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A Novel Three Coding-pitches Triangular Patterns Phase-shifting Profilometry based on Modified Sunzi Theorem

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Abstract

In order to realize the high precision real-time measurement for the static objects, three coding-pitches triangular patterns phase-shifting profilometry based on modified Sunzi Theorem (ST) was proposed in this paper. First, according to Sunzi Theorem, the triangular pattern is designed. This Sunzi Theorem was adopted to unwrap three wrapped coordinate acquired from triangular patterns fringe images. The greatest common divisor of coding-pitches is obtained by the used projector. In unwrapping processing, the wrapped coordinate is considered as remainder of the absolute coordinate modulo of the coding-pitch. Generally there are two kinds of rounding policies for the remainder in real number set. If the rounding policy selection is wrong, the rounding result of remainder containing decimal fraction error which can lead the unwrapping result to quite away from the correct one. In order to solve the high sensitivity of the decimal fraction error, the decimal fraction difference is recognized as the criterion of the rounding policy selection for real number erroneous remainder. To verify the presented method in this paper, a 3D shape measurement experimental system is constructed using projector and camera. The experiment results shown that, a maximum standard deviation of measurement error to the rule 3D objects is 0.62mm, and the complex surface reconstruction with different surface reflectivity can be realized very well.

Keywords: *Phase-shifting Profilometry, Triangular Patterns, Three Coding-pitches, Sunzi theorem*

1. Introduction

Structured light (SL) technique is one of most widely used techniques for generating three dimensional (3D) shape measurements [1]. Because of its numerous advantages, such as non-contact, low-cost, high-resolution, phase-shifting profilometry (PSP) has been widely used in such fields as the manufacturing inspection, medical sciences, reverse engineering, MEMS component characterization, *etc*., [2]. The advantage of PSP is that it is less sensitive to the surface reflectivity variations and the object can be measured point by point [3]. For PSP, a phase unwrapping algorithm is usually necessary to obtain continuous phase maps [4].

The spatial unwrapping is one solution for phase unwrapping. Since it assumes that the phase difference between neighboring pixels is less than π, and spatial unwrapping basically only works well on smooth surfaces [5]. An alternative solution for the height-step object is PSP combining with Grey-code patterns and frequently regarded as the powerful solution for 3D shape measurement. In this case, the pattern period's number used to create the wrapped map is usually determined by the patterns amount of Grey-code, since GC essentially codes the areas within every pattern period [6]. The Multi-Wavelength PSP (MWPSP) based on

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more than two wavelengths is employed to obtain the higher accuracy 3D shape measurement. Three-Wavelength Heterodyne Phase Shift Profilometry (TWHPSP) based on three-wavelength is popularly used, because the unambiguity measurement range (UMR) of the dual-wavelength PSP improved no significant beyond than any of the wavelength used [7, 8]. The excess fractions is another unwrapping method for unwrapped phase map acquired by three-wavelength PSP [9], but this method has some defect, such as large amount computing, computing time expensive, etc. An appealing approach was originally introduced by Gushov and Solodkin [10]. This approach is based on the Sunzi Theoretic Approach (ST) where only two patterns, with a larger number of periods, are considered, which is in theory sufficient to provide highly accurate 3D results. Unfortunately, the conventional Sunzi theorem is very sensitive for phase error, which is usually exist in obtaining wrapped phase map. Many robust algorithms [11, 12] were presented, but these are based on search algorithms and application in density 3D shape measurement is restricted.

The phase map was calculated using the standard phase-shifting formula and three sinusoidal disturbed fringe images for every image pixel [3, 4, 5]; thus the processing time-consuming is expensive. Alternatively, a linear gray-scale pattern, such as triangular patterns [13], can obviously reduce time-consuming. Therefore, the accurate and time-consuming play the significantly larger roles for MWPSP. Fast obtaining wrapped phase map and improving the Sunzi theorem sensitivity of wrapped phase error are vital for the success of MWPSP to measure objects with larger surface reflectivity variations.

In this research, we propose a novel three coding-pitches triangular patterns phase-shifting profilometry based on Sunzi theorem. It should be noted that wavelength as defined in this paper is different from that defined in physics. The wavelength here indicates the coding-pitch of single triangular fringe stripes, *i.e.*, number of projected image pixels per fringe stripe. The results of experiments are demonstrated that the presented profilometry significantly reduced the 3D shape measurement time-consuming and increased the accurateness of unwrapping phase. The main work of this paper is given as follows. Section 2 gives the principle of the novel phase coding method. Section 3 demonstrates the presented method through experiments. Section 4 summarizes this presented method.

