International Journal of Signal Processing, Image Processing and Pattern Recognition
Vol.7, No.2 (2014), pp.51-60
http://dx.doi.org/10.14257/ijsip.2014.7.2.05

An Image Fusion Algorithm for ECT based on Tikhonov Algorithm and Wavelet Transform

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Abstract

Electrical capacitance tomography (ECT) image reconstruction is a typical ill-posed problem. Tikhonov regularization algorithm is one of the popular methods to solve ill-posed problem. Because functional for the standard Tikhonov regularization algorithm is over-smoothness. The reconstruction image could miss edge information and the quality is not ideal. This paper proposes an image fusion algorithm based on Tikhonov regularization algorithm and wavelet transform, which can correct the edge of the reconstruction image. The simulation proves that it can enhance edges and improve the quality of the reconstruction image.

Keywords: *ECT system; image reconstruction; standard Tikhonov regularization algorithm; wavelet transform*

1. Introduction

Electrical capacitance tomography (ECT) technology is widely used in the measurement of multiphase flow, it has many advantages such as high speed and low cost. So ECT has a good prospect in the measurement of multiphase flow. There is difficulty to reconstruct image for ECT system because of the soft field characteristic of the ECT's capacitive sensor and the equation of ECT's forward problem is a typical ill-posed problem. Tikhonov regularization algorithm can efficiently solve the ill-posed problems, which is widely used in image reconstruction of ECT in recent years. It was initiated by Andrey Nikolayevich Tikhonov, a Russian mathematician. Its basic principle is to constrain ill-conditioned equations, and to get approximate solution from acceptable solutions. The essence of Tikhonov regularization algorithm is to alleviate the severity of the ill-posed equation by changing condition number of the matrix. The regularization parameters choice is the key for Tikhonov regularization algorithm. The usual practice is to use iterative method. But the iterative method has two main shortcomings: firstly, the iterative method increases the computation in imaging process. Secondly, the edge of reconstructed image is excessive smoothing and image details are lost.

In this paper, to overcome the shortcomings of iterative method, a new processing method is proposed. The new method is wavelet fusion algorithm which based on Tikhonov regularization algorithm. The simulation results show that this method can preserve the edges of image effectively, improve the quality of the reconstruction image and achieve the purpose which is to enhance the display effect of the reconstructed image.

ISSN: 2005-4254 IJSIP

2. ECT Imaging Principle Based on Tikhonov Regularization Algorithm

The ECT system is composed of three parts: capacitive sensor, data acquisition system and imaging computer. The measured capacitance value will change with the change of the distribution of medium in the pipeline. So we can obtain media distribution according to the actual value of the measured capacitance[1].The ECT system model of direct problems with matrix is as follows[2].

$$C = SG \tag{1}$$

Among formula (1), C is normalized capacitance vector, S is normalized sensitivity matrix and G is permittivity. Permittivity represents the image gray value in the image reconstruction.

The inverse problem of ECT system is to reconstruct the distribution of permittivity in medium by measuring data (capacitances). The permittivity values are mapped to the image gray level values, and inverse problem is transformed into solving the gray value of pixels in imaging area. Because the number of pixels is greater than the number of measurement data, the formula (1) belongs to the morbid equation. Regularization algorithm can be used to solve this formula. Tikhonov regularization algorithm is a typical representation [3]. The main idea of this algorithm is to transform formula (1) into the problem of solving the minimum value, and to solve the objective function (2).

$$\min J(G) = \left\| SG - C \right\|^2 + a \left\| G \right\|^2 \tag{2}$$

We can get the solution of the standard Tikhonov regularization algorithm by minimizing the equation (3)[4].

$$G = (S^T S + aI)^{-1} S^T C \tag{3}$$

The key issue of Tikhonov regularization algorithm is to choose the regularization parameters. From the approximation theory perspective, too large regularization parameters bring large deviation between auxiliary problems and the original problem. From the numerical stability perspective, too small regularization parameters cannot reduce the ill-posed problem. Generally, the choice of the regularization parameter is determined according to the experience [5].

