

Simulation Method of Random Ocean Waves Based on Fractal Interpolation

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Abstract

Distinguished with traditional computer realistic graphics, qualities of real time and fidelity are regarded as significant indexes of weighing system validity for real-time simulation of ocean wave simultaneously. Based on ocean waves' fractal character and the linear superposition method of Linear Ocean Wave Theory, the paper puts forward the simulation algorithm of ocean surfaces and realizes the real-time simulation of 3D ocean waves. In the course of programming, the character of real time is researched and corresponding simplified means are presented. The approaches and key techniques of ocean wave real-time simulation put forward in the paper better satisfy the requirements of realistic rendering.

Keywords: wave simulation, fractal interpolation, midpoint displacement, real time, opengl

1. Introduction

In recent years, simulations of natural sceneries as ocean waves, waterfalls, rain, snow, smoke and flames have been increasingly becoming one of the most challenging research directions in computer graphics. Their simulations are more and more widely applied to a variety of areas as computer game, movie and television, advertising and visual simulation. With their irregularity and non-repeatability in time or space, ocean waves are more complicated and difficult to be simulated compared with other natural environments.

Currently, simulation studies on ocean waves are broadly grouped into 4 categories [1]: geometric model-based approach [2], physical model-based approach [3, 4], statistical model-based approach [5, 6] and texture-based approach[7]. The geometric model is represented by the approach put forward and adopted by Perchy, which simulates waves' geometrical shapes through the linear superposition of sine function and quadratic function. This type of approach is quite simple and it is easy to control the wave forms. However, the wave forms are too regular, have obvious artificial traces and therefore are often used in games or cartoons; the physical model is based on Navie-Stokes (Equation N-S)-the basic equation of fluid physics. Fluctuations in sea surface are dominated by Equation N-S; so many researchers have adopted Equation N-S to create the sea surface model and tried to obtain the sophisticated ocean wave simulation by approximately solving this equation. But, it has an enormous amount of calculation and the real-time is not easy to be guaranteed. The statistical model is to simulate ocean waves by superposing a series of sine and cosine waves and then combining the physical model, with the superposed value representing the wave amplitude.

Many statistical models adopt Fast Fourier Transformation Algorithm to compute the superposed value. These data models are extremely complex and have large amount of computation, so they are not suitable for large-scale simulation of ocean surfaces; the texture-based approach is not a real movement, so it has less sense of reality and is unsuitable for a closer observation.

Due to the disadvantages of the above approaches, based on ocean waves' fractal features and by way of multi-directional linear superposition, the paper presents the way to obtain ocean wave elevation data, based on the data, establish curved ocean wave surfaces and achieve the real-time and dynamic display of three-dimensional ocean waves.

The rest of this paper is organized as follows, Section 2 introduces Midpoint Displacement method based on fractal theory, Section 3 analyzes the mathematical model of ocean waves, Section 4 proposes simulation algorithm of ocean fractal surfaces, Section 5 presents several approaches and key techniques of ocean waves real-time and dynamic display, Section 6 gives flow chart of simulating ocean wave and effect figure, Section 7 presents the conclusion.

2. Fractal Theory and Midpoint Displacement Method

The ocean surface is a complicated random rough one. It is made by superposing small-scale ripples on large-scale and approximately periodic waves. That is to say, it is related to the large scale and similar to the small scale. Moreover, its directions in space are of approximate isotropy. The superposable ocean surface presents a fractal feature.

Based on the ocean surface fractal characteristics, the paper has relation to the mathematical model, Fractal Brownian Motion [8]. Fractal Brownian Motion provides a method of interpolating values among known data points and is very effective to the generation of convergence among known ocean surface data points. Brownian Motion can be generated through a variety of methods, among which Random Midpoint Displacement [9] is the popular way.