2. Principle of Three Coding-pitches Triangular Patterns PSP based ST

PSP is a triangulation based, 3D reconstruction technique based on measuring the fringe distortion in a series of phase shifting fringe images captured by camera from an angle relative to the projector, when some patterns are projected by projector, reflected off the 3D object. PSP measurement system is shown as Figure 1.

2.1. Principle of Triangular Patterns PSP

In the Triangular Patterns PSP (TPPSP), only three triangular patterns phase-shifted by one third of the pitch are used to reconstruct the 3-D object. The gray-scale distribution of the three-step phase-shifted triangular fringe patterns are shown in Figure 2. The gray-scale formula for the two shifted triangular patterns is formulated as (1), (2).

$$I_j^p\left(n^p,\ m^p\right)=\begin{cases} I_{\min}+\dfrac{2I_m}{T}\operatorname{mod}\left(n^p+\dfrac{j-1}{3}T,T\right) & \operatorname{mod}\left(n^p+\dfrac{j-1}{3},T\right)\le\dfrac{T}{2} \\ I_{\max}-\dfrac{2I_m}{T}\operatorname{mod}\left(n^p+\dfrac{j-1}{3}T,T\right) & \dfrac{T}{2}<\operatorname{mod}\left(n^p+\dfrac{j-1}{3},T\right)<T \end{cases} \tag{1}$$

$$I_m = I_{\max} - I_{\min} \tag{2}$$

Where, $I_j^p(n^p, m^p)$ are the gray-scales of projector pixel (n^p, m^p) in three-step shifting patterns, respectively and $j = 1, 2, 3$. T is the coding-pitch of one triangular. $I_{\max}$ And $I_{\min}$ are the minimum and maximum gray-scales of the triangular patterns. $\mathrm{mod}(n^p, T)$ Equals the remainder of n^p congruent modulo T.

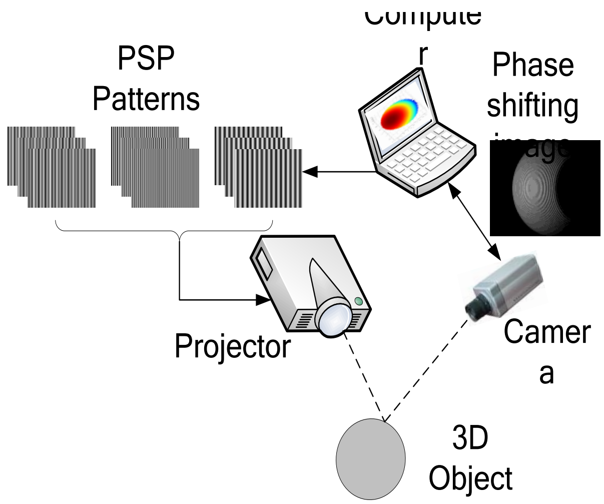

Figure 1. PSP Measurement Systems

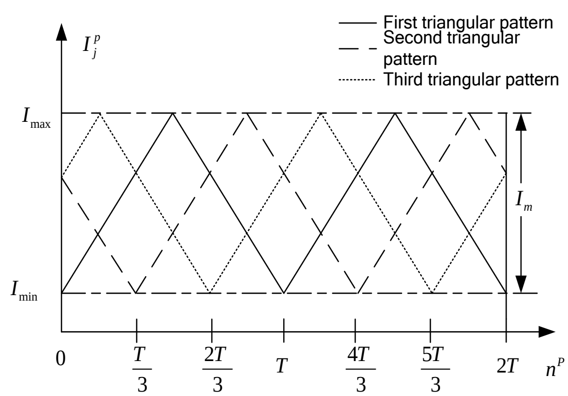

Figure 2. Gray-Scale Distribution of the Three-Step Phase-Shifted Triangular Patterns

$I_j^c(n^c, m^c)$ definite as the gray-scales of camera image pixel (n^c, m^c) in the fringe images. The matching pixel in projector image to (n^c, m^c) could be found. And its row coordinate n^p (namely absolute coordinate as follow) is wrapped into one coding-pitch as n^r. n^r Is wrapped coordinate and calculates according formula (3). The camera image plane coordinates (n^c, m^c) is omitted in formula (3).