However, the edges of images which are reconstructed by Tikhonov regularization algorithm are irregular. Aiming at this question, this paper introduces wavelet decomposition algorithm which is used to fusion images and can improve the imaging quality of the ECT system.

3. Wavelet Fusion Algorithm Based on Tikhonov Regularization Algorithm

In the image reconstruction, the image reconstructed by Tikhonov regularization algorithm has better quality in its center. For multiple targets, the shape of the reconstructed image is greatly improved, but the quality of its boundary need to be further improved [6]. Our algorithm firstly decomposes the images which are reconstructed by Tikhonov regularization algorithm using wavelet, and then fusion the processed images. The algorithm step is as follows:

1. Decompose the original image by wavelet. In this step we will get a series of sub images.

2. Choose features in the transform domain and then create fuse images.
3. Fuse the images again with inverse transform.

3.1. Decompose Image with Wavelet

The image is different from the one-dimensional signal, and any point (x, y) has a gray value of image counterpart. When image's coordinate continuously changes, it can be defined as a continuous change of two-dimensional signal $f(x, y)$. In order to analyze the details and the edge of the image reconstructed by Tikhonov regularization algorithm, we need to carry on multi-resolution decomposition processing to the image. If the two-dimensional space is separable, it can be decomposed into the tensor product of two one-dimensional spaces: $V_j(x_1)$ and $V_j(x_2)$.Support that $\varphi(x)$ is a scale function, and $\psi(x)$ is a wavelet function. Two-dimensional multi-resolution decomposition can be divided into two steps. First of all, the image is analyzed by $\varphi(x_1)$ and $\psi(x_1)$ along the x_1 direction and $f(x_1, x_2)$ is decomposed into two parts: smooth approximation and the detail section. And then the two parts are analyzed by $\varphi(x_1)$ and $\psi(x_1)$ along the x_2 direction respectively. So, we get four routes output. The output which was processed by $\varphi(x_1)$ and $\varphi(x_2)$ is $A_1 f(x_1, x_2)$, which is the first level of smoothing approximation of $f(x_1, x_2)$. The other outputs are $D_1^{(1)} f(x_1, x_2)$, $D_1^{(2)} f(x_1, x_2)$ and $D_1^{(3)} f(x_1, x_2)$ which are used as detail functions. The formula is summarized as follows when we are doing first order analysis (j=1).

$$\begin{cases} A_1 f(x_1, x_2) = \left\langle f(x_1, x_2), \varphi_{1k_1}(x_1)\varphi_{1k_2}(x_2) \right\rangle \\ D_1^{(1)} f(x_1, x_2) = \left\langle f(x_1, x_2), \varphi_{1k_1}(x_1)\psi_{1k_2}(x_2) \right\rangle \\ D_1^{(2)} f(x_1, x_2) = \left\langle f(x_1, x_2), \psi_{1k_1}(x_1)\varphi_{1k_2}(x_2) \right\rangle \\ D_1^{(3)} f(x_1, x_2) = \left\langle f(x_1, x_2), \psi_{1k_1}(x_1)\psi_{1k_2}(x_2) \right\rangle \end{cases} \tag{4}$$

The analysis process is shown in Figure 1.

