

The Method of Quaternions Wavelet Image Denoising

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Abstract

Based on traditional wavelet image denoising model, studies the quaternion, the application of wavelet transform in image denoising, quaternions are given three on the wavelet transform domain image denoising model and algorithm: (1) based on quaternion wavelet hidden markov tree model of image denoising; (2) based on quaternion wavelet transform non-gaussian bivariate distribution image denoising algorithm of bayesian statistical model; (3) based on quaternion mixing statistical model of wavelet image denoising. Experiments show that: in this paper, the denoising effect of these methods, in both the peak signal-to-noise ratio and the visual effects are better than many classic denoising algorithms.

Keywords: *Quaternion analytic signal, Quaternion wavelet transform (QWT), Image denoising, Donoho's threshold*

1. Introduction

R Quaternion (Quaternions) by the British mathematician Hamiltonian (w. r. Hamilton) in 1843, is considered to be a plural, but in a long time, didn't get very good application. Since the 1980 s, with the development in the field of computer graphics and image processing, the quaternion theory again found its application value. Wavelet analysis is quickly developed in the late 1980 s a new branch of mathematics, it is developed on the basis of the Fourier transform of a new method of time-frequency analysis, has been widely used in the field of signal analysis and image processing, *etc.*, Image denoising is a basic problem in image processing, digital watermarking is an important branch in the field of information hiding [1-2].

When one dimensional real area expanded to two-dimensional complex domain, we can use the plural to represent the two-dimensional plane vector, the plane vector synthesis, scaling and rotation transformation can use the plural addition and multiplication for convenient operation, so, naturally, the problem is that the 3 d or more in the high dimensional space can be like expansion of real number, on the basis of complex domain to add one or several new elements, get a new number set, one of the elements similar to complex domain do add, subtract, multiply, divide, and meet the plural those usually operation laws, including the addition and multiplication commutative law and associative law, multiplication for additive distributive law and so on, furthermore, from the geometric, want to use this number, as expressed in complex number plane vector adjustable rotation, to represent the 3 d or higher dimensions of scaling and rotation. So, there are many mathematicians in the history of trying to explore a three-dimensional complex, Irish mathematician was first discovered,

Hamilton to based on planar complex three-dimensional plural, make it real and complex structure and various operation properties, it is not possible. Because from the perspective of algebra, the common complex domain to expand into three-dimensional complex domain doesn't exist, from the point of view of geometry, need four variables to determine the scale in 3 d space rotation transform, the direction of the shaft need to be determined two variables, need a variable to determine the Angle of rotation, scaling and need a variable to determine. Because 3 d complex only three variables, thus can meet such requirement. Hamilton after years of study, he found himself was forced to make two concessions conditions, the first condition is that it is this new number four is required, and the second condition is that it must sacrifice the multiplication commutative law. On this basis, in 1843, he developed the so-called "four dimensional complex", namely the quaternion, and in 1857 published the quaternion notes. Complex is made up of real number plus the element I, similarly, the quaternion is a real plus three elements I, j, k, to be exact, quaternion is plural cannot exchange, relative to the plural for two-dimensional space vector, quaternion represents the elements of a four dimensional space. Before quaternions multiplication, multiplication is exchange, Hamilton created the first non-commutative law of algebra, the algebra liberated from the bondage of traditional real number operation and he found this caused a qualitative leap in the field of algebra [3-6].

In signal processing, and the classical Fourier transform to spread out the signal for a linear combination of the different frequency of harmonic function, which can well describe the frequency of the signal characteristics, therefore, for stationary signal processing and analysis, Fourier transform has played a strong advantage, but because of the Fourier transform is a kind of pure frequency domain analysis method, it can't for local analysis on airspace (time domain), so for non-stationary signal, cannot give the local characteristics of the time-frequency domain. In order to overcome the shortcomings, puts forward the concept of adding window Fourier transform, the idea is that by introducing a time localization "function" window, to add window handle of the signal, thereby improving the deficiency of the Fourier transform, but due to its window size and shape are fixed, not essentially make up the defects of Fourier transform. And wavelet transform in time domain and frequency domain has good localization ability at the same time, there is a change of time - frequency window, it is compared with Fourier transform, can be effectively extracted from signal time-frequency characteristics of local information to multi-scale decomposition, therefore, wavelet transform has been hailed as a "mathematical microscope" laudatory name.