$$n^r = \begin{cases} \frac{T}{6}\frac{I_2^C - I_3^C}{I_2^C - I_1^C} & I_1^C < I_3^C < I_2^C \\ \frac{T}{6}\frac{I_1^C + I_2^C - 2I_3^C}{I_2^C - I_3^C} & I_3^C < I_1^C < I_2^C \\ \frac{T}{6}\frac{3I_1^C - I_2^C - 2I_3^C}{I_1^C - I_3^C} & I_3^C < I_2^C < I_1^C \\ \frac{T}{6}\frac{3I_1^C - 4I_2^C + I_3^C}{I_1^C - I_2^C} & I_2^C < I_3^C < I_1^C \\ \frac{T}{6}\frac{5I_3^C - I_1^C - 4I_2}{I_3^C - I_2^C} & I_2^C < I_1^C < I_3^C \\ \frac{T}{6}\frac{5I_3^C - 6I_1 + I_2^C}{I_3^C - I_1^C} & I_1^C < I_2^C < I_3^C \end{cases} \tag{3}$$

Three difference coding-pitches are selected to obtain the absolute coordinate. The unwrapping algorithm details as follow.

2.2. Wrapped Coordinate Unwrapping Algorithm Based Modified ST

Let n^p be a positive integer, $0 < T_1 < T_2 < \cdots < T_k$ be k module, and $n_1^r, n_2^r, \cdots, n_k^r$ be the k remainders of n^p, *i.e.*, as depicted in

$$n^p \equiv n_i^r \left(\text{mod } T_i\right) \text{ or } n^p = f_i T_i + n_i^r \tag{4}$$

Where f_i is unknown integer and named folding integer? n^p and n_i^r are said to be congruent modulo T_i, $1 \le i \le k$. If and only if $n^p < \text{lcm}\{T_i\}$ (where lcm{} denotes the Least Common Multiple), n^p can be uniquely reconstructed from k remainders. If the entire modules T_i are co-prime, then ST has a simple formula as follow

$$n^p \equiv \left(\sum_{i=1}^{k} n_i^r t_i t_i'\right)\left(\text{mod } T\right) \tag{5}$$

Where $T = \text{lcm}\{T_i\} = \prod_{i=1}^{k} T_i$, t_i defines as T/T_i, t_i' is modular multiplicative inverse of t_i, *i.e.*

$$t_i t_i' \equiv 1\left(\text{mod } T_i\right) \tag{6}$$

If any pair module T_i have the greatest common divisor t (t=gcd{T_i}), in this case, the entire modules have gcd t. Let $P_i = T_i/t$, then all P_i are co-prime. Define $P = \text{lcm}\{P_i\}$, and $p_i = P/P_i$. Notes that p_i and P_i are co-prime, and the modular multiplicative inverse exists, which is denoted by p_i'. Define a_i @floor$\left(n_i^r/t\right)$, floor () denotes rounding to the nearest integer towards zero, and then

$$n_i^r = a_i t + n_c = a_i t + a_c t \tag{7}$$

where $n_c = \mathrm{mod}(n_i^r,\ t)$ is the common remainder of n_i^r modulo t for $1 \le i \le k$, a_c is the decimal fraction, *i.e.*, $0 \le a_c < 1$. Define $n_0^p\ @\mathrm{floor}\left(n^p/t\right)$. Then if only if $0 \le n_0^p < P$, n_0^p can be uniquely reconstructed as

$$n_0^p \equiv \left(\sum_{i=1}^{k} a_i\, p_i\, p_i'\right)(\mathrm{mod}\ P) \tag{8}$$

Therefore, n^p can be uniquely reconstructed as

$$n^p = n_0^p t + n_c \tag{9}$$

If n^p and n_i^r are positive real. And defines as f_i and T_i must be integer, and formulas (7), (8), (9) could generalization to real.

In nature, wrapped coordinate have errors. If the entire modules T_i are not co-prime, define E_i as the error of n_i^r , and the i th erroneous remainder (in this research as the wrapped coordinate) be $\hat{n}_i^r\ @ n_i^r + E_i$. Define e_i is real quotient of E_i/d , *i.e.*, $e_i = E_i/d$. Assume $|e_i| < \gamma$, and $|E_i| < \Gamma$, *i.e.*, $\Gamma = \gamma t$. The ST is very sensitive to the error of remainder. The well-known problem in ST is the entire $\hat{n}_i^r$ are rounded and then used in (5) or (8). Consider that, due to error, just one of the wrapped coordinate is slightly above an integer, when it should actually be just below that integer. Evidently, rounding (discarding a fractional part) gives a wrong input in (5) or (8), which is even magnified in the computation with the corresponding coefficient e_i or E_i . Consequently, the absolute coordinate n^p will be quite apart from the correct one.

