

An Optimization Sparse Representation Algorithm based on Log-Gabor

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Abstract

In this paper, we have proposed an optimized sparse representation algorithm based on Log-Gabor (Sparse Representation-based Classification Based on Log-Gabor, Log-GSRC), which applies local features information of samples to the sparse representation method. Actually, SRC (Sparse Representation-based Classification) is using a linear correlation between the samples of one class which can be assumed that these samples exist in a subspace, and also can be linear represented with each other. It is a global representation and it ignores the local features information of the samples, while in the case of there are a smaller number of training samples per class, SRC will obtain an inaccurate classification result which may correspond to one and more classes in the process of sparse decomposition. However, the Log-GSRC combines global and local features information of the samples and also improves the robustness of SRC. The experimental results clearly showed that Log-GSRC has much better performance than SRC and also has much higher recognition rates than SRC in face recognition.

Keywords: Compressive Sensing, Sparse Representation-based Classification, Gabor, Log-Gabor

1. Introduction

In recent years, with the rapid development of computer and information technology, more and more information data increases in exponential. These information data are usually high-dimensional data in reality, such as biomedical image data, large-scale text data, and face image data. As in image classification, such as face recognition, sign detection and so on, has always been the research hotspot of image processing and computer vision field. These images data usually contain high-dimensional feature information, and these high-dimensional data often lead to the "curse of dimensionality", so dimension reduction will become the research focus of many researchers, many algorithms have been proposed by researchers, such as PCA [1, 2], KPCA [3], 2DPCA [4, 5], LDA [6-8], Manifold Learning [9-11] and so on.

Signals are existed with analog form in real nature world. However, the tools we have carried out to process and transport the information are digitized, which requires us to use the way by sampling the analog signal into a digital signal that can be processed. The traditional method of sampling signals or images is the classical Nyquist sampling theorem, and Shannon sampling theorem is the basic theory of image compression. Nyquist sampling theorem requires the sampling frequency must be greater or equal than twice of the highest frequency of the signal, in order to ensure to recover the original signal without distortion.

In the field of signal processing, image representation has been a very basic problem, how to effectively utilize the sparsity of the signal and image data is a hot topic to many researchers. As the leader of the research in this area, Candés and Donoho proposed Compressive Sensing theory in 2006 [12-14], which is a challenge to the traditional Shannon-Nyquist sampling theorem. In Compressive sensing, it directly perceives and restores the information of the compressed data, breaks the constraints and limitations of the Shannon's theorem.

In Figure 1, compressive sensing theoretical framework as follows:

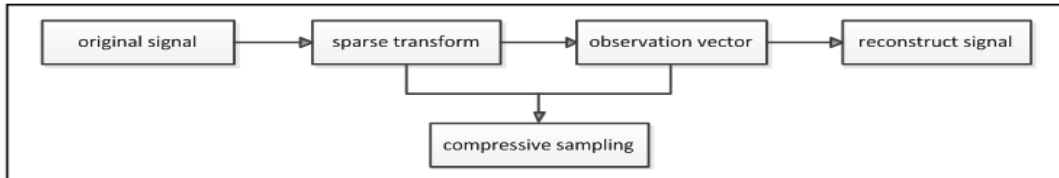


Figure 1. Compressive Sensing Theoretical Framework

In the past period of time, many researchers made a lot of new optimization algorithms based on the research of Candés and Donoho, to further develop the compressive sensing and sparse representation theory. At present, the sparse representation has also been widely used in many areas, and also has made great significance. In 2009, Wright proposed a face recognition method based on sparse representation which pushed this theory into a climax, he constructed a classifier based on sparse representation (Sparse Representation-based Classification, SRC) [15], and applied it in face recognition issues successfully. He utilized ℓ_1 -norm to represent the testing samples with training samples set, and then find the training samples of the same class, so as to solve the face recognition and classification problem. SRC has the best recognition rate by far in ORL, AR and Yale standard face datasets, the experimental results show that SRC has better robustness in varying expression and illumination, as well as occlusion and noise.

SRC is actually using a linear correlation between the samples of one class which can be assumed that these samples exist in a subspace, and also can be linear represented with each other. It is a global representation that to solve the testing samples to be represented globally in all training samples, but it ignores the local information of the samples. SRC builds sparse representation dictionary based on the overall training samples' characteristic, however, in the case of there are a smaller number of training samples per class, SRC will obtain an inaccurate classification result which may correspond to one and more classes in the process of testing samples sparse decomposition.