In acquiring, the process of transmission and storage, usually polluted by noise, noise of the image is not only affected the image visual effect, but also bring difficulty to the subsequent processing. De-noising purpose is to keep the important information of original image premise, as far as possible to reduce or eliminate the noise. The shortage of the traditional denoising method is to make signal transformation after the entropy increase and cannot get the signal correlation, etc., because wavelet transform has low entropy, correlation, the time-frequency multi-resolution and the characteristics of the wavelet base selection diversity, widely used in image denoising. Research by quaternions wavelet is a new type of wavelet, make up for the deficiency of the traditional wavelet, has the advantages of good translation invariance. First and the evaluation standard of denoising algorithm and wavelet image denoising threshold value, then the quaternion is studied the application of wavelet transform in image denoising, in the ordinary real wavelet denoising model, on the basis of improving the classification criterion, generalize the threshold function, gives several quaternion

based wavelet transform domain image denoising model and corresponding algorithm, simulation experiments or on a SAR image, has obtained the good effect.

2. Related Works

The essence of image denoising is to estimate the original image from the noise image g f , for \hat{f} after image denoising, we hope it is the best estimate the original image f . For denoising algorithm at present there are mainly two kinds of methods: to evaluate the performance of subjective evaluation and objective evaluation. Subjective evaluation is directly employing eye observation after denoising image, based on visual perception in the judgment of image quality; the evaluation method is simple and intuitive, but easily affected by man-made factors. Objective evaluation standard mainly USES the mean square error (MSE) and peak signal-to-noise ratio (PSNR) for evaluation ^[7-9] \hat{f} f , respectively, and the original image denoising image, $M \times N$ for image size. For the practical application, the original image is unknown, such as SAR image coherent spot, suppress coherent spot work is blind restoration process, so used to experimental objective evaluation standard is not suitable for denoising, can adopt the following criteria:

1) depending on the number of equivalent (ENL), ENL theoretically and depending on the number of the SAR image is equal, ENL value, the greater the said image smoothing degree is higher, the coherent plaque inhibition, the better.

ENL is defined as:

$$ENL = A \left(\frac{\mu}{\sigma} \right)^2 \quad (1)$$

Among them σ respectively and μ homogenous area of standard deviation and average SAR images, for the intensity and range image $4/\pi$ -lis equal to 1 and 4, respectively.

2) the average \bar{Y} of the original image with spot image after the suppression of the ratio of average \bar{X} , the smaller the gap with the value 1, shows that spot image radiation characteristics remain after the suppression, the better, that is:

$$PM = \bar{Y} / \bar{X} \quad (2)$$

3) mean square error of STD, its reaction to the reconstruction of deviation. In this paper, by adopting the method of subjective evaluation and objective evaluation method for combining denoising image quality and algorithm performance evaluation method.

In this paper, we study of the additive noise model, set up the original image for the f , g observation image and Gaussian white noise for epsilon, has: $g = f + \text{epsilon}$. Quaternions after wavelet transform, still for the additive noise model in wavelet domain $w = m + n$, including of g , f , epsilon quaternion after wavelet transform of w , m , n , respectively. Because epsilon obedience is independent identically distributed Gaussian noise, noise in the wavelet domain n is still independent identically distributed Gaussian noise, and the variance is constant.

Noise variance is an important parameter of noise statistical characterization, whether can accurately estimate the denoising performance has a great influence. Usually adopt famous proposed by Donoho robustness value estimation method to estimate the noise variance. In the image denoising method based on wavelet domain, the wavelet threshold denoising method is a simple and effective method. Images after wavelet transform, most of the energy is focused on a small number of large amplitude coefficient, these coefficients represent the image edge details such as; And the noise of

the image are mainly corresponds to the amplitude of the smaller factor. So we can set a certain threshold, and in some on the processing of the wavelet coefficient threshold function. Wavelet threshold denoising method is the core of the problem is that the size of the threshold determination and the selection of threshold function.