Generally, there are two policies when computing an integer apart from the rational wrapped coordinate: either rounding to the nearest integer or to the nearest integer towards zero. The key issue is how to assure that the entire wrapped coordinates are rounded towards the same corresponding value on the absolute coordinate axis and that, consequently, the rounding process provides the correct integer to be used in (5) or (8).

Assume γ =0.25 or Γ =0.25t, and the Decimal Fractional Difference (DFD) of two wrapped coordinate could be regard as criterion for selection the rounding policies. In this case, no deviation decoding scope of ST depicted in shading part exclude dotted circle of Figure 3, it is limited by the gcd of coding-pitches.

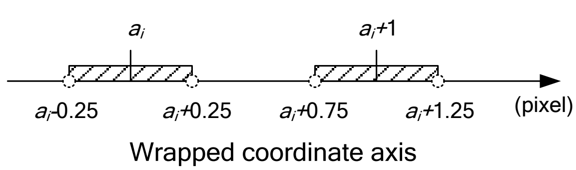

Figure 3. No deviation Decoding Scope of Wrapped Coordinate

If $\hat{n}_i^r$ and $\hat{n}_j^r$, $1 \le i \ne j \le k$, are two erroneous wrapped coordinates of k , DFD Δb_{ij} define

$$\begin{cases} \Delta b_{ij} = \hat{n}_i^r - \mathrm{floor}\left(\hat{n}_i^r\right) - \hat{n}_j^r + \mathrm{floor}\left(\hat{n}_j^r\right) & \gcd\{T_i\} = 1 \\ \Delta b_{ij} = \hat{r}_i - \mathrm{floor}\left(\hat{r}_i\right) - \hat{r}_j + \mathrm{floor}\left(\hat{r}_j\right) & \gcd\{T_i\} = t \ne 1 \end{cases} \tag{10}$$

Where $\hat{r}_i$ is a real, define as $\hat{r}_i \square \hat{n}_i^r / t$. If the absolute value of Δb_{ij} is not more than 0.5, the rounding policies is rounding to the nearest integer the nearest integer towards zero, alternative rounding to the nearest integer.

2.3. Optimum Coding-Pitch Selection in Unwrapping Algorithm Based Modified ST

As discuss above, the co-prime coding-pitch should obtain the bigger unambiguity measurement range (UMR) and improve anti-noise of ST using only few shorter coding-pitch.

But in the co-prime coding-pitch must be odd, and the accuracy of wrapped coordinate under odd coding-pitch is lower than the even coding-pitch [14]. In addition to the odd or even number coding-pitch, the amount of pixels including in one pitch for triangular pattern should be multiple of three. In this case, the gray-scales of pattern is the integer, the error of coding is reduced to minimum *i.e.*, 0. In summary, the gcd of the coding-pitches is the multiple of six. The gray-scales is limited to 256 by 8 bit in coding pattern, taking into account minimizing the sensitivity of ambient noise, and the gcd of coding-pitches is selected as 6.

The off-the-shelf digital video projector usually has the projector image resolution 1024 $\times$768 pixels. The maximum pixels of projector mean to the biggest UMR. The absolute coordinate is selected as 1024. In this selection and taking into account the gcd of coding-pitches, the product of the remaining prime factor in coding-pitches dividing by gcd is more than 1024/6 *i.e.*, 170.667. The three root of this product is rounding to the nearest integer towards zero, and the minimum remaining prime factor is 5. The other two remaining prime factor is selecting co-prime larger than 5. Clearly these are 6 and 7. And the triangular pattern three coding-pitches are respectively 30, 36 and 42. The unwrapping formula as follow

$$n_0^p \equiv \left(\sum_{i=1}^{k} \hat{r}_i \; p_i p_i' \right) (\mathrm{mod}\ P) \equiv 6 \times \left[(\hat{r}_1 \times 3 \times 42 + \hat{r}_2 \times 4 \times 35 + \hat{r}_3 \times 5 \times 30) \,\mathrm{mod}(210) \right] \quad (11)$$

The average of difference between wrapped coordinate and the rounding integer is defined as the estimates of n_c . The estimates of n^p , *i.e.*, $\hat{n}^p$ can calculate with (9).