Midpoint displacement is use of the process to simulate the ocean wave that interpolation within two or more points. This method has the capacity of high-speed and increasing the details of the shape. Its interative formula is:

$$P_{mid} == \frac{P_i + P_{i+1}}{2} + \Delta \times Rand() \quad (1)$$

Where $\Delta_n = \left(\frac{1}{2}\right)^{nH} \sigma$. P_i and P_{i+1} respectively represents a segment of the two endpoints. Δ determines the roughness of the resulting fractal. It is to control the disturbance of the current level. $Rand()$ is a random function. n represents the number of iterations. H is a floating-point number in the range 0.0 to 1.0. It represents the complicated of ocean wave. With H increasing, fractal result is smooth. With H falling, fractal result is rough. σ responses the regional fractal characteristics of the ups and downs. As long as we have taken a sea area characteristic parameters H and σ , we can use Midpoint Displacement simulate the ocean wave that has the multi-resolution and more details and levels.

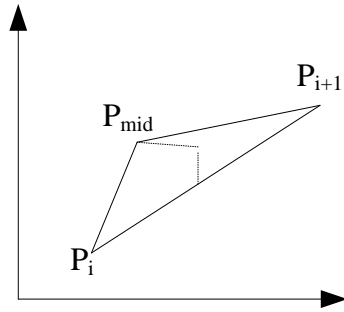


Figure 1. Disturbed Partition on Middle of Line

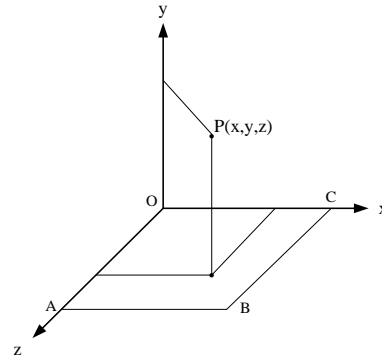


Figure 2. Three-dimensional Mid-point Displacement Principle

According to different surface net-constructing approaches, the often-used approaches that the two-dimensional random Midpoint Displacement algorithm simulates include triangle edge subdivision, diamond-square subdivision and so on. There among, the triangle edge subdivision approach [10] means evenly dividing each edge of a triangle, linking up each mid-point to transform one triangle into 4 small triangles and then adding one Gaussian random offset to each mid-point. Each triangle is subdivided at different time and with unpredictable sequence and there is no information transfer among triangles, so this approach is irrelative to the sequence of subdivision, which results in the so-called “crease problem”. That is to say, unnatural linear trajectories are left on the waterscape surface and can not be removed by local smoothing. To eliminate “the crease phenomenon”, we use the diamond-square subdivision approach [11, 12] to establish the fractal curved ocean wave surface.

By utilizing the diamond-square subdivision approach to generate a curved ocean wave surface with Fractal Brownian Motion feature, we can use the idea of mid-point displacement migration to generate the point’s third coordinate on a two-dimensional basal surface, thereby completing the expected ocean wave modeling process. In the three-dimensional space O-XYZ, As Figure 2, if we choose the square OABC on surface XOZ as a basal surface and suppose spatial point P (x, y, z) to be horizontally projected within the basal surface, then its height coordinate Y can be generated with the mid-point displacement approach.

Suppose the heights of the four angular points of the square basal surface OABC are known and their values are respectively Y_A, Y_B, Y_C, Y_O , As Figure 3, then the height value Y_M at the center point M of the square can be determined through the mean value of the four angular points’ height values plus a random amount. Then we have:

$$Y_M = (Y_A + Y_B + Y_C + Y_O)/4 + \alpha$$

Where, Y_M is the height value at the center point of the square basal surface and α is the random offset at the center point.

Draw two opposite-sided parallel lines DE and GH , then we can divide the square into smaller squares. To determine the height values at the four angular points of each small square, we also need to acquire the height values Y_D, Y_E, Y_G, Y_H at the mid-points of the square’s four sides on the basal surface. These height values on the basal surface

can be acquired by obtaining the even value from the known height value that is nearest to it. That is:

$$Y_D = (Y_O + Y_A)/2 + \alpha$$

$$Y_G = (Y_O + Y_C)/2 + \alpha$$

$$Y_E = (Y_C + Y_B)/2 + \alpha$$

$$Y_H = (Y_A + Y_B)/2 + \alpha$$

The α is the corresponding random offset but the four α can not same. But they are in a same range.