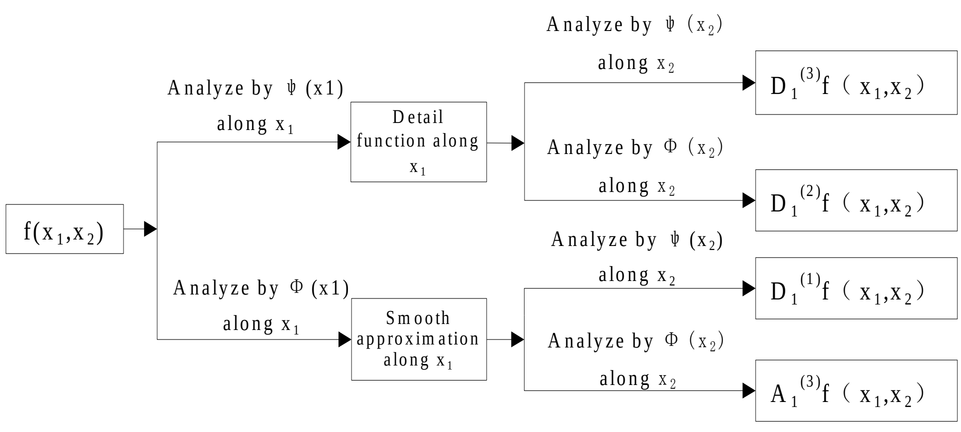


Figure 1. The Wavelet Decomposition of Image

The formula is summarized as follows when we are doing j order analysis.

$$\begin{cases} A_j f(x_1,x_2) = \left\langle f(x_1,x_2), \varphi_{jk_1}(x_1)\varphi_{jk_2}(x_2) \right\rangle \\ D_j^{(1)} f(x_1,x_2) = \left\langle f(x_1,x_2), \varphi_{jk_1}(x_1)\psi_{jk_2}(x_2) \right\rangle \\ D_j^{(2)} f(x_1,x_2) = \left\langle f(x_1,x_2), \psi_{jk_1}(x_1)\varphi_{jk_2}(x_2) \right\rangle \\ D_j^{(3)} f(x_1,x_2) = \left\langle f(x_1,x_2), \psi_{jk_1}(x_1)\psi_{jk_2}(x_2) \right\rangle \end{cases} \tag{5}$$

As we see from equation 5, while doing j order analysis, we decompose the image into four sub-images as $A_j f(x_1,x_2)$, $D_j^{(1)} f(x_1,x_2)$ and $D_j^{(2)} f(x_1,x_2)$, $D_j^{(3)} f(x_1,x_2)$. $A_j f(x_1,x_2)$ represents smooth approximation(low frequency part) of the original image. $D_j^{(k)} f(x_1,x_2)$ represents detail image(high frequency part) of the original image. $D_j^{(1)} f(x_1,x_2)$ represents the high frequency component in the vertical direction of the original image. $D_j^{(2)} f(x_1,x_2)$ represents the high frequency component in the horizontal direction of the original image. $D_j^{(3)} f(x_1,x_2)$ represents the high frequency component in the diagonal direction of the original image. If we decompose $A_j f(x_1,x_2)$ repeatedly, we can get n order decomposition. The decomposition (n=3) is shown in Figure 2.

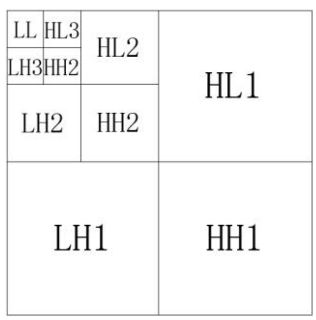


Figure 2. The Decomposition (n=3) of Image

As we can see from the Figure 3, at each level of decomposition, the image is decomposed into smooth approximation (LL), vertical component (LH), horizontal component (HL) and diagonal component (HH). We can get decomposition for the next layer by decomposing smooth approximation (LL)[7].

When we decompose the image which is reconstructed by Tikhonov regularization algorithm, less decomposition level brings fewer details so that the good effects could not be achieved. While, more decomposition level brings more computation power so that it costs more time to decompose image. This paper will solve and verify the best decomposition level of the image.

3.2. ECT Image Fusion Method Based on Wavelet Decomposition

After having decomposed the image by wavelet, we select the characteristics on transform domain, and then create fused images. The original image is decomposed into high frequency component and low frequency component by Wavelet decomposition. Low frequency component represents approximate characteristics of the original image, and expresses the central area of the image reconstructed by Tikhonov regularization algorithm. High frequency component represents the great changes point and expresses the edge of the image reconstructed by Tikhonov regularization algorithm. To solve the problem that the edge of image reconstructed is not clear, we can choose appropriate method to fusion the high frequency component to enhance the quality of image edge effectively.

