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# Carrier Phase Difference Positioning with Kalman Filter

Xiao-dong Su, Yu-ru Zhang, Hai-tao Jiang, Ming Zhao and Jun-ling Li

*School of Computer and Information Engineering*
*Harbin University of Commerce*
*hsdsxd@163.com*

***Abstract***

*Today Global Positioning System (GPS) is the most important system of positioning in the world and is used in different industries. Carrier phase has a substantial accuracy, but its main problem is that it is an indirect measurement which only computes the displacement. In order to realize precision positioning, this paper proposes an improved satellite positioning algorithm based on carrier phase difference and Kalman filter. The second-difference positioning mathematical model aiming at satellite navigation signal carrier phase on a single fiducial station is established. For the integer ambiguity influence of carrier phase, it presents the mathematical model to eliminate. The Kalman filter is constructed, so that it is more improvement of system stability. Then, it realizes positioning by computer simulation, and the results show that the accuracy of the algorithm is higher.*

***Keywords:*** *Positioning Precision, Carrier Phase, Kalman Filter, Integer Ambiguity*

## 1. Introduction

Once upon a time when there were no sign of today's technology, primitive men used constellations for their positioning and navigation. Then it was compass's turn until now when different global navigation systems like Global Positioning System (GPS) conduct the responsibility for navigation and positioning. In addition to GPS which is designed by the USA Department Of Defense (DOD), other navigation systems like the Russian GLONASS, the Chinese Beidou Satellite Navigation System, GALILEO, etc. are designed over the past years [1, 2].

GPS has many uses in different industries and its receivers are produced in various prices and accuracies. There are two types of single-frequency and dual-frequency receivers in which the former uses only one carrier band while the latter is able to use a second one, too.

At present the only way of positioning a kinematic receiver in real-time with centimeter level positioning accuracy is by differential mode that is a relative positioning of two receivers [3, 4]. To achieve a higher accuracy in positioning with GPS, many researches have been done on differential GPS and hybrid systems [5–8]. In some other research efforts too, it has been tried to combine GPS with other navigation systems to attain a higher accuracy [9–11].

Due to deviations in positioning data, it is mandatory to filter positioning data. Kalman Filter (KF) will be used for this purpose. KF was created by Kalman in 1960 [12]; a recursive based filter which estimates the state of a dynamic system from a sequence of incomplete and corrupted evaluations. In this filter according to measurements an accurate estimation from the system states is obtained. To make an accurate estimation from the distribution, KF uses only the mean and distribution

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covariance. While in general to compute a probability distribution, we need many distribution parameters. Even in case that there were such a probability the other problematic matter is the huge amount of data to be used which imposes a high load of computation to the estimation process.

KF, which is an important concept in control theory and control system engineering, is used in a wide range of engineering functions from radar to computer optic. This filter has a crucial role in the development of GPS. It is also used in noise modeling optimization [13].

Satellite positioning divides into carrier code observation, carrier phase observation and Doppler observation. In view of the requirements of low speed dynamic environment to satellite positioning precision in landslide monitoring and driver training, with the analysis of positioning error cause and the accuracy compare of different positioning technique, this paper proposes the fundamental positioning of carrier phase observation. By deal with the simple difference and second-difference, the influence of ephemeris error, satellite clock error and receiver clock error are erased. The integer ambiguity model which reduces double differencing carrier phase eliminates the phase integer ambiguity influence. It adopts the accurate value to amend positioning coordinate value, in combination with Kalman filter algorithm, and the positioning precision and stability is verified improvement.

Advantages of the proposed KF in our paper are that they are simple, low cost, and easy to design. They have the structure complexity less than for hardware implementation. They also require less memory than for software implementation.

## 2. Research on Modeling Carrier Phase in Second-Difference

The wave length of the satellite carrier signal in distance measurement is about 19cm, while the alignment error of the receiver carrier phase reaches 1% of the wave length, thus the positioning precision of phase measurement pseudo range is much larger than that of the positioning of code measurement. Distance variation can be observed in the changing carrier phase until the high-frequency modulation sinusoid signal transmitted from the navigational satellite is received.

Assume that the phase of the carrier signal launched by satellite $j$ at $t_j$ is $\varphi^j(t_j)$ and becomes $\varphi_k(t_k)$ when the signal reaches receiver $k$ at $t_k$, with the wave length $\lambda$, and if the error is neglected, besides carrier frequency is regarded as constant during transmission, then the distance from $j$ to $k$ is

$$\rho = \lambda[\phi_k(t_k) - \phi^j(t_j)] \quad (1)$$

Figure 1 shows the carrier phase measurement. $\rho = |AB| = c\tau_{AB} = \lambda[\varphi(t_B) - \varphi(t_A)] = \lambda(N + \Delta\varphi)$ , where $c$ is the propagation velocity of electromagnetic wave, $\lambda$ is carrier wavelength, $N$ is the period of carrier for $\rho$ , $\Delta\varphi$ is the phase which is less than one period.