Consider the sparse representation discriminant information utilized by the SRC, we proposed an optimized sparse representation algorithm based on Log-Gabor [16] (Sparse Representation-based Classification Based on Log-Gabor, Log-GSRC). The Log-GSRC algorithm applies local features of samples to the sparse representation method, the algorithm combines global and local features of the samples, and in view of the local features of Log-Gabor has better robustness to changes in illumination, expression, gesture, and local deformation conditions, so as to further improve the recognition rate.

2. Sparse Representation-based Classification (SRC)

These signals' energy of sparse representation is concentrated in a small number of atoms, and these signals are expressed as a linear combination of a few atoms which belong to the over-complete dictionary.

Given a set $D = \{d_k, k = 1, 2, \dots, K\}$, the element d_k of D is the unit vector of the Hilbert space $H = \mathbb{R}^N$, where $K \leq N$; we define the set D as an over-complete dictionary or redundant dictionary, and each element d_k is called an atom or basis function.

Given a signal $x \in \mathbb{R}^N$, sparse representation decomposes the signal x into a linear combination of the series of basis signal $\{d_i \in \mathbb{R}^N, i = 1, \dots, n\}$,

$$x \approx \sum_{i=1}^n \alpha_i d_i \quad (1)$$

where $n \leq N$, and $\alpha = [\alpha_1, \dots, \alpha_n]^T$ is defined as sparse vector. Sparse representation model as can be illustrated by Figure 2:

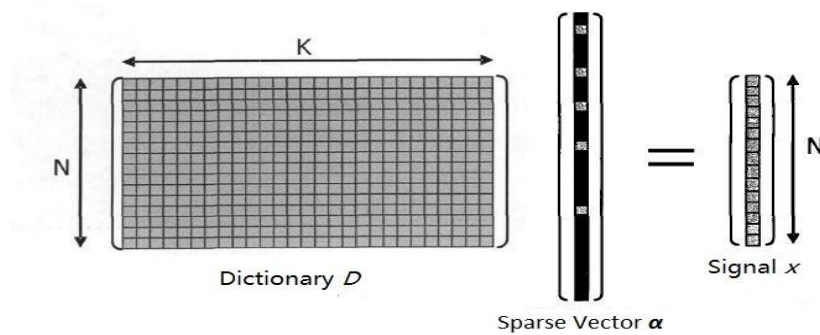


Figure 2. Sparse Representation Model

The goal of sparse representation is to decompose the signal into a linear combination of a few atoms. Because of the over-complete dictionary's redundancy, signal x 's sparse representation is not unique, over-complete sparse representation ultimate goal is to find the sparsest one of the entire sparse vector α , namely:

$$\arg \min_{\alpha} \|\alpha\|_0 \quad \text{Subject to } x = D\alpha \quad (2)$$

where $\|\cdot\|_0$ is defined as l^0 -norm, denotes the number of non-zero entries of sparse vector α . As to solve the above expression is an NP-hard problem, Many researchers have proposed many approximation algorithm to transform l^0 -norm problem into optimization problems under different constraints. Most of them utilize l^1 -norm to approximate l^0 -norm to solve l^0 -minimization problem:

$$\arg \min_{\alpha} \|\alpha\|_1 \quad \text{Subject to } x = D\alpha \quad (3)$$

Usually in real applications, but also need to consider the signal noise, generally we will add Gaussian white noise into image processing, and then we relax constraints of the formula above to an unconstrained convex optimization problem as flows:

$$\arg \min_{\alpha} \|\alpha\|_1 \quad \text{Subject to } \|x - D\alpha\|_2 \leq \varepsilon \quad (4)$$

where ε is the error tolerance. The SRC algorithm [15] is summarized as flows:

Algorithm 1 Sparse Representation-based Classification (SRC)