Low frequency component represents the general trend of the image, and expresses the central area of the reconstructed image. We use the pixel gray value selection algorithm to fuse low frequency region. The pixel gray value selection algorithm includes maximum and minimum pixel gray value selection algorithm. Suppose that we use $P_n(x,y)$ (n=1, 2, 3) to represent the original images. The size of the image is M*N pixel, and $R(x,y)$ represents the fused image. The maximum pixel gray value selection algorithm can be expressed as follows.

$$R(x_i,y_j)=\max(P_1(x_i,x_j),P_2(x_i,y_j),P_3(x_i,y_j)) \quad i=1,2,\cdots,M;\ j=1,2,\cdots,N \tag{6}$$

The minimum pixel gray value selection algorithm can be expressed as follows[8].

$$R(x_i,y_j)=\min(P_1(x_i,x_j),P_2(x_i,y_j),P_3(x_i,y_j)) \quad i=1,2,\cdots,M;\ j=1,2,\cdots,N \tag{7}$$

The central area of ECT's flow pattern is universally solid. If we choose those points which have great gray value, the quality of the reconstruction image can be improved. So we use the maximum pixel gray value selection algorithm to fusion the low frequency component so that it can strengthen the display effect of central area. The high frequency component represents the edge of image whose brightness is large variable. For the algorithm to fusion the edge of the image, we need to calculate the mean of edge of three images thus it can reduce error. So we choose the average gray value algorithm. It can be expressed as follows.

$$R(x_i,y_j)=(P_1(x_i,x_j)+P_2(x_i,y_j)+P_3(x_i,y_j))/3 \quad i=1,2,\cdots,M;\ j=1,2,\cdots,N \tag{8}$$

We use Mean Square Error (MSE) to evaluate the image quality while the number of images changes. The curve of MSE is shown in Figure 3.

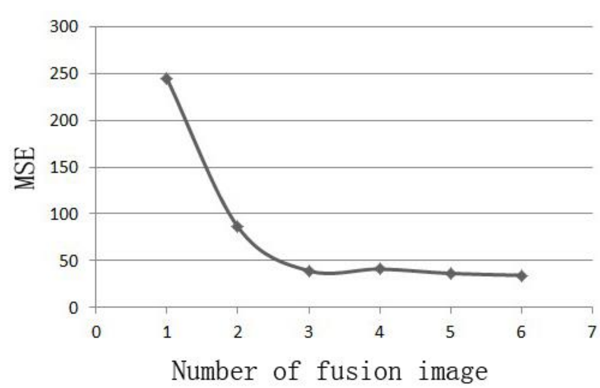


Figure 3. Relationship Curve of Fusion Image Number and Mean Square Error

From the Figure 4, it can be seen that the mean square error tends to be stable from the 3th fusion image. Taking the typical center flow as an example, the fused images are shown in Figure 4.

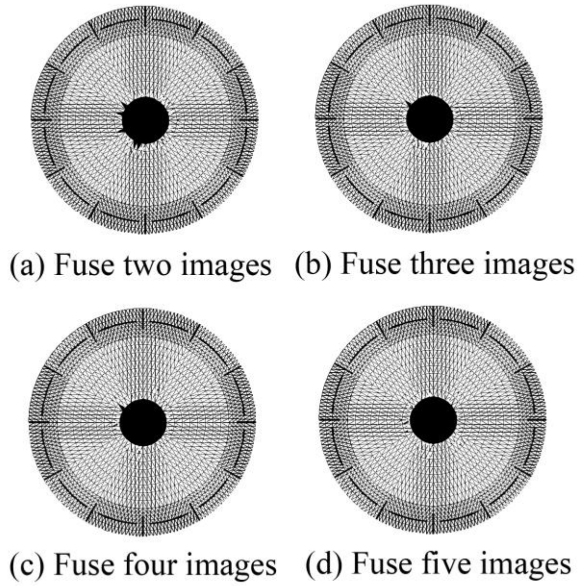


Figure 4. Fusion Effect for Different Number of Images

The time for fusing different number of images is shown in Figure 5.

