

## An Improved Gaussian Mixture Model based on NonLocal Information for Brain MR Images Segmentation

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### Abstract

*Brain image segmentation is an important part of medical image analysis. Due to the effect of imaging mechanism, MR images usually intensity in homogeneity, which is also named as bias field. Traditional Gaussian Mixed Model (GMM) method is hard to obtain satisfied segmentation results with the effect of noise and bias field. We propose a novel model based on GMM and nonlocal information. The improved method coupled segmentation and bias field correction that can manage the bias field while segmenting the image. In order to obtain a smooth bias field, we employed the Legendre Polynomials to fit it and merged it to the EM framework. We also use the non local information to deal with the noise and preserve geometrical edges information. The results show that our method can obtain more accurate results and bias field.*

**Keywords:** MRI, GMM, Bias field, Non local information

### 1. Introduction

Medical imaging technology has been used to prevent a qualitative and quantitative analysis of brain tissue [1]. Among many biological imaging methods, the magnetic resonance imaging (MRI) technology is very effective for the soft tissues such as the brain tissues due to its special imaging mechanism. The segmentation of magnetic resonance (MR) images is important for the study of many diseases, i.e. brain diseases [2].

Many image techniques have been used for image segmentation, *i.e.*, thresholding method, the region growing method, the active contour model and the clustering method. The traditional active contour models can obtain continuous results, however, they usually only uses the edge information which makes them sensitive to the noise and weak boundaries. Furthermore, the initial contour must be placed near the edges of the region of interesting. Because of the ambiguity of the internal brain organization and the inherent uncertainty of MR images, the active contour model is hard to obtain satisfied results. Recently, the fuzzy c-means (FCM) [3] is widely used for brain MR image segmentation. However, this method only considers intensity of image, which makes it sensitive to the noise. Gaussian Mixed Model (GMM) [4, 6] is another widely used method and can obtain more satisfied results than the FCM methods. The traditional GMM is still sensitive to the noise. Further, this method is sensitive to the intensity in homogeneity.

In this paper we proposed an improved GMM method based on nonlocal information [7] to reduce the effect of the noise, which can estimate the bias field when segmenting the images. In order to obtain smooth bias field, we used the Legendre polynomial [5] to fitting the bias field and integrated it into the EM framework.

## 2. Methods

### 2.1. Traditional Gaussian Mixed Model

For most MR images, the distribution of the intensity obeys Gaussian distribution. Gaussian Mixed Model can estimate distribution of each class. Let  $I = \{x_1, x_2, \dots, x_N\}$  denote an image with  $N$  pixels, which can be divided into  $c$  classes. Modeling each class by a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ , the probability density for class  $j$  can be written as:

$$p(x_i | \Gamma_i = j, \theta_j) = \frac{1}{\sqrt{2\pi}\sigma_j} \exp\left(-\frac{(x_i - \mu_j)^2}{2\sigma_j^2}\right), j = 1, 2, \dots, c. \quad (1)$$

where  $\Gamma_i, i = 1, 2, \dots, c$ , is the tissue class at position  $i$  and  $\theta_j = (\mu_j, \sigma_j)$  is the parameters of the  $j$ th class. Then the overall probability density of  $x_i$  is:

$$p(x_i | \Theta) = \sum_{j=1}^c p(x_i | \Gamma_i = j, \theta_j) p(\Gamma_i = j) \quad (2)$$

where  $\Theta = (\theta_1, \theta_2, \dots, \theta_c)$  is the Gaussian distribution parameters of the mixed model.  $p(\Gamma_i = j)$  is the priori probability of  $j$ th class. The probability density for image  $I$  can be written as:

$$p(I | \Theta) = \prod_{i=1}^N p(x_i | \Theta). \quad (3)$$

The maximum likelihood estimates for the parameters  $\mu_j$  and  $\sigma_j$  can be obtained by maximization of  $p(I | \Theta)$ .