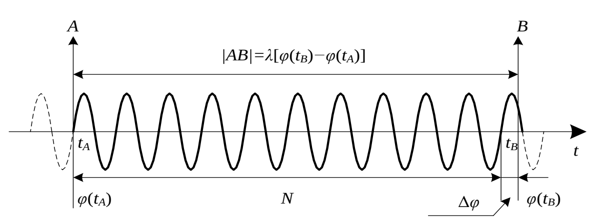


**Figure 1. Carrier Phase Measurement**

Since an almost synchronous carrier signal with respect to the receipt signal can be produced by the receiver, measurement of the phase difference between these two signals enables distance acquisition. But not as the phase $\Delta\varphi$ of less than one period that is readily available, the satellite carrier signal $\varphi^j(t_j)$ is hard to be measured directly, that's the reason why the integral carrier period $N$ is introduced.

If satellite $j$ is continuously observed by receiver $k$, then there will be correlation between the carrier phases via repeated measurement with the unknown integer $N$. Figure 2 shows that at $t_i$, moment, $k$ starts to measure its range from $j$,

$$\rho_i = \lambda[N_k^j(t_i) + \Delta\varphi_k^j(t_i)] \tag{2}$$

Where, $\Delta\varphi_k^j(t_i)$ is the phase less than one period obtained by the receiver at $t_i$ and $N_k^j(t_i)$ is the initial integer ambiguity.

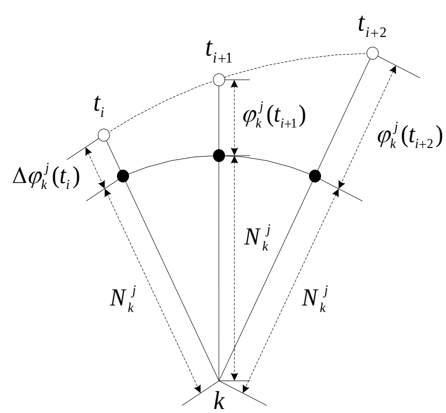


**Figure 2. Continuously Observed Carrier Phase**

If the satellite is tracked on end at $t_i$ with counting the carrier integer cycle, and the corresponding value reaches $\Delta N_k^j(t_{i+1})$ at next moment $t_{i+1}$, then the distance from the satellite to the receiver can be described as

$$\rho_{i+1} = \lambda[N_k^j(t_i) + \Delta N_k^j(t_{i+1}) + \Delta\varphi_k^j(t_{i+1})] \tag{3}$$

Where apparently, the unknown whole integer $N_k^j(t_i)$ measured at first time is contained, while the integral and decimal parts of the carrier wave obtained directly by the receiver is denoted as $\Delta N_k^j(t_{i+1})$ and $\Delta\varphi_k^j(t_{i+1})$ respectively, satisfying

$$\varphi_k^j(t_{i+1}) = \Delta N_k^j(t_{i+1}) + \Delta\varphi_k^j(t_{i+1}) \tag{4}$$

Which serves as the observed value of carrier phase at $t_{i+1}$? When the carrier phase measurement is carried out n times via the receiver, the pseudo range from the satellite to the receiver can be written as

$$\left.\begin{array}{l} \rho_{i+1} = \lambda[N_k^j(t_i) + \varphi_k^j(t_{i+1})] \\ \rho_{i+2} = \lambda[N_k^j(t_i) + \varphi_k^j(t_{i+2})] \\ \quad\quad\ldots\ldots \\ \rho_{i+n} = \lambda[N_k^j(t_i) + \varphi_k^j(t_{i+n})] \end{array}\right\} \tag{5}$$

Function (5) implies that when the first observation is achieved, $N_k^j(t_i)$ consists in the subsequent continuous measures for the phase. If the receiver has no counting missing during the measurement, then all the phase observation contain $N_k^j(t_i)$, simply noted as $N_k^j$, and for the receiver, $N_k^j$ is an unknown quantity.

### 2.1. Research on Modeling the Carrier Phase in Single Difference Positioning

In Figure 2, the base station is set as *B* and the subscriber station is set *U*, both observing satellite *j*. The observation noise is neglected and the phase measure pseudo range of these two stations from *j* is given by

$$\left.\begin{aligned} \rho_b^j &= R_b^j + c(d\tau_b - d\tau^j) + d\rho_b^j - d\rho_{b,ion}^j + d\rho_{b,trop}^j \\ \rho_u^j &= R_u^j + c(d\tau_u - d\tau^j) + d\rho_u^j - d\rho_{u,ion}^j + d\rho_{u,trop}^j \end{aligned}\right\} \tag{6}$$

The subscript "*b*" denotes base station, "*u*" subscriber station; $\rho_*^j$ is the phase measure pseudo range of the subscriber station *, $\rho_*^j = \lambda N_*^j + \lambda\varphi_*^j$; $R_*^j$ represents the real range from * to *j*; $d\tau_*$ and $d\tau^j$ are the clock skews of *'s and *j*'s respectively; $d\rho_*^j$ is pseudo range error aroused by *j*'s ephemeris error; $d\rho_{*,ion}^j$ is *'s ionosphere lead, besides $d\rho_{*,trop}^j$ is *'s troposphere delay.