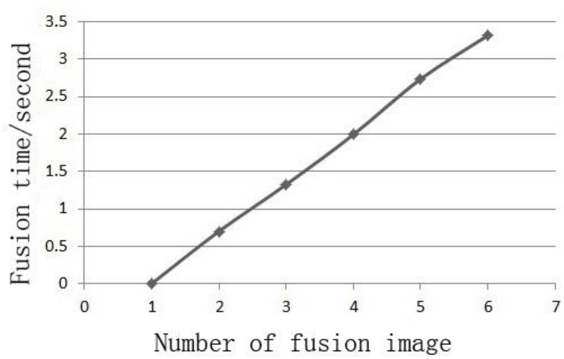


Figure 5. The Time Cost to Fuse Different Number of Images

We can find that the fusion time is close to linear. Too many images can't bring image resolution enhancement but bring more calculation. The image is decomposed into 3N high frequency components and 1 low frequency component when we are doing N layer decomposition to ECT image reconstructed by Tikhonov regularization algorithm. The steps of the image fusion are shown as follows.

1. Decompose the three images with selected wavelet function and then construct wavelet pyramid image.
2. Fuse the high frequency component and low frequency component by using fusion method in the formula, and get fused wavelet pyramid image.
3. Process inverse transform on the fused image and get the final image.

The fusion process is shown in Figure 6.

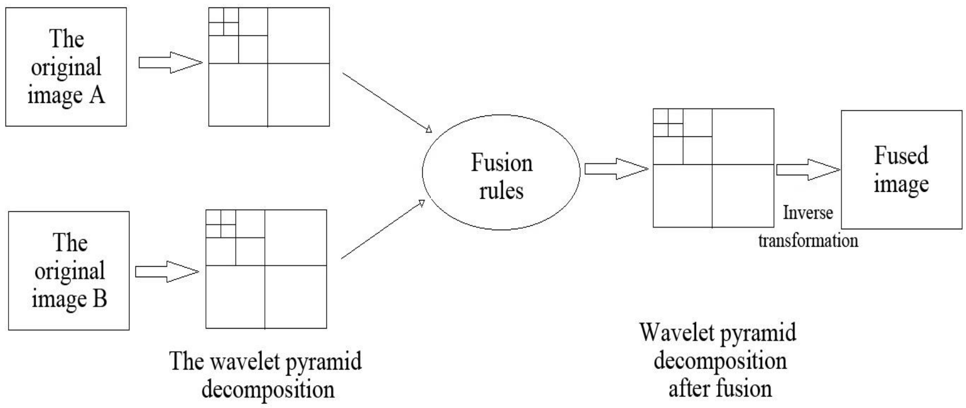


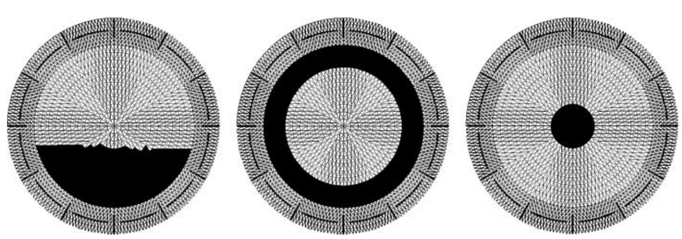
Figure 6. Image Fusion Process

4. Data Simulation and Result Analysis

In order to verify the validity of the results, we carried out simulation for the three most common flow patterns of ECT (circular flow, central flow and laminar flow) with the MATLAB 7 environment.

Calculate the 66 measured capacitances of each flow pattern and determine the parameters of the Tikhonov regularization algorithm by using the L- curve method. And then do wavelet decomposition and image fusion for three random selected images from 20 groups of images which we reconstructed for each flow pattern. Coiflet (Coif 3) wavelet function is used as the wavelet function and then use three-level wavelet decomposition. Average pixel gray value selection algorithm is used to fuse the low frequency components and the maximum pixel gray value selection algorithm is used to fuse the high frequency components.