In the wavelet threshold denoising method, the determination of threshold size is an important issue. Threshold value is too small can't effectively filter out noise, after denoising image smoothness of difference; Too much threshold can "kill" a number of important wavelet coefficients, after denoising image will become fuzzy. So the choice of the threshold size should be valid for noise suppression and detail on the protection for compromise. Several kinds of commonly used threshold are given below.

(1) the global threshold Global threshold is Donoho first proposed wavelet threshold:

$$T = \sqrt{2 \ln(M \times N)} \sigma \quad (3)$$

Which is suitable for image size, $M \times N$ sigma for noise standard deviation.

(2) Sure threshold

In 1995, Donoho and put forward the famous Sure threshold [10], it is obtained by minimizing Stanley unbiased risk estimate Sure risk function:

$$SURE(T, w) = N - 2\#\{i : |w_i| \leq T\} + \sum_{i=1}^N (|w_i| \wedge T)^2 \quad (4)$$

Where N is a number of subband coefficients, (#) to satisfy the number of elements in the conditions, \wedge said to take the minimum value

3) the BayesShrink threshold

Chang, etc will be generalized gaussian distribution as prior distribution of wavelet coefficient, by minimizing the Bayes risk function, the famous BayesShrink threshold:

$$T_{Bayes} = \frac{\sigma_n^2}{\sigma} \quad (5)$$

Among them σ respectively and σ_n^2 the standard deviation of noise and subband coefficient of standard deviation.

(4) Map Shrink threshold

Liu and Moulin hypothesis of Laplace function for the distribution function of wavelet coefficients, using the maximum a posteriori probability (MAP) estimation theory, got Map Shrink threshold:

$$T_{MAP} = \sqrt{2} \frac{\sigma_n^2}{\sigma} \quad (6)$$

Traditional quaternion wavelet domain denoising defect HMT model

Crouse in 1998 put forward by hidden markov tree (HMT) model ([CNB98]), by hidden markov chain relationship between the correlation of wavelet coefficients between father and son. Then many scholars combines wavelet transform and statistical model and applied to image denoising, but traditional transform domain HMT model there are shortcomings due to real orthogonal wavelet has no translation invariance, makes the HMT model based on traditional wavelet domain algorithm to get the images often have a ringing in edge distortion phenomenon, based on this, we are given based on quaternion wavelet transform domain hidden markov tree model (Q - HMT), and applied to image denoising, due to the quaternion wavelet has the advantages of translational invariance, thus obtained the certain effect.

3. A Quaternion based on Wavelet Design

3.1. Q - HMT Model and Parameter Estimation

HMT model using hidden markov tree structure reflect the correlation of wavelet coefficients between the scales, the use of markov chain on the multi-scale modeling of wavelet coefficients corresponding to the number of hidden states. HMT the hidden states of wavelet coefficients of large and small, big state of wavelet coefficients with large variance of gaussian distribution to model; Small state with small variance of the wavelet coefficients of gaussian model. Actual signal after quaternions wavelet transform coefficient between displays continuity and clustering characteristics, at the same time, the statistical results show that after the transformation of the coefficient of peak, long trailing edge density non-gaussian distribution characteristics [7]. Mentioned in the state of the two zero mean gaussian mixture model can be a good approximation of the actual image wavelet coefficient distribution. In this paper the Q - HMT model, each wavelet coefficients using the state of the two gaussian mixture model to represent, and each domain of wavelet coefficients corresponding to state variables it four variables on the next layer is linked together, the relationship between the coefficient of only by the father and son the state transition probability between state variables of the embodiment. Under normal circumstances, the state of two Gaussian mixture model of random probability density function of wavelet coefficient W consists of two parts: the probability of state variable S function $P_s(m)$ and the conditions of the Gaussian probability density function

$$f_{w|s}(W|S=m), \quad f_w(w) = \sum_{m=1}^2 p_s(m) f_{w|s}(W|S=m) \quad (7)$$

Where

$$f_{w|s}(W|S=m) = \frac{1}{\sqrt{2\pi\sigma_m^2}} \exp\left(-\frac{(w-\mu_m)^2}{2\sigma_m^2}\right) \quad (8)$$