The distance from base station to subscriber station is less than the height of satellite, providing that the distance is below 1000km, thus it's assume that $d\rho_b^j \approx d\rho_u^j$, $d\rho_{b,ion}^j \approx d\rho_{u,ion}^j$, $d\rho_{b,trop}^j \approx d\rho_{u,trop}^j$. Function (6) can be deduced as

$$\lambda(\varphi_b^j - \varphi_u^j) = (R_b^j - R_u^j) + c(d\tau_b - d\tau_u) - \lambda(N_b^j - N_u^j) \tag{7}$$

Where the terms of single difference is specified as $\Delta(*)_{ub}^j = (*)_b^j - (*)_u^j$, then function (7) can be further deduced as

$$\lambda\Delta\varphi_{ub}^j = \Delta R_{ub}^j + c\Delta d\tau_{ub} - \lambda\Delta N_{ub}^j \tag{8}$$

Where $\Delta N_{ub}^j$ is the single difference ambiguity of carrier phase.

The linearization of function (8) gives rise to function (9), in which $\boldsymbol{I}_u^j$ is the direction vector from * to *j*, $\boldsymbol{I}_u^j = \left[\frac{X^j - \hat{x}_u}{\hat{\rho}_u^j}, \frac{Y^j - \hat{y}_u}{\hat{\rho}_u^j}, \frac{Z^j - \hat{z}_u}{\hat{\rho}_u^j}\right]$, $\Delta\boldsymbol{x}_u$ is the correction to the *'s general coordinates, and the real range from * to *j* is $R_b^j = |\boldsymbol{X}^j - \boldsymbol{x}_b|$, with the corresponding approximation $\hat{\rho}_u^j$ described as

$$\lambda\Delta\varphi_{ub}^j = \boldsymbol{I}_u^j\Delta x_u + \hat{\rho}_u^j - R_b^j + c\Delta d\tau_{ub} - \lambda\Delta N_{ub}^j \tag{9}$$

$$\hat{\rho}_u^j - \left|\boldsymbol{X}^j - \hat{\boldsymbol{x}}_u\right| = \sqrt{(X^j - \hat{x}_u)^2 + (Y^j - \hat{y}_u)^2 + (Z^j - \hat{z}_u)^2} \tag{10}$$

In function (10), $\boldsymbol{X}^j$ and $\boldsymbol{x}_u$ are respectively the position vectors of j and U, with regard to ECEF, namely $\boldsymbol{X}^j = [X^j, Y^j, Z^j]^T$, $\boldsymbol{x}_u = [x_u, y_u, z_u]^T$. The error equation is formulated as

$$\Delta v_{ub}^j = \boldsymbol{I}_u^j\Delta\boldsymbol{x}_u + c\Delta d\tau_{ub} - L_{ub}^j \tag{11}$$

 

Where $\Delta v_{ub}^{j}$ is the phase observation error of single difference, and $L_{ub}^{j} = \lambda(\Delta N_{ub}^{j} + \Delta\phi_{ub}^{j}) + (R_{b}^{j} - \hat{\rho}_{u}^{j})$ is constant.

Function (11) is the representation of the mathematical model of the observation error of carrier phase in single difference positioning. This model can eliminate the effects of satellite's clock skew and ephemeris error, besides evidently decrease the interference made by the refraction error of troposphere and thermosphere, but fails in vanishing receiver's clock skew.

## 2.2. Research on Modeling the Carrier Phase in Double Difference Positioning

On the basis of single-difference model, double-difference location introduces a trail difference between satellites, as shown in Figure 3 and Figure 4. It selects the satellite *h* which has the most altitude angle as reference satellite, and proposes a difference between the observation equations pertaining to other satellites and reference satellite.

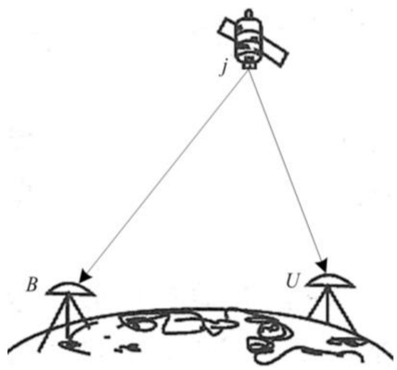


**Figure 3. Carrier Phase Single Difference**

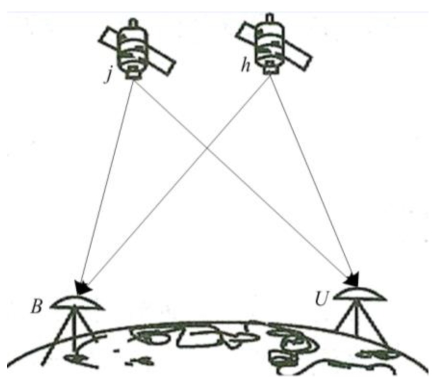


**Figure 4. Carrier Phase Double Difference**

One of the most exacting tasks required of Naval pilots is the landing of high performance airplanes aboard an aircraft carrier. This operation demands precise control and any gross excursion from the glideslope will require the pilot to execute a wave off manipulation.