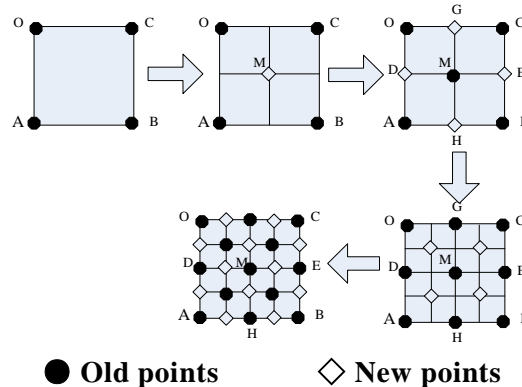


Figure 3. Twice Diamond-Square Iterative Sketch Map

After the above process, the original square's basal surface is divided into four smaller squares and the height values at the four angular points of each square are also known. That is to say, based on the 4 originally known height values and through calculation, there are 5 new height values. Thus, we not only make detailed analysis on the basal surface, but also increase the density of three-dimensional points.

Repeat the above process on each small square, we can obtain smaller square grids and meantime generate more three-dimensional elements with heights. In this way, after many iterations, when the density of three-dimensional elements reaches the requirement, we can acquire the geometric object's data model with Fractal Brownian Motion features.

3. Linear Mathematical Model of Ocean Surface Waves

The ocean wave research theory indicates, in the linear wave theory[13], the ocean wave can be regarded as a stationary and random process, which may be superposed by numbers of (which should be infinite in theory) cosine waves with different cycles and different random initial phases:

$$\eta(x,t) = \sum_{n=1}^{\infty} a_n \cos(k_n x - \omega_n t + \varepsilon_n) \quad (2)$$

Where, $\eta(x,t)$ is the instantaneous height of the fluctuating water surface at the fixed point x , which is relative to the still water surface at the moment t . a_n is the vibration amplitude of composition wave No. n . x , t are respectively the position and time, ε_n is

the initial phase of composition wave No. n and here it is taken as the random variable in $(0, 2\pi)$. ω_n, k_n are respectively the wave frequency in radians per second, and wave number of composition wave No. n :

$$\omega_n = \frac{2\pi}{T_n}, \quad k_n = \frac{2\pi}{L_n} \quad (3)$$

Where, T_n, L_n are respectively the wave period and wave length of composition wave No. n . The wave period T_n is the time it takes two successive wave crests or troughs to pass a fixed point. The wave length L_n is the distance between two successive wave crests or troughs at a fixed time.

Wave frequency ω is related to wave number k by the dispersion relation :

$$\omega^2 = gk \operatorname{th}(kd) \quad (4)$$

where d is the water depth and g is the acceleration of gravity.

Two approximations are especially useful: one, deep-water approximation is valid if the water depth d is much greater than the wave length L . In this case, $d \gg L$, $kd \gg 1$, and $\operatorname{th}(kd) = 1$. two, shallow-water approximation is valid if the water depth is much less than a wavelength. In this case, $d \ll L$, $kd \ll 1$, and $\operatorname{th}(kd) = kd$.

Ocean wave spectrum $S(\omega)$ is described as:

$$S(\omega) = \frac{1}{\Delta\omega} \sum_{\omega}^{\omega+\Delta\omega} \frac{1}{2} a_n^2 \quad (5)$$

Where $S(\omega)$ represents the average energy within the frequency interval $\Delta\omega$. If we set $\Delta\omega=1$, The equation (5) is representative of the energy within the unit frequency, namely, energy density.

According to (2), we can only describe the wave of the fixed point that can change over time. The real sea is three-dimensional. Its energy is distributed not only within a certain frequency range but also within a very wide range of directions.

So, ocean wave may be superposed by numbers of cosine waves, and the direction of propagation is perpendicular to the wave crest and toward the positive x direction:

$$\eta(x, y, t) = \sum_{n=1}^{\infty} a_n \cos(k_n x \cos \theta_n + k_n y \sin \theta_n - \omega_n t + \varepsilon_n) \quad (6)$$

Where $a_n, \omega_n, \varepsilon_n$ respectively figures amplitude of the wave, frequency, initial phase. θ_n refers to an angle crossed by x, y horizontal plane and x -axis ($-\pi \leq \theta_n \leq \pi$). k_n refers to the wave number.