1. *Input: a matrix of training samples $D = \{d_k, k = 1, 2, \dots, K\}$ for K classes, a test sample $x \in \mathbb{R}^N$, and an optional error tolerance $\varepsilon > 0$.*
2. *Normalize the columns of D to have unit l^2 -norm.*
3. *Solve the l^1 -minimization problem:*

$$\arg \min_{\alpha} \|\alpha\|_1 \quad \text{Subject to } x = D\alpha \quad \text{or} \quad \arg \min_{\alpha} \|\alpha\|_1 \quad \text{Subject to } \|x - D\alpha\|_2 \leq \varepsilon$$
4. *Compute the residuals $r_i(x) = \|x - D\alpha\|_2$ for $i = 1, 2, \dots, K$.*
5. *Output identity(x) = $\arg \min_i r_i(x)$.*

3. Gabor filters and Log-Gabor filters Theory

3.1. Gabor Filters

In 1940s, Dennis Gabor [17] proposed a Gabor filter, and later J. Daugman [18] utilized Gabor filter to analog simple-cell receptive field with directional selectivity, and extended it to the 2D-Gabor filter, through to mammals the information processing mechanism of the visual cortex studies show that most of the visual cortex of simple cells is very similar to the response of an image signal filtering and 2D-Gabor filter, and 2D-Gabor wavelet theory has been proven that it can discover the local structure of the image data. The information corresponds to a spatial frequency, position and directional selectivity, which means that the face image applies. 2D-Gabor defined as follows:

$$g(x) = \frac{\|k_j\|^2}{\sigma^2} \exp\left\{-\frac{\|k_j\|^2 \|x\|^2}{2\sigma^2}\right\} \left\{ \exp(ik_j \cdot x) - \exp\left(-\frac{\sigma^2}{2}\right) \right\} \tag{5}$$

$$k_j = \begin{pmatrix} k_{jx} \\ k_{jy} \end{pmatrix} = \begin{pmatrix} k_v \cos \phi_u \\ k_v \sin \phi_u \end{pmatrix} \tag{6}$$

where x is defined as a coordinate of a given position image, k_j is the filter center frequency, σ is the size of Gaussian window, k_v is the wavelength of texture, ϕ_u is the directional selectivity of filter.

3.2. Log-Gabor Filters

Gabor filter has a good directional and frequency selectivity for to obtain localized frequency information, however they have two main limitations. The maximum bandwidth of a Gabor filter is limited and does not contain a DC Component in the transfer function of a high bandwidth even-symmetric Gabor filter. Later, in 1987, Field proposed Log-Gabor filter, it is not affected by bandwidth limitations, and has excellent communication skills in the high frequency components. The experiments show that the Log-Gabor has better performance than Gabor in representing natural images. Log-Gabor function is defined as follows:

$$G(f, \theta) = \exp\left\{-\frac{\left(\log\left(\frac{f}{f_0}\right)\right)^2}{2\left(\log\left(\frac{\sigma_f}{f_0}\right)\right)^2}\right\} \exp\left\{-\frac{(\theta - \theta_0)^2}{2\sigma_\theta^2}\right\} \tag{7}$$

where f_0 is the filter's center frequency, θ_0 is filter's direction, σ_f is used to determine the size of the radial bandwidth, σ_θ is used to determine the size of the angular direction bandwidth.

3.3 Log-Gabor Features of Face Image

We utilize face image and Log-Gabor filter's convolution to obtain the Log-Gabor features of face image. Where $I(x, y)$ denotes as a gray-scale image, then the convolution of face image and Log-Gabor filter is defined as follows:

$$G_{u,v}(x, y) = I(x, y) \otimes \psi_{u,v}(x, y) \quad (8)$$

where $\psi_{u,v}(x, y)$ is defined as the Log-Gabor filter with scaling u and direction v , \otimes is convolution; $G_{u,v}(x, y)$ is the result of convolution. There are 4 scaling parameters $u \in (0, 1, 2, 3)$ and 6 direction parameters $v \in \{0, 1, 2, 3, 4, 5\}$ used here, and corresponding to the filter center frequency and direction is $(\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24})$ and $(0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{5\pi}{6})$, finally we obtain 24 filters to extract facial image feature. An example is shown below:

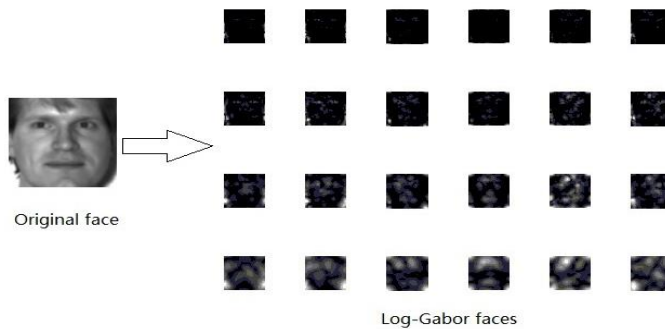


Figure 3. Left is the Original Face Image; We Use 4 Scaling Parameters and 6 Direction Parameters, thus Generating 24 Log-Gabor Faces Right

4. Sparse Representation-based Classification Based on Log-Gabor (Log-GSRC)

We review the image sparse representation theory framework and applications, as well as Gabor and Log-Gabor filter theory in the first few chapters, and have deeply studied the mathematical theoretical basis of SRC. Finally, we analyze and spread the solving optimization method of SRC which improves results ever deduced by SRC.

As we know, SRC is actually using a linear correlation between the samples of one class, and we can suppose that these samples are all in a subspace and meanwhile can be linear represented with each other. SRC ignores the local information of the samples and builds sparse representation dictionary based on the overall training samples' feature, but in the case of there are a smaller number of training samples per class, SRC will obtain an inaccurate classification result. In this chapter, we have proposed an optimized sparse representation algorithm based on Log-Gabor (Sparse Representation-based Classification Based on Log-

Gabor, Log-GSRC). The Log-GSRC algorithm introduces local features of samples to the sparse representation method, the algorithm combines global and local features of the samples, and in view of the local features of Log-Gabor has better robustness to changes in illumination, expression, gesture, and local deformation conditions.

Assuming $C = \{c_1, c_2, c_3, \dots, c_M\}$ is a vector of object classes, the number of samples of per class is n_1, n_2, \dots, n_M ; we use an m -dimensional $\mathbf{d}_{i,j} \in \mathbb{R}^m$ vector to denote the feature of per face sample, and the vector is processed by the Log-Gabor wavelet, where $\mathbf{d}_{i,j} \in \mathbb{R}^m$ is defined by the j^{th} sample of the i^{th} class. Suppose the Log-Gabor feature vector of testing sample is $\mathbf{T} \in \mathbb{R}^m$. The Log-GSRC algorithm is summarized as flows:

Algorithm 2 Sparse Representation-based Classification Based on Log-Gabor (Log-GSRC)

1. Utilize formula (7) and (8) to compute the Log-Gabor feature of per sample $I(x, y)$, then get the feature value $g(z)$ where $z = (x, y)$ of $I(x, y)$. Array all of $g(z)$ to get column vector $\mathbf{d}_{i,j} \in \mathbb{R}^m$ ($i = 1, 2, \dots, M, j = 1, 2, \dots, n_i$) which defined as the Log-Gabor feature vector of per sample.
2. From Step 1, we can get n_i m -dimensional vector which defined as $\mathbf{d}_{i,1}, \mathbf{d}_{i,2}, \dots, \mathbf{d}_{i,n_i} \in \mathbb{R}^m$ then we utilize $\mathbf{a}_i = (\mathbf{d}_{i,1}, \mathbf{d}_{i,2}, \dots, \mathbf{d}_{i,n_i})^T$ to denote one column of over-complete matrix D . Using the above $\mathbf{a}_i = (\mathbf{d}_{i,1}, \mathbf{d}_{i,2}, \dots, \mathbf{d}_{i,n_i})^T$, we can obtain over-complete redundancy dictionary $D = \{\mathbf{a}_i, i = 1, 2, \dots, M\}$, then utilize K-SVD algorithm [19] to train this over-complete redundancy dictionary, at last we can get the optimized over-complete redundancy dictionary D .
3. Testing sample \mathbf{T} can be regarded as a linear combination of training samples, is defined as
$$\mathbf{T} = \mathbf{d}_{1,1}\alpha_1 + \mathbf{d}_{1,2}\alpha_2 + \dots + \mathbf{d}_{M,n_M}\alpha_n,$$
 where $n = n_1 + n_2 + \dots + n_M$ and $M \ll n$, $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)^T$ denotes as sparse vector. Consider the noise, can be written as $\mathbf{T} = D\boldsymbol{\alpha} + \boldsymbol{\varepsilon}$, where $\boldsymbol{\varepsilon}$ is defined as an optional error tolerance.
4. We utilize Homotopy[20, 21] algorithm to solve the problem $\mathbf{T} = D\boldsymbol{\alpha} + \boldsymbol{\varepsilon}$, as same to l^1 -minimization problem $\arg \min_{\boldsymbol{\alpha}} \|\boldsymbol{\alpha}\|_1$ Subject to $\|\mathbf{T} - D\boldsymbol{\alpha}\|_2 \leq \boldsymbol{\varepsilon}$.