The linearized single-difference observation equation of reference satellite *h*:

$$\lambda\Delta\varphi_{ub}^{h} = \boldsymbol{I}_{u}^{h}\Delta\boldsymbol{x}_{u} + \hat{\rho}_{u}^{h} - R_{b}^{h} + c\Delta d\tau_{ub} - \lambda\Delta N_{ub}^{h} \quad (12)$$

Introduce difference between (9) and (12):

$$\lambda(\Delta\varphi_{ub}^{j} - \Delta\varphi_{ub}^{h}) = (\boldsymbol{I}_u^j - \boldsymbol{I}_u^h)\Delta\boldsymbol{x}_u + (\hat{\rho}_u^j - \hat{\rho}_u^h) - (R_b^j - R_b^h) - \lambda(\Delta N_{ub}^j - \Delta N_{ub}^h) \tag{13}$$

Where $\nabla\Delta(*)_{ub}^{hj} = \Delta(*)_{ub}^{j} - \Delta(*)_{ub}^{h}$. Simplify the function (13):

$$\lambda\nabla\Delta\varphi_{ub}^{hj} = (\boldsymbol{I}_u^j - \boldsymbol{I}_u^h)\Delta\boldsymbol{x}_u + (\hat{\rho}_u^j - \hat{\rho}_u^h) - (R_b^j - R_b^h) - \lambda\nabla\Delta N_{ub}^{hj} \tag{14}$$

The coordinate corrective value should be calculated by equations set via function (14). For $\nabla\Delta v_{ub}^{hj}$ is double-difference phase observation error:

$$\nabla\Delta v_{ub}^{hj} = (\boldsymbol{I}_u^j - \boldsymbol{I}_u^h)\Delta\boldsymbol{x}_u - L_{ub}^{hj} \tag{15}$$

Where $\nabla\Delta\varphi_{ub}^{hj}$ is double-difference phase observation value, $\nabla\Delta N_{ub}^{hj}$ is double-difference phase integer ambiguity (known assumption), $L_{ub}^{hj} = \lambda(\nabla\Delta N_{ub}^{hj} + \nabla\Delta\varphi_{ub}^{hj}) - (\hat{\rho}_u^j - \hat{\rho}_u^h) + (R_b^j - R_b^h)$ is constant term of double-difference respectively.

If two station observe $n$ satellites simultaneously, and 3 unknown location coordinate corrective value of subscriber station as unknown number, the matrix is

$$\boldsymbol{V} = \boldsymbol{AX} - \boldsymbol{L} \tag{16}$$

Where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_u^2 - \boldsymbol{I}_u^1 \\ \boldsymbol{I}_u^3 - \boldsymbol{I}_u^1 \\ \vdots \\ \boldsymbol{I}_u^n - \boldsymbol{I}_u^1 \end{bmatrix}, \quad \boldsymbol{L} = [L_{ub}^{12}, L_{ub}^{13}, \cdots, L_{ub}^{1n}]^{\mathrm{T}}, \quad \boldsymbol{X} = \Delta\boldsymbol{x}_u, \quad \boldsymbol{V} = [\nabla\Delta v_{ub}^{12}, \nabla\Delta v_{ub}^{13}, \cdots, \nabla\Delta v_{ub}^{1n}]^{\mathrm{T}}$$

Double-difference positioning model relieves the receiver clock error of subscriber station and improves positioning precision. However, it should be approached to phase integer ambiguity for accomplishing carrier phase double-difference positioning.

### 2.3. Research on Modeling the Carrier Phase Integer Ambiguity Elimination

At present, receiver can't provide double-difference phase integer ambiguity. For the above issues, $\nabla\Delta N_{ub}^{hj}$ regards as unknown parameter, function (14) is simplified:

$$\nabla\Delta v_{ub}^{hj} = (\boldsymbol{I}_u^j - \boldsymbol{I}_u^h)\Delta\boldsymbol{x}_u - \lambda\nabla\Delta N_{ub}^{hj} - L_{ub}^{hj} \tag{17}$$

Where $L_{ub}^{hj} = \lambda\nabla\Delta\varphi_{ub}^{hj} - (\hat{\rho}_u^j - \hat{\rho}_u^h) + (R_b^j - R_b^h)$ is the constant term.