In order to generate multiple-directional wavelets in the plane surface and superpose them to form ocean waves, the energy of ocean wave spectrum $S(\omega)$ is expanded in all directions, thereby resulting in the forming of the ocean wave's directional spectrum $S(\omega, \theta)$. Density function of directional spectrum $S(\omega, \theta)$ is defined as follows[14]-[15]:

$$\sum_{\Delta\omega} \sum_{\Delta\theta} \frac{1}{2} a_n^2 = S(\omega, \theta) d\omega d\theta \quad (7)$$

By front equations, we can obtain a_n :

$$a_n = \sqrt{2S(\omega_n, \theta_n) \Delta\omega \Delta\theta} \quad (8)$$

Thus, (6) and (8) can be combined to write:

$$\eta(x, y, t) = \sum \sum \sqrt{2S(\omega_n, \theta_m) \Delta\omega_n \Delta\theta_m} \times \cos(k_n x \cos \theta_m + k_n y \sin \theta_m - \omega_n t + \varepsilon_{nm}) \quad (9)$$

Where, $S(\omega, \theta)$ is the directional spectrum and can generally be written as the following form:

$$S(\omega, \theta) = S(\omega)G(\omega, \theta) \quad (10)$$

Where, $S(\omega)$ is the ocean wave's frequency spectrum and $G(\omega, \theta)$ is the ocean wave's direction distribution function:

$$\int_{\theta_{\min}}^{\theta_{\max}} G(\omega, \theta) d\theta = 1 \quad (11)$$

Where, θ_{\min} , θ_{\max} refers to the ocean wave's direction distribution range. $\varepsilon_{nm} \in 2\pi[0,1]$ refers to the random numbers distributing evenly in $[0,2\pi]$. While simulating ocean waves, the selection of frequency range $[\omega_L, \omega_H]$ depends on the energy that is permitted to be respectively omitted at each side of the spectrum's high and low frequency. The frequency intervals can be divided with equal frequency method or equal energy method (each frequency band has the same energy). The representative of every composition wave should be randomly selected within every interval to avoid that waves repeatedly emerge with $2\pi/\Delta\omega$ as a cycle. The division of directions generally adopts the bisection method, namely:

$$\Delta\theta = \frac{\theta_{\max} - \theta_{\min}}{N} \quad \theta_n = \frac{\theta_n + \theta_{n+m}}{2} \quad (12)$$

According to (9), we can successfully simulate the ocean wave's data and better satisfy the requirement of simulating the ocean wave three-dimensionally and dynamically. However, we have also noticed, while using the multi-directional spectrum superposition method to calculate the data of three-dimensional ocean waves, the equations are comparatively complex and call for comparatively larger amount of calculation, failing to satisfy the requirement of real-time rendering. For this reason, we can simplify (9) to a certain degree without considering the directional spectrum. That is to say, θ is fixed. In this way, the amount of calculation can be greatly reduced. By default, we can set $\theta=0$.

4. Simulation Algorithm of Fractal Ocean Surfaces

Based on ocean waves' fractal character and Linear Ocean Wave Theory, we can obtain small amount of data by the linear superposition method. Rely on the data obtained in a real-time manner, we can generate a curved ocean wave surface by use of the fractal interpolation. That is a highly realistic multi-resolution simulated ocean wave.

To simulate the random three-dimensional ocean surface, according to the diamond-square subdivision approach of the Random Midpoint displacement method, we put forward the following algorithm. In this algorithm, we introduce a two-dimensional array of height values, with the dimension being the n th power of 2 plus 1 and map the index (x, z) to the height y . Then the array only need store the height value y .

The basic process of this algorithm is as follows: After initializing the data, according to the input array dimension n , generate different arrays and assign initial heights to the 4 angles, namely, $array[0, 0]$, $array[0, n-1]$, $array[n-1, 0]$ and $array[n-1, n-1]$, then carry out the following process:

Step one: Generate a random amount, form squares with all the adjacent 4 points with assigned values and assign values to the central elements of the squares, with the value being the even value of the square's 4 vertex elements plus the random amount; Step two: Assign values to the mid-point elements of each square's each edge, with the assigned value being the even value of the element's adjacent array element whose value has been assigned plus the random amount; Step three: Let the random amount multiplied by 2^{-H} , $0 < H < 1$. Repeat the above three steps until all the array elements are assigned values.