Given a set of scale for the wavelet coefficient of J , assumptions and respectively the wavelet coefficient of random variables and state variables, in which $I = 1, \dots, J$, $J = 1, \dots, N$ (N scale for the wavelet coefficient of the I number). Q - HMT by parameter $\theta = [P_{S_i}(m), \xi_{i,\rho(i)}^{m,r}, \mu_{i,m}, \sigma_{i,m}^2]$ for the model, and to describe, which the probability of the root node aggregation function $\xi_{i,\rho(i)}^{m,r} = P_{S_i|s_{\rho(i)}}(m|s_{\rho(i)}=r)$, $P_{S_i}(m)$ are given in the state of r , sub-state $s_{\rho(i)}$ in the conditional probability of m , S_i is the state transition probability.

Figure 1 shows the two-dimensional wavelet domain hidden markov tree model diagram.

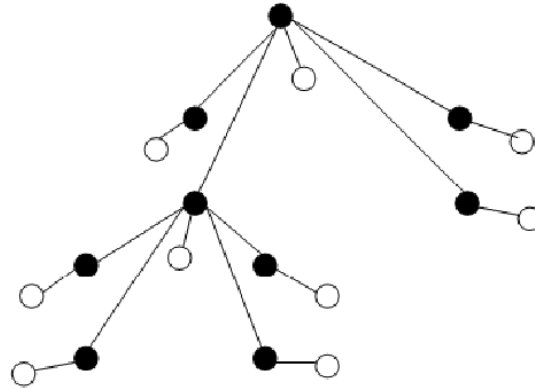


Figure 1. Two-Dimensional Wavelet Domain Hidden Markov Tree Model

(4) According to the literature, there are three kinds of hidden markov model standard algorithm: likelihood function calculation, the path of state estimation and model training, they also apply for HMT model. In general, do not know the specific parameters of the model, can only be observed according to the first wavelet coefficients W , training the model parameters. If you know that the wavelet coefficients of state variable, then by maximum likelihood (ML) estimation is easy to get the model parameters $\theta = \arg \max_{\theta} f(W|\theta)$. But not all, of the actual observation data is also, state variables is unknown, so need to use the EM algorithm to estimate parameters [7]. The main steps are divided into E and M step:

Initial conditions: select an initial model estimation θ^0 , set the iteration number $l = 0$;

Step1 E step: the forward - backward fast algorithm, the hidden state variables of the probability of the joint posterior probability distribution $p(S|W, \theta^l)$, method is very great value $E_s [\ln f(W, S|\theta) | W, \theta^l]$;

Step2 M step: to update model parameters $\theta^{l+1} = \arg \max_{\theta} E_s [\ln f(W, S|\theta) | W, \theta^l]$;

Step3 $l = l + 1$, if convergence criterion to stop, or back to step E.

In quaternion Q - HMT wavelet domain model, the noise of image wavelet coefficients as a mixture of two gaussian source and image through quaternion after wavelet transform, the coefficients are unrelated, with distribution, variance σ_n^2 of the noise interference. Image denoising based on Q - HMT is to quaternion with noise image wavelet decomposition coefficient estimation formula is:

$$w_i = y_i + n_i \quad (9)$$

Which w_i is including the quaternion wavelet coefficient of noise image, including y_i is not noise, n_i quaternion wavelet coefficients of wavelet coefficients for noise, and then using the bayesian estimation method to estimate the actual image wavelet coefficients of the wavelet coefficients.