When $n$ satellites are observed simultaneously, $\nabla\Delta\boldsymbol{N} = [\nabla\Delta N_{ub}^{12}, \nabla\Delta N_{ub}^{13}, \cdots, \nabla\Delta N_{ub}^{1n}]^{\mathrm{T}}$ is defined as ambiguity vector, the matrix is:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_u^2 - \boldsymbol{I}_u^1 & -\lambda & 0 & \cdots & 0 \\ \boldsymbol{I}_u^3 - \boldsymbol{I}_u^1 & 0 & -\lambda & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ \boldsymbol{I}_u^n - \boldsymbol{I}_u^1 & 0 & 0 & \cdots & -\lambda \end{bmatrix}, \quad \boldsymbol{L} = [L_{ub}^{12}, L_{ub}^{13}, \cdots, LN_{ub}^{1n}]^{\mathrm{T}}, \quad \boldsymbol{X} = \begin{bmatrix} \Delta\boldsymbol{x}_u \\ \nabla\Delta\boldsymbol{N} \end{bmatrix}^{\mathrm{T}}$$

The equation can't be solved because of $n$-1 observation equations have $n$+2 unknown parameters, included 3 coordinate corrective values and $n$-1 double-difference phase integer ambiguity. However, there will be $m$ ($n$-1) observation equations if receiver observed $m$ epoch continuously.

## 3. Research on Modeling Kalman Filter

Multipath effect error is another main error source in satellite navigation system. Multipath signal of superposition proposes received signal fading and cycle slip, even signal lock-lose. Multipath error is composed of constant part and periodic part which can be dispelled for extending observation time. However, there is no longer observation for moving object dynamic prisioning. For the above issues, Kalman filter is proposed fro error correction.

Kalman filter are composed of State equation and observation equation:

$$\left.\begin{aligned} \boldsymbol{x}_k &= \boldsymbol{\Phi}_{k,k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{\omega}_{k-1} \\ \boldsymbol{z}_k &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \qquad k \geq 1 \end{aligned}\right\} \tag{18}$$

Where, $\boldsymbol{x}_k$ is the predictive value of coordinate corrective value at $k$ time, namely ( $\nabla\Delta x_{ub}^{kj}, \nabla\Delta y_{ub}^{kj}, \nabla\Delta z_{ub}^{kj}$ ) is predictive state. $\boldsymbol{z}_k$ is the coordinate corrective value at $k$ time, namely ( $\Delta\hat{x}_u, \Delta\hat{y}_u, \Delta\hat{z}_u$ ) is estimated value. Dynamic noise $\boldsymbol{\omega}_k$ and measurement noise $\boldsymbol{v}_k$ are zero-mean white noise sequence by separate, for all $k$ and $j$, the basic statistical characteristics of model are:

$$\left.\begin{aligned} \mathrm{E}[\boldsymbol{\omega}_k] = \boldsymbol{0}, &\quad \mathrm{Cov}\ (\boldsymbol{\omega}_k, \boldsymbol{\omega}_j) = \boldsymbol{Q}_k\delta_{kj} \\ \mathrm{E}[\boldsymbol{v}_k] = \boldsymbol{0}, &\quad \mathrm{Cov}\ (\boldsymbol{v}_k, \boldsymbol{v}_j) = \boldsymbol{R}_k\delta_{kj} \\ &\quad \mathrm{Cov}\ (\boldsymbol{\omega}_k, \boldsymbol{v}_j) = \boldsymbol{0} \end{aligned}\right\} \tag{19}$$

$$\delta_{kj} = \begin{cases} 1, k = j \\ 0, k \neq j \end{cases}$$

Suppose the statistical characteristics of initial state are:

$$\mathrm{E}[\boldsymbol{x}_0] = \mu_0\ ,\ \mathrm{Var}(\ \boldsymbol{x}_0) = \mathrm{E}[(\boldsymbol{x}_0 - \boldsymbol{\mu}_0)(\boldsymbol{x}_0 - \boldsymbol{\mu}_0)^{\mathrm{T}}] = \boldsymbol{P}_0 \tag{20}$$

And

$$\mathrm{Cov}\ (\boldsymbol{x}_0, \boldsymbol{\omega}_k) = \boldsymbol{0}\ ,\ \mathrm{Cov}\ (\boldsymbol{x}_0, \boldsymbol{v}_k) = \boldsymbol{0}$$

$\hat{\boldsymbol{x}}_k$ Which launched by Kalman filter recurrence formula is the unbiased estimation, namely $\mathrm{E}[\hat{\boldsymbol{x}}_k] = \mathrm{E}[\boldsymbol{x}_k]$. Error variance matrix is beneficial to guarantee prisitioning unbias and robustness.