The simulation was carried out with 12-electrode ECT system. The image reconstructed by wavelet fusion algorithm based on Tikhonov regularization algorithm was compared with that reconstructed by standard Tikhonov regularization algorithm. Results of the simulation for common flow such as circular flow, central flow and laminar flow are shown respectively in Figure 7, 8, 9. The white parts are the distribution of low dielectric constant whose permittivity is 1. The black parts are the distribution of high dielectric constant whose permittivity is 3.5.



(a) Laminar Flow (b) Circular Flow (c) Central Flow

Figure 7. The Original Image of The Three Typical Flow Regimes

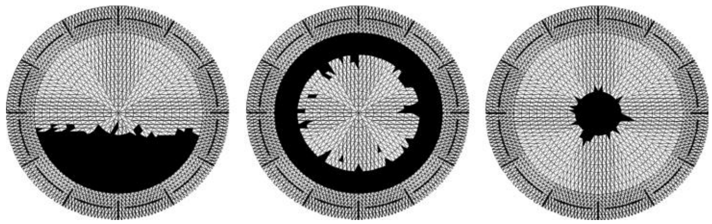


Figure 8. The Image Reconstructed by Standard Tikhonov Regularization Algorithm

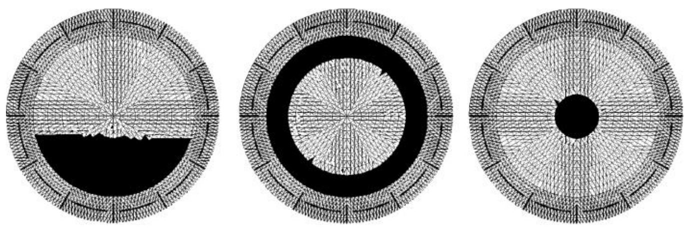


Figure 9. The Image Reconstructed by Wavelet Fusion Algorithm

The result shows that the algorithm in this paper has better imaging results than the standard Tikhonov regularization algorithm. In order to evaluate the imaging quality accurately, we evaluate the image using the following 4 parameters: mean-square error (MSE), normalized mean square error (NMSE), signal-to-noise ratio (SNR) and signal-to-noise ratio (PSNR)[9].

The values of four kinds of evaluation criteria are calculated by Matlab as shown in Table 1, Table 2 and Table 3.

Table 1. The Comparison Result of Laminar Flow

Reconstruction algorithm	MSE	NMSE	SNR	PSNR
Standard Tikhonov regularization algorithm	312.5253	0.0096	14.5390	23.1820
Our algorithm	35.1517	0.0011	24.0284	32.6713

Table 2. The Comparison Result of Circular Flow

Reconstruction algorithm	MSE	NMSE	SNR	PSNR
Standard Tikhonov regularization algorithm	705.8905	0.0236	11.3570	19.6434
Our algorithm	80.1342	0.0027	24.8062	32.6713

Table 3. The Comparison Result of Central Flow

Reconstruction algorithm	MSE	NMSE	SNR	PSNR
Standard Tikhonov regularization algorithm	244.4148	0.0065	14.2021	24.2495
Our algorithm	39.1265	0.0010	22.1587	32.2061

From these data, high quality ECT images were obtained by the wavelet fusion algorithm based on Tikhonov regularization algorithm. The mean-square error and the normalized mean square error descended 6 times, meanwhile signal-to-noise ratio increased 0.67 times and peak signal-to-noise ratio increased 0.3 times. Consequently, the wavelet fusion algorithm based on Tikhonov regularization algorithm is more efficient in ECT image reconstruction.

5. The Conclusion

This paper discusses applying Tikhonov regularization algorithm and wavelet fusion to the ECT image reconstruction. The simulation results are given to demonstrate the availability of the method. The images reconstructed by the method in this paper are evaluated. Compared with the standard Tikhonov regularization, this method can effectively optimize blurred edge of the reconstruction image and improve the image quality of ECT system. But how to apply wavelet transform to remove image noise while fusing images should be further studied, the study in the future will mainly focus on it.

Acknowledgements

This paper was supported by Scientific Research Fund of Heilongjiang Provincial Education Department under NO:12531107.

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