lena.

It can be observed that the proposed algorithm generates the image with the highest visual quality for the synthesized image. As is shown in Figure 9, the edges of blocks and bars are very clear, without blurring. Besides, the contours of characters and circles reconstructed by proposed algorithm are smoother than those recovered by other methods.
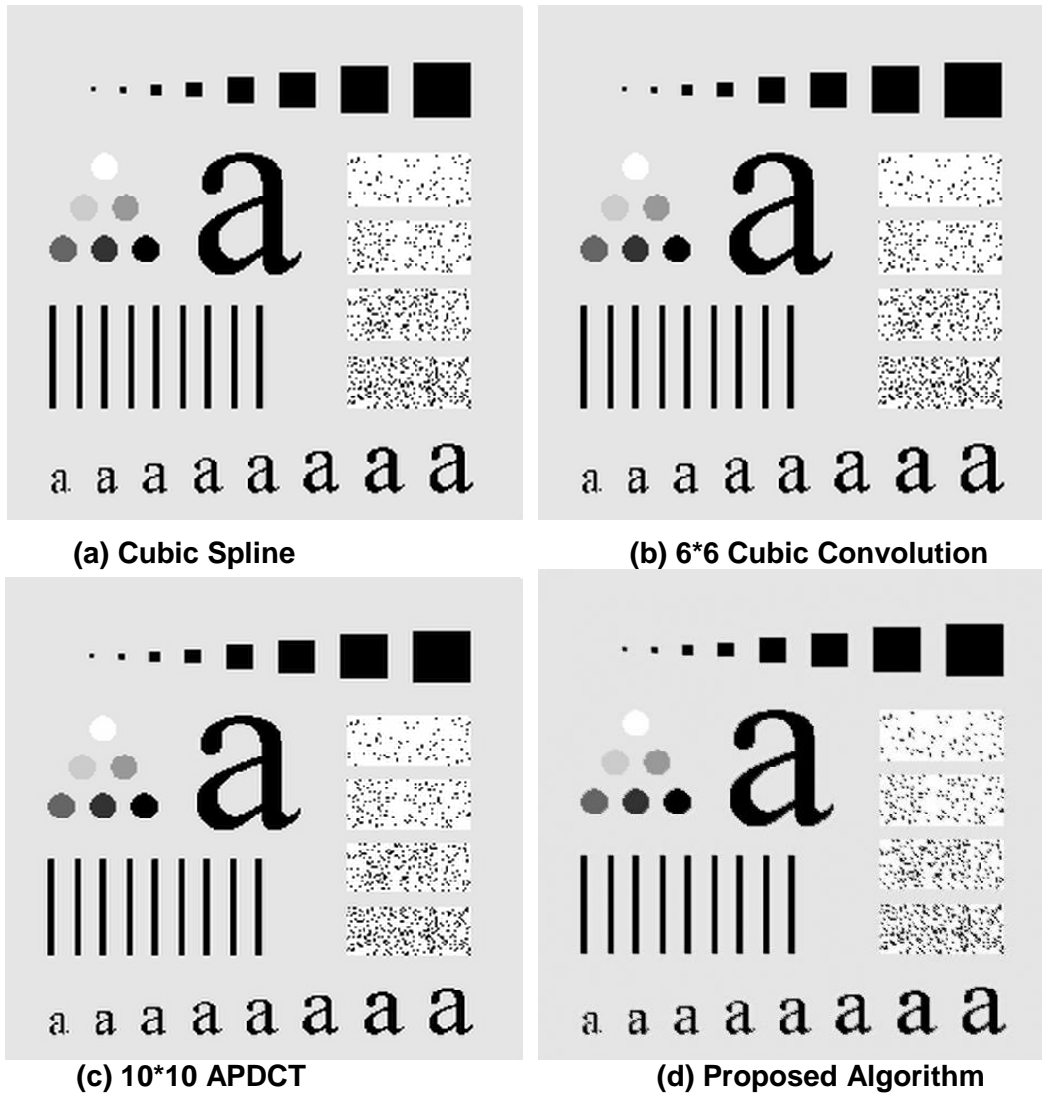
(a) Cubic Spline

(b) 6*6 Cubic Convolution

(c) 10*10 APDCT

(d) Proposed Algorithm

Figure 9. Comparison of the Interpolated Images of a Synthesized Image

The computation complexity of the proposed approach is between the conventional linear schemes and adaptive methods and it is easy to be implemented with parallel structure so as to save more computation time.

Table 1. Objective Comparison of Interpolation

PSNR/dB	Cameraman	Lena	Barbara	Mandrill	synthesized
Cubic Spline	25.14	33.89	24.02	22.15	17.60
6*6 Cubic Convolution	25.33	**34.02**	24.11	22.24	17.65
10*10 APDCT (N=6)	25.51	33.95	23.90	22.13	17.55
Proposed Algorithm	**27.80**	33.50	**24.58**	**22.57**	**18.03**

6. Concluding Remarks

The proposed interpolation algorithm combines the multi-scale and multi-directional decomposition of images with the directionally extended APDCT interpolation kernels. The

2-D interpolation kernels with different directional features are designed specifically for different directional sub-band images. As a non-adaptive interpolation approach without edge or direction detection, it has been proven to be able to preserve image details efficiently by suppressing the artifacts of blurring. For the sake of saving computation, the directional extension of APDCT kernel is limited to four most typical directions in this algorithm. The concept of directional extension of linear interpolation kernel can be further researched and applied to specific applications.

Acknowledgements

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