A case study of observing 5 satellites continuously between epoch of 2 o'clock and 2 o'clock zero minute 30 seconds, there are 8 observation equations as $\lambda\nabla\Delta\varphi_{ub}^{hj} = (\boldsymbol{I}_u^j - \boldsymbol{I}_u^h)\Delta\boldsymbol{x}_u + (\hat{\rho}_u^j - \hat{\rho}_u^h) - (R_b^j - R_b^h) - \lambda\nabla\Delta N_{ub}^{hj}$ , 3 coordinate corrective values and 4 double differencing phase integer ambiguities as unknown values. And so forth, multi-sets coordinate corrective values ( $\Delta x_{ux}$ , $\Delta x_{uy}$ , $\Delta x_{uz}$ ) are proposed by observation equations. The relationship between coordinate corrective values and observation point outline coordinates, coordinates is fllowing: $\Delta x_{ux} = \hat{x}_u - x_u$ , $\Delta x_{uy} = \hat{y}_u - y_u$ , $\Delta x_{uz} = \hat{z}_u - z_u$ . The location coordinate of subscriber station is amended by solving equations above. The fusion of carrier phase double differencing positioning model and Kalman filter equation is developed by $\Delta x_{ux}$ , $\Delta x_{uy}$ , $\Delta x_{uz}$ introducing function (18).

For (18), $\boldsymbol{x}_k = (x_1, x_2, \cdots, x_n)^{\mathrm{T}}$ is $n$×1 dimensional vector, where $n$ is number of observation satellites, respectively, transfer matrix $\boldsymbol{\Phi}_k = \begin{pmatrix} a_1 & & & \\ a_2 & & \boldsymbol{I}_{(n-1)} & \\ \vdots & & & \\ a_n & 0 & \dots & 0 \end{pmatrix}$ is $n$×$n$ dimensional vector. $a_1$, $a_2$, …, $a_n$ are the weighted coefficient. For $i$=1，2，⋯，$n$; $a_i$=1. $\boldsymbol{I}_{(n-1)}$ is ($n$-1) unit vector, $\boldsymbol{\Gamma}_k = (a_1, a_2, \cdots, a_n)^{\mathrm{T}}$ is $n$×1 dimensional control matrix, respectively, $\boldsymbol{H}_k = (1, 0, \cdots, 0)$ is 1×$n$ observation matrix.

Introduce filter recurrence:

$$\hat{\varphi}_k = \hat{\varphi}_{k/k} = \hat{\varphi}_{k/k-1} + \boldsymbol{K}(\boldsymbol{\Omega}_k - \boldsymbol{H}_k \hat{\varphi}_{k/k-1}) \tag{21}$$

The option of undetermined corrected gain matrix must be make the error variance matrix minimum. The iterative formula of error equation is $\boldsymbol{P}_k = \boldsymbol{P}(\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k)\boldsymbol{P}_{k/k-1}$, where $\boldsymbol{P}_k \geqslant 0$, and symmetric. Corrected gain matrix $\boldsymbol{K}_k$ is solved by taking the derivative and extremum of $\boldsymbol{P}_k$ .

The accuracy location of subscriber station at $k$ time:

$$\boldsymbol{x}_u(k) = \hat{\boldsymbol{x}}_u(k) - \Delta \boldsymbol{x}_u \tag{22}$$

Where, $\hat{x}_u(k)$ =[ $\hat{x}_u(k), \hat{y}_u(k), \hat{z}_u(k)$ ]is the outline coordinate of observation station which calculated by station distance and satellite coordinate, respectively, $\Delta \boldsymbol{x}_u = [\Delta x_{ux}, \Delta x_{uy}, \Delta x_{uz}]$ is the coordinate corrective value of carrier phase double-difference and Kalman filter.

## 4. Positioning Experiment and Analysis

To solve the three-dimensional coordinate and receiver clock error, we need to observe 4 satellites at least, namely $j \geqslant 4$. What is more, to develop carrier phase double-difference positioning as shown in Figure 5, it selects the satellite $h$ which has the most altitude angle as reference satellite. It chooses the adverse simple-base station difference to verify how adaptable the positioning experiment can be.

The geographic coordinate of base station B as known condition takes a participation of difference positioning. For $i$=1,2,…,$n$, $U_i$ is the subscriber station receive satellite navigation signal and carrier phase information of base station B. Positioning results are real-time proposed by establishing carrier phase difference observation model.

WGS-84 coordinate system is used in experiment, and data is obtained from NOAA (National Oceanic and Atmospheric Administration) http://www.ngs.noaa.gov/CORS. Base station $B$ is the ID: bjfs0000 observation station in Fangshan, and location coordinate is (2148743.0840, 4426640.2014, 4044655.3498). Subscriber station $U$ is the ID: chan01 observation station in Changchun, and location coordinate which calculated as unknown value is (-2674431.9143, 3757145.2969, 4391528.8732).