In the algorithm, H can determine the fractal result's roughness. If necessary, H can be reduced to make the surface rougher. The array values can be regarded as the height values, with which we can draw the ocean surface. Verified by the rear ocean surface output effect figure, the ocean surface generated with the above algorithm can effectively eliminate the crease problem.

The elevation data obtained with this algorithm has a certain degree of smoothness in the overall trend, but partially it still has the problems that elevation data's variation range is too wide and the ocean surface's fluctuations appear too big. To achieve better visual effect, it still needs certain smoothing. In order to make the elevation data partially smooth, the paper adopts the average approach. It supposes one elevation point $array[i, j]$, adds it with the 8 elevation points around it according to a certain percentage and substitutes the elevation value of this point with the newly obtained value, thereby avoiding the fierce changes of partial elevation data. Secondly, the randomness of the elevation data generated with the fractal interpolation algorithm enables the data to distribute both in the plus and minus ranges, traverse the whole elevation data and order the elevation data less than a certain height value (usually choosing zero) to be zero. Thus, the three-dimensional ocean surface has some flat range and meets the real situation of the ocean surface. Lastly, the paper traverses and normalizes the whole elevation data.

5. Technology of Real-time and Dynamic Display

In such applications as simulation or virtual reality, in order to intuitively observe the ocean surface and related simulation process, we often need display the dynamic and real-time ocean surface. However, the ocean surface has a large amount of data and the hardware environment is usually a PC, so the three-dimensional and dynamic display of the ocean surface is the focal and difficult point. We have to maximize the "lifelikeness" of the simulation process on the premise of meeting the simulation result's accuracy. Through comparative studies, some of the following approaches are to be adopted to improve the display performance of ocean waves.

5.1. Separate Generation and Display of Ocean Wave Data

Generate ocean wave data first and then save the ocean wave data that meet the requirement as backup ocean wave data files. In the course of three-dimensional display and

simulation, open the needed ocean wave data file and directly invoke the ocean wave data from the memory to display, which can separate the time-consuming ocean wave data generation process from the three-dimensional display process to improve the display speed and reuse each ocean wave datum, thereby improving efficiency.

5.2. View-dependent LOD Model

LOD model is one of the commonly used approaches to improve the graphic running speed. It is easy to construct a view-dependent LOD model if we adopt the fractal approach to model ocean waves. The specific approach is as follows: First, construct a relatively coarse grid, then as required use comparatively more interpolation iterations in the regions closer to the view point to refine the grid and use comparatively less interpolation iterations in the regions farther from the view point to reduce the amount of calculation.

5.3. Display List

To speed up graphic rendering, OpenGL provides an interface for a set of display list [16]. The display list is a group of stored OpenGL commands, which are carried out in order when the display list is cited. Having been compiled, these commands have high performance efficiency and can effectively improve OpenGL's graphic rendering performance.

Virtually, the display list is the high-speed cache for a series of commands instead of a dynamic database in the memory, so it neither needs memory management nor occupies memory resources, thereby greatly improving the graphic rendering performance. Before being displayed, the curved surface grids of ocean wave simulation are all pre-stored in the display list.

5.4. Texture Mapping Technique

Texture mapping technique is one of the techniques to enhance the graphic realism in OPENGL [16, 17] and is a technique to vividly express the object surface's details by mapping 2D figures into 3D objects. When the dynamic ocean surface is being simulated, it is impossible for any of the present imaging system to directly simulate the movement of each water particle in the ocean surface. The most commonly used approach is the texture mapping technique, with which we take the images of some ocean state in the real world as the texture source and stick them to each grid of the data field. In this way, we can not only effectively enhance the simulation effect's sense of reality, but also greatly reduce the amount of calculation and improve the simulation's real-time effect.

5.5. Use of Double-buffer Technique

Executing a drawing command in the double-buffed window is actually to construct a scene in the one-dimensional picture and then quickly switch to the window view. To use the double-buffer technique in OPENGL, first, use function SetPixelFormat() to