EM algorithm is one of the widely used methods for maximum likelihood estimation method for solving the model parameters from incomplete data. EM algorithm is simple and stable but it is dependent on the initialization and easily trapped into local optima. So we first use the fuzzy c-means to get the initial parameter  $\Theta^{(0)}$  and then calculate the prior probability  $P^{(0)}$ . The whole EM algorithm can be written as:

Step 1. E-step:

$$p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k)}) = \frac{p(x_i | \Gamma_i = j, \theta_j^{(k)}) p^{(k)}(\Gamma_i = j)}{\sum_{j=1}^c p(x_i | \Gamma_i = j, \theta_j^{(k)}) p^{(k)}(\Gamma_i = j)} \quad (4)$$

$j = 1, 2, \dots, c$

Obviously,  $x_a$  is more likely regarded as class  $\Gamma_i = m$  when  $p(\Gamma_a = m | x_a, \theta_m) \geq p(\Gamma_a = n | x_a, \theta_n), m \neq n$ .

Step 2. M-step:

$$p^{(k+1)}(\Gamma_i = j) = \frac{\sum_{i=1}^N p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k+1)})}{N} \quad (5)$$

$$\mu_j^{(k+1)} = \frac{\sum_{i=1}^N x_i p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k)})}{\sum_{i=1}^N p(\Gamma_i = j | x_i, \theta_j^{(k)})} \quad (6)$$

$$(\sigma_j^2)^{(k+1)} = \frac{\sum_{i=1}^N p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k)}) (x_i - \mu_j^{(k+1)})^2}{\sum_{i=1}^N p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k)})} \quad (7)$$

## 2.2. Improved Gaussian Mixed Model

The observed MRI signal  $J$  is the product of the true signal  $I$  generated by the underlying anatomy and spatially varying field factor  $B$ , and an additive noise  $n$  :

$$J = (I + n) \cdot B \quad (8)$$

Given the observed signal  $J$ , the problem is to estimate the true image  $I$ . To simplify the computation, we ignore the noise and take the logarithmic transform of both sides.

$$\log J = \log((I + n) \cdot B) = \log(I + n) + \log B \quad (9)$$

Where  $B$  is the bias field, which can be estimated by using Legendre polynomial functions:

$B = \sum_{k=1}^N c_k \varphi_k(I)$ . The Legendre polynomial functions can make the bias field smooth. In this

paper, we set  $N = 4$ . Then the improved GMM can be written as:

$$p(x_i | \Gamma_i = j, \theta_j, C) = \frac{1}{\sqrt{2\pi} \sigma_j} \exp \left\{ - \frac{(x_i - \mu_j - \sum_{k=1}^4 c_k \varphi_k(x_i))^2}{2\sigma_j^2} \right\} \quad (10)$$

$$j = 1, 2, \dots, c$$

and

$$p(x_i | \Theta, C) = \sum_{j=1}^c p(x_i | \Gamma_i = j, \theta_j, C) p(\Gamma_i = j) \quad (11)$$

with  $C = \{c_k\}$  is the bias field parameters. The parameters  $\mu_j$ ,  $\sigma_j$  and  $c_k$  can be calculated

by maximizing the likelihood  $\prod_{i=1}^N p(x_i | \Theta, C)$  :

$$\mu_j = \frac{\sum_{i=1}^N \left( x_i - \sum_{k=1}^4 c_k \varphi_k(x_i) \right) p(\Gamma_i = j | x_i, \Theta, C)}{\sum_{i=1}^N p(\Gamma_i = j | x_i, \Theta, C)}, \quad (12)$$

$$\sigma_j^2 = \frac{\sum_{i=1}^N p(\Gamma_i = j | x_i, \Theta, C) \left( x_i - \mu_j - \sum_{k=1}^4 c_k \varphi_k(x_i) \right)^2}{\sum_{i=1}^N p(\Gamma_i = j | x_i, \Theta, C)}. \quad (13)$$

### 2.3. Improved GMM based on Non Local Information

Gaussian mixture model is based on the gray scale information, which makes it sensitive to the noise. Local spatial information was introduced to reduce the effect of the noise by using neighbor information. Because the neighborhoods of each pixel contain both target and non-target points when corrupted by noise, we need to increase the weight of the target point to avoid over-segmentation. We utilize non local information into the prior probability:

$$p^{(k+1)}(\Gamma_i = j | x_i, \theta_j^{(k)}) = \frac{p(x_i | \Gamma_i = j, \theta_j^{(k)}) (p^{(k)}(\Gamma_i = j) * w(i, q))}{\sum_{j=1}^c p(x_i | \Gamma_i = j, \theta_j^{(k)}) (p^{(k)}(\Gamma_i = j) * w(i, q))} \quad (14)$$

$$j = 1, 2, \dots, c$$

where  $w(i, q)$  is a non local mean,  $W_i^r$  denotes the neighbor of pixel  $i$  with size  $r \times r$ .  $w(i, q)$  is defined as:

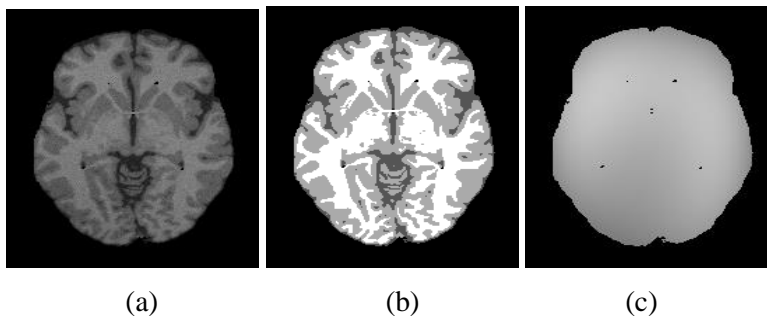
$$w(i, q) = \frac{1}{Z(i)} \exp\left(-\frac{\|v(M_i) - v(M_q)\|_{2,a}^2}{h^2}\right) \quad (15)$$

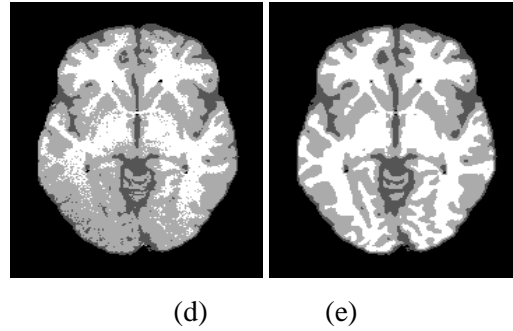
$$Z(i) = \sum_j \exp\left(-\frac{\|v(M_i) - v(M_q)\|_{2,a}^2}{h^2}\right) \quad (16)$$

Where,  $0 \leq w(i, q) \leq 1$  and  $\sum w(i, q) = 1$ ,  $\|v(M_i) - v(M_q)\|_{2,a}^2$  is a weighted Euclidean distance of intensity gray level vectors  $v(M_i)$  and  $v(M_q)$ .  $M_q$  denotes a square neighborhood centered at a pixel  $q$  which is usually fixed as  $r \times r$ .  $h$  is a degree which controls the decay of the function,  $Z(i)$  is the normalizing constant. The pixel with a similar gray level neighborhood to  $v(M_i)$  have larger weights. The non local means not only use the adjacent pixels, but the similar configuration in a whole neighborhood. It performed well at image denoising.

### 3. Implementation and Results

Experimental results on brain MR images are presented in Figure 1, 2. The brain MR image from brain web database of McGill University. Figure 1 shows the results of MR image with 3% noise and 80% INU. Due to the bias field, the traditional GMM failed to find the results while our method obtains satisfied results similar to the ground truth. In Figure 2 shows the segmentation on the data with noise level 5% and INU level 80%. From the results we can find our method can reduce the effect of the noise.





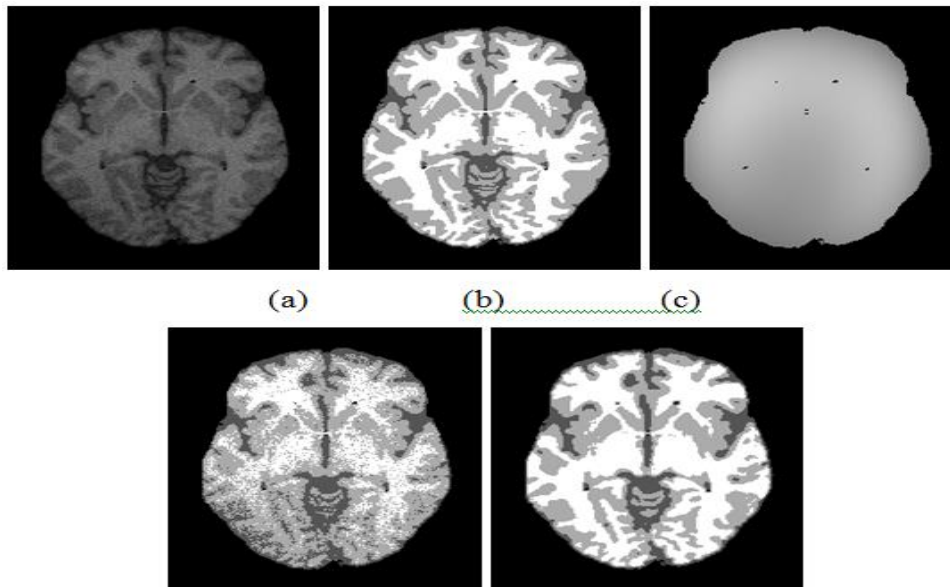
**Figure 1. 3% Noise and 80% INU (a) Original Image (b) Ground Truth (c) Estimated Bias Field use our Method (d) Result of our Traditional GMM (e) Result of our Method**