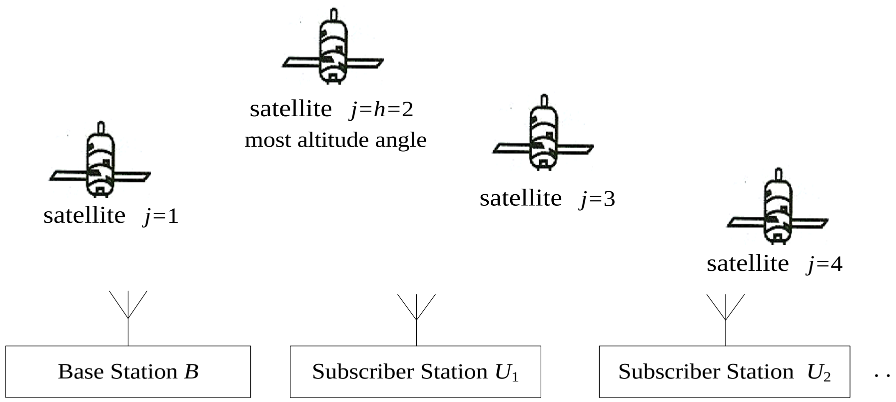


**Figure 5. Carrier Phase Double-Difference Positioning**

Positioning precision is introduced via 2 times distance root mean square error (2DRMS). The liner distance of $U$ and $B$ is following as:

$$\left.\begin{aligned}(\overline{\sigma}_{ubx})^{\frac{1}{2}} &= \left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|\rho_{bx}-\hat{\rho}_{uxi}\right|-\overline{\rho}_{ubx}\right)^2\right)^{\frac{1}{2}}\\(\overline{\sigma}_{uby})^{\frac{1}{2}} &= \left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|\rho_{by}-\hat{\rho}_{uyi}\right|-\overline{\rho}_{uby}\right)^2\right)^{\frac{1}{2}}\\(\overline{\sigma}_{ubz})^{\frac{1}{2}} &= \left(\frac{1}{n}\sum_{i=1}^{n}\left(\left|\rho_{bz}-\hat{\rho}_{uzi}\right|-\overline{\rho}_{ubz}\right)^2\right)^{\frac{1}{2}}\end{aligned}\right\} \tag{23}$$

Rohde & Schwarz GNSS simulator is used to generate raw GPS data such as pseudo range, integrated carrier phase, Doppler shift and satellite ephemeris. The GNSS simulator in the R&S_SMBV100A includes the ability to simulate realistic transmission conditions through the use of multipath signal generation and modeling of various atmospheric effects. The motion of an aircraft containing GNSS receiver, simulator is used to model effects that impact GNSS receiver performance, such as atmospheric conditions, multipath reflections, antenna characteristics and interference signals.

Table 1 is the accuracy of experiment and Figure 6 is the 2DRMS longitude and distance accuracy. Results show that positioning precision is the centimeter level, with better effect. With observation number $n$ increasing, mean accuracy will also improve, however, a little extend. In consequence, selecting suitable $n$ is a good choice.

**Table 1. Positioning Precision**

| $n$=20 | | | $n$=100 | | | $n$=500 | | |
|---|---|---|---|---|---|---|---|---|
| $(\overline{\sigma}_{ubx})^{\frac{1}{2}}$ | $(\overline{\sigma}_{uby})^{\frac{1}{2}}$ | $(\overline{\sigma}_{ubz})^{\frac{1}{2}}$ | $(\overline{\sigma}_{ubx})^{\frac{1}{2}}$ | $(\overline{\sigma}_{uby})^{\frac{1}{2}}$ | $(\overline{\sigma}_{ubz})^{\frac{1}{2}}$ | $(\overline{\sigma}_{ubx})^{\frac{1}{2}}$ | $(\overline{\sigma}_{uby})^{\frac{1}{2}}$ | $(\overline{\sigma}_{ubz})^{\frac{1}{2}}$ |
| $1.91\times10^{-5}$ | $2.06\times10^{-5}$ | $1.78\times10^{-5}$ | $1.78\times10^{-5}$ | $2.00\times10^{-5}$ | $1.68\times10^{-5}$ | $1.74\times10^{-5}$ | $1.93\times10^{-5}$ | $1.66\times10^{-5}$ |
| longitude error 0.076m<br>latitude error 0.082m<br>height error 0.071m<br>mean accuracy 0.077m | | | longitude error 0.071m<br>latitude error 0.080m<br>height error 0.067m<br>mean accuracy 0.073m | | | longitude error 0.069m<br>latitude error 0.077m<br>height error 0.066m<br>mean accuracy 0.071m | | |

 