5. Experimental Results

In this section, we perform experiments on benchmark face databases to demonstrate the improvement of Log-GSRC over SRC. We evaluated the performance of the proposed algorithm on two representative facial image databases: ORL database [22] and AR database [23]. For each subject, we randomly selected one part of the images for training, and used the other for testing.

5.1. ORL Database

The ORL Database of faces comes from <http://www.cl.cam.ac.uk/research/dtgb/attarchive/facedata-base.html>. These images were taken at different times from April 1992 to April 1994 by the AT&T Laboratories Cambridge; there are 10 different images of each of 40 distinct subjects. The image size is 92×112 , the image background is black. For each subject, we randomly selected half of the images for training (*i.e.*, 5 images per subject), and used the other half for testing. All of the samples keep the original format, without any pretreatment. We have carried out two sets of experiments, and experimental results are shown as follows:

Table 1. ORL Database Recognition Rate

Algorithm	Subject Number			
	10	20	30	40
SRC	98.0%	94.0%	90.7%	91.5%
Log-GSRC	96.0%	96.0%	97.3%	97.0%

Table 2. ORL Database Recognition Rate

Algorithm	Training Sample Number			
	2	3	4	5
SRC	83.5%	84.0%	89.5%	91.5%
Log-GSRC	89.0%	92.5%	97.5%	97.0%

From Table 1 and Table 2, we can get that Log-GSRC has much higher recognition rates than SRC with the increasing number of training sample classes and also under the same condition Log-GSRC has much better performance than SRC.

5.2. AR Database

The AR Database of faces comes from <http://www2.ece.ohio-state.edu/~aleix/ARdatabase.html>. This face database was created by Aleix and Robert. It contains over 4,000 color images corresponding to 126 individuals. In this experiment, we chose a subset of AR Database consisting of 50 male subject and 50 female subjects. For each subject, contains 14 images with only facial expressions and illumination changes. We select the first seven images for training and another seven images for testing. The size of original images is 165×120 , we prune the image size into 50×40 . The experimental results are shown as follows:

Table 3. AR Database Recognition Rate

Algorithm	Subject Number				
	20	40	60	80	100
SRC	60.7%	68.2%	73.3%	72.0%	69.6%
Log-GSRC	63.6%	73.9%	81.2%	77.4%	74.0%

Table 4. AR Database Recognition Rate

Algorithm	Training Sample Number					
	2	3	4	5	6	7
SRC	62.0%	64.4%	63.6%	65.8%	69.3%	69.6%
Log-GSRC	64.1%	65.3%	66.1%	70.0%	74.1%	74.0%

From Table 3 and Table 4, we also can see that Log-GSRC has much better performance than SRC under the same condition.

6. Conclusions

In this paper, we have proposed an optimized sparse representation algorithm based on Log-Gabor (Sparse Representation-based Classification Based on Log-Gabor, Log-GSRC), which applies local features of samples to the sparse representation method. As you know, SRC is actually using a linear correlation between the samples of one class which can be assumed that these samples exist in a subspace, and also can be linear represented with each other. It is a global representation that to solve the testing samples to be represented globally in all training samples, but it ignores the local information of the samples, however, the Log-GSRC combines global and local features of the samples. We evaluated the proposed method based on public face data set. The experimental results clearly showed that Log-GSRC has much better performance than SRC, meanwhile, also has much higher recognition rates than SRC in theory and in real world face recognition.

Acknowledgements

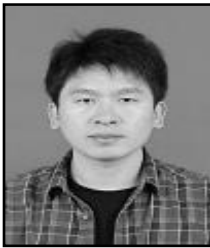
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