This unwrapping operation is performed for every camera pixel $\left(n^c,\ m^c\right)$, and the correspondence projector pixel $\hat{n}^p$ of these is acquired. The 3D world coordinates of the scanned object can, therefore, be derived through triangulation with the projector [15] where, under the assumption that the camera and projector are accurately modeled using the pinhole lens model such that their perspective matrices, C and P , are given by

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{pmatrix} = \begin{bmatrix} f_n^p & \tan\alpha^p \cdot f_n^p & n_0^p \\ 0 & f_m^p & m_0^p \\ 0 & 0 & 1 \end{bmatrix} \cdot \left(R^p, T^p\right) \qquad (12)$$

$$C = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \end{pmatrix} = \begin{bmatrix} f_n^c & \tan\alpha^c \cdot f_n^c & n_0^c \\ 0 & f_m^c & m_0^c \\ 0 & 0 & 1 \end{bmatrix} \cdot \left(R^c, T^c\right) \qquad (13)$$

where $\left(R^p, T^p\right)$ and $\left(R^c, T^c\right)$, named the extrinsic parameters of projector and camera respectively, are the rotation and translation which relates the 3D world coordinate system to the camera coordinate system and the projector coordinate system. $\left(n_0^p,\ m_0^p\right)$ and $\left(n_0^c,\ m_0^c\right)$ are respectively the coordinates of the principal point in the digital image coordinate system of projector and camera. f_n^c and f_m^c are the scale factors in camera image m and n axes. f_n^p and f_m^p are the same scale factors. α^p and α^c are the parameter describing the skew of the two image axes, respectively.

The 3D world coordinates X_w, Y_w and Z_w are calculated as

$$\begin{bmatrix} X^{\mathrm{w}} \\ Y^{\mathrm{w}} \\ Z^{\mathrm{w}} \end{bmatrix} = \begin{bmatrix} C_{11} - m^c C_{31} & C_{12} - m^c C_{12} & C_{13} - m^c C_{33} \\ C_{21} - n^c C_{31} & C_{22} - n^c C_{32} & C_{23} - n^c C_{33} \\ P_{11} - \hat{n}^p P_{31} & P_{12} - \hat{n}^p P_{32} & P_{13} - \hat{n}^p P_{33} \end{bmatrix}^{-1} \cdot \begin{bmatrix} m^c C_{34} - C_{14} \\ n^c C_{34} - C_{24} \\ \hat{n}^p P_{34} - P_{14} \end{bmatrix} \tag{14}$$

3. 3D Shape Measurement System and Experimental Results

In order to verify the performance of the presented method in this paper, we programmed the 3D shape measurement experimental system of Figure 4. The system is composed of a Hitachi F22 camera with Computar M3Z1228C-MP camera lens and an Acer H7531D digital video projector. Camera images have a resolution of 1360 × 1024 pixels. The baseline between the camera and the projector is approximately 400 mm. The angle between the optical axes of the camera and the projector is of about 30 degree. With the linear calibration method based on orthogonal three coding-pitches triangular patterns, the calibration results are as follows

$$C = \begin{pmatrix} -2.42 & -0.07 & 1.07 & -37.74 \\ 0.02 & -2.63 & -0.11 & 54.86 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix} \text{and } P = \begin{pmatrix} -2.68 & -0.04 & -0.07 & -125.02 \\ 0.05 & -2.73 & -0.55 & 118.16 \\ 0.00 & 0.00 & 0.00 & 1.00 \end{pmatrix}$$

Figure 5 shows the gray-scales response curve of this 3D shape measurement system. From this Figure, between 70 and 190, the linearity is well, and the minimum and maximum gray-scales of the triangular patterns are selected respectively.

To generating one wrapped coordinate map, using sinusoidal pattern phase-shifting, must be using arctangent processing, and needs to determine the results, translation and expand. The time consuming is expensive. But using triangular pattern is linearity, time-consuming, 0.3861s in experiments, is few. Comparing with sinusoidal pattern, it takes 3.3029s in the same processing. And the time efficiency in measurements using the presented method is well.