Denoising algorithm is mainly composed of five steps, namely:

Step1 to five layer QWT transformation, the original image wavelet coefficients w_i obtained;

Step2 Q - HMT model is established, the parameters of the model for θ ;

Step3 using EM algorithm of wavelet coefficients of real and three imaginary part for training respectively, get parameter θ^k , $k = 1, 2, 3, 4$;

Step4 of no noise wavelet coefficients to estimate:

$$\hat{y}_i^k = E[y_i | W, \theta^k] = \sum_{m=1}^2 p(s_i = m | W, \theta^k) \times \frac{\sigma_{i,m}^2}{\sigma_{i,m}^2 + \sigma_n^2} w_i^k \quad (10)$$

After Step 5 to estimate the wavelet coefficients of QWT inverse transformation, after denoising image.

Statistical methods are widely used in image denoising, mainly from the following two points to consider: (1) from the perspective of nonparametric statistics processing, set up a specific estimator, apply it to a large number of images; (2) from bayesian statistics, suppose that a priori probability model, and then calculated by the bayesian estimation. Generalized gaussian distribution is often used in prior model, but the model only based on scale, ignoring the correlation between the wavelet coefficient of scale. Actually is related to the wavelet coefficients, such as subband coefficients of spatial clustering and scale the transitivity of correlation, etc.). Then, this paper puts forward dual non-gaussian statistical model to simulate the quaternion statistical distribution of wavelet coefficient, this model is composed of two variable and an argument, is a good way to reflect the correlation between the wavelet coefficients layer.

Below we think an equals 3, then a simplified non gaussian distribution as follows:

$$p_m(m|a) = \frac{3}{2\pi\sigma^2} \exp\left[-\sqrt{\frac{3}{\sigma^2}}(m_1^2 + m_2^2)\right] \quad (11)$$

According to the derivative can get bivariate shrinkage function:

$$\hat{m}_1 = \frac{\left(\sqrt{w_1^2 + w_2^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)}{\sqrt{w_1^2 + w_2^2}} w_1 \quad (12)$$

To introduce w_1 and m_1 neighborhood coefficient w_3 and m_3 , improved binary contraction model:

$$\hat{m}_1 = \frac{\left(\sqrt{w_1^2 + w_2^2 + w_3^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)}{\sqrt{w_1^2 + w_2^2 + w_3^2}} w_1 \quad (13)$$

4. Simulation Results and Analysis

In simulation experiments, testing 512×512 Lena image, to join the mean zero, variance of the gaussian white noise. Will the proposed denoising method (*i.e.*, the quaternion HMT models in the wavelet domain denoising method, shorthand for the Q - HMT) with real wavelet hard threshold method (Donoho 's HT), the HMT model of real wavelet domain denoising method and the HMT based on dual tree complex wavelet denoising method (C - LCHMM) has obtained the good effect.



(a) The original image



(b) With noise image



(c) LAWML



(d) BayesShrink



(e) Proposed method

Figure 2. The Noise Variance of 20 Barbara Image Denoising Algorithm of Rendering



(a) With noise image



(b)HT



(c) LAWML



(d)BayesShrink



(e)Proposed method

Figure 3: The Noise Variance of 25 Lena Image Denoising Algorithm of Renderin

5. Conclusion

This article first introduces the denoising performance evaluation standard and image wavelet threshold denoising method, and then gives the quaternion based on wavelet transform domain hidden markov tree model for image denoising (Q - HMT), non gaussian distribution model and hybrid statistical model of image denoising algorithm, good results have been achieved on the subjective and objective. Finally for SAR image, on the basis of the introduction of additive model, through quaternion wavelet transform, using improved coefficient classification criterion, the coefficients are divided into two categories: important coefficient and the important factor, proposed to improve the Donoho threshold and the new threshold function, and it deals with the important coefficient, estimate the excluding of quaternion wavelet transform coefficient, and the coherence of SAR image is obtained. Coherent speckle noise suppression experiments of real SAR image, on the objective indicators and visual effect, the proposed method is superior to the current many methods.

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