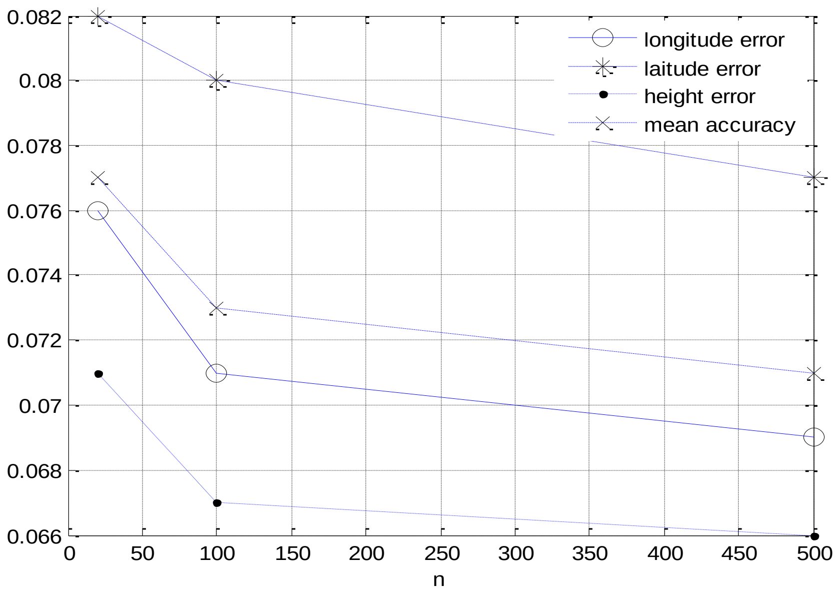


**Figure 6. 2DRMS Longitude and Distance Accuracy**

## 5. Conclusion

This paper proposes an improved satellite positioning algorithm based on carrier phase difference and Kalman filter. The second-difference positioning model aiming at satellite navigation signal carrier phase on a single fiducial station is established. Simulation results show that the improved accuracy of the algorithm is higher.

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## References

[1] M. R. Mosavi, M. Soltani, I. Azad and J. EmamGholipour, "Position Estimation in Single-Frequency GPS Receivers Using Kalman Filter with Pseudo-Range and Carrier Phase Measurements", vol. 72, **(2013)**, pp. 2563–2576.

[2] S. Purivigraipong and S. Hodgart, "Resolving Integer Ambiguity of GPS Carrier Phase Difference", IEEE Transactions of Aerospace and Electronic Systems, vol. 46, no. 2, **(2010)** April.

[3] J. M. Codol and A. Monin, "Improved Triple Difference GPS Carrier Phase for RTK-GPS Positioning", IEEE Statistical Signal Processing Workshop (SSP), **(2011)**.

[4] L. Zhiqiang, W. Jiexian and W. Hu, "An Approach for Kinematic Precise Point Positioning Based on Precise Velocity Estimation", Journal of Astronautics, vol. 33, no. 3, **(2012)**.

[5] W. Qian, "Based on Carrier Phase Differential GPS/DR Integrated Positioning Algorithm", Computer Knowledge and Technology, vol. 9, no. 5, **(2013)**.

[6] W. Wanqing, "Algorithms Research of Integrated GPS、DR Land Vehicle Navigation System Based on Difference of Carrier Phase", Institute of Geodesy and Geophysics Chinese Academy of Sciences, vol. 6, **(2004)**.

[7] M. R. Mosavi, "Wavelet neural network for corrections prediction in single-frequency GPS users", Neural Processing Letters, vol. 33, no. 2, **(2011)**, pp. 137–150.

[8] Z. Wen, P. Henkel and C. Gunther, "Reliable estimation of phase biases of GPS satellites with a local reference network", IEEE conference on ELMAR, **(2011)**, pp. 321–324.
[9] J. Yu and X. J. Chen, "Application of extended Kalman filter in ultra-tight GPS/INS integration based on GPS software receiver", IEEE conference on vehicular green circuits and systems, **(2010)**, pp. 82–86.
[10] P. Defraigne, A. Harmegnies and G. C. Petit, "Time and frequency transfer combining GLONASS and GPS data", IEEE joint conference on frequency control and the European frequency and time forum, **(2011)**, pp. 1–5.
[11] X. Zhao, Y. Qian, M. Zhang, J. Niu and Y. Kou, "An improved adaptiveKalman filtering algorithm for advanced robot navigation system based on GPS/INS", IEEE conference on mechatronics and automation, **(2011),** pp. 1039–1044.
[12] M. S. Grewal and A. P. Andrews, "Applications of Kalman filtering in aerospace 1960 to the present", IEEE Magazine on Control Systems, vol. 30, no. 3, **(2010)**, pp. 69–78.
[13] D. M. Kim and J. Suk, "Optimal tracking filter considering both correlated/white measurement noise", IEEE conference on control, automation and systems, **(2011)**, pp. 1424–1428.

## Authors

**Xiao-dong Su**, he is a professor of Harbin University of Commerce and he is the member of council of the China Software Industry Association, vice president of Electronics Experiment Teaching Research Council of Universities in Heilongjiang, member of Heilongjiang Computer Society Education and Training Professional Committee. His recent research interests are in information detection and estimation, wireless communication theory and technology, management information systems.

**Yu-ru Zhang**, she is a professor of School of Computer and Information Engineering, Harbin University of Commerce. Her recent research interests are in intelligent information processing and fault detection.

**Hai-tao Jiang**, he is an associate professor of Harbin University of Commerce. He received a M.E. degree in Software Engineering from Beijing University of Posts and Telecommunications, Beijing, China, 2010. His recent research interests are in intelligent control, digital signal processing and embedded technology.

 