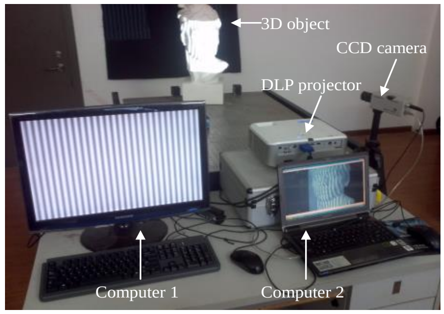

Figure 4. The Setup of 3D Shape Measurement System

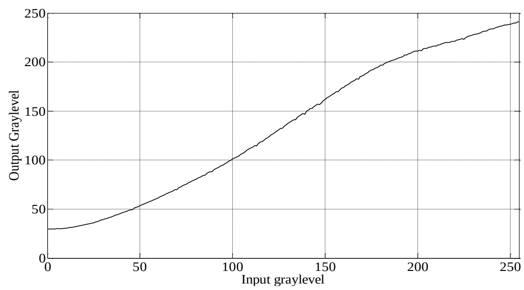

Figure 5. Measurement System Gray Level Response Function Curve

In order to evaluate the accuracy, we have reconstructed a white plane. The fringes image of the white plane, with 42 pixels in a coding-pitch, is shown in Figure 6. Ideally, all the acquired 3D points should lie on the plane, but in practice they do not. Hence, a common practice to evaluate accuracy in this case is to interpolate the plane from the entire 3D points and then to compute the errors to the interpolated plane. The error of reconstructed plane positioned as Figure 5 is shown in Figure 6. In Table 1 we provide the mean and Standard Deviations (SD) of difference position in the 3D world coordinate system calibration volume.

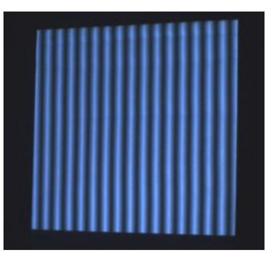

Figure 5. The White Plane Fringe Image with Plane Positioned 42 Pixels Coding-Pitch

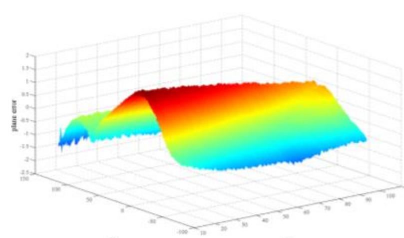

Figure 6. The Error of Reconstructed as Figure 5

As part of the qualitative evaluation of the presented method in this paper, a complex curve surface, as shown in Figure 7, was reconstructed shown as Figure 9. From Figure 9, noted that the reconstructed surface shows the vivid detail of 3D object. Uniform absolute coordinate in the projector digital image coordinate system is show in Figure 8.

Figure 7. Complex Curve Surfaces

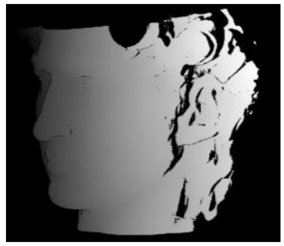

Figure 8. Uniform Absolute Coordinate

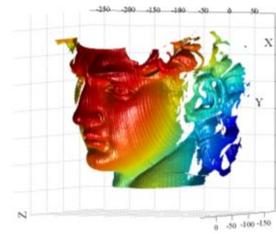

Figure 9. Reconstructed using the Presented Method Surface in this Paper

Table 1. The Measure Result of White Plane (Unit: mm)

3D world coordinate	-150.00	-100.00	-50.00	0.00	50.00	100.00	150.00
Mean of measure	-150.29	100.17	-50.09	0.04	50.06	100.10	150.15
SD of error	0.62	0.54	0.47	0.40	0.42	0.46	0.49

Another part of the qualitative evaluation of the presented method in this paper, many different material surfaces, such as textiles, colored paper and plastic surface as shown in Figure 10, were reconstructed and wrapped with their texture shown as Figure 11. From Figure 11, noted that the reflectivity of different material surface could not be affected the reconstructed result of using the presented method. And our method is not sensitivity to the reflectivity.

Figure 10. Difference Material Surfaces

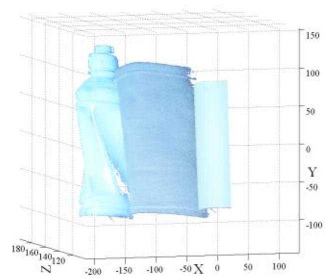

Figure 11. Reconstructed Surface using the Presented Method

4. Conclusion

A novel three coding-pitches triangular patterns phase-shifting profilometry based on modified Sunzi theorem is presented in this paper. The presented method is based on one of most high accuracy and high speed structured light pattern, *i.e.*, three wavelength triangular phase-shifting. In phase unwrapping, we have adopted a modification of Sunzi Theorem. The result of 3D shape measurement using programmed in this paper shown that there are some merits in the presented method, such as robust phase unwrapping, which is applicable to the 3D object surface of complex curve and difference material.

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