In order to measure the result of the experiment, we use the Jaccard Similarity (JS) to exam the accuracy of segmentation.

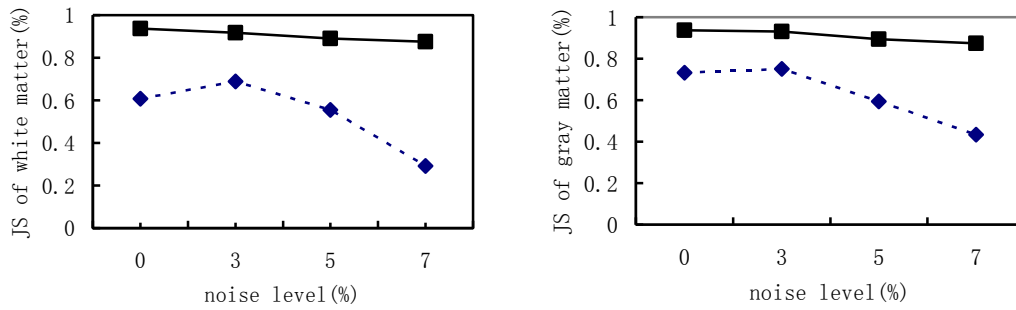
$$J(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2} \quad (17)$$

$S_1$  is the result of segmentation where  $S_2$  is the ground truth. The better results have a higher JS.

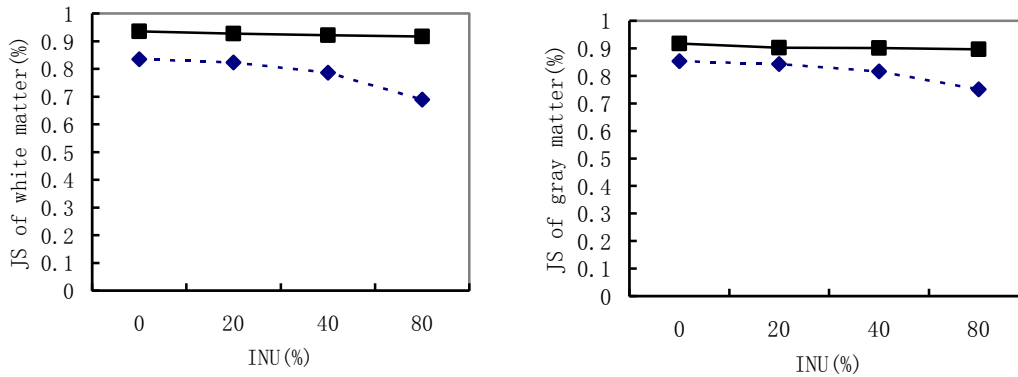
Figure 3 and Figure 4 shows the accuracy of segmenting WM and GM. The accuracy reduced when the image combined with noise level increased from 0% to 9% and the INU level increased from 0% to 80%. We can found that the traditional GMM had a poor performance on the segmentation of white matter when the INU level increased. Our method was stable when the INU level and noise level changed.



**Figure 2. 5% Noise and 80% (a) Original Image (b) Ground Truth (c) Estimated Bias Field use our Method (d) Result of our Traditional GMM (e) Result of our Method**



**Figure 3. JS for Different Noise Level under 80% INU Level, Solid Line is our Method and Broken Line is Traditional GMM Method**



**Figure 4. JS for Different INU Level under 3% Noise Level, Solid Line is our Method and Broken Line is Traditional GMM Method**

#### 4. Conclusions

In this paper, we proposed a method for MR segmentation and bias field correction using an improved Gaussian mixed model by using non local information to reduce the effect of the noise. The improved method can segment images meanwhile estimate the bias field. The results show that the presented method can accurately segments the brain MR images corrupted with bias field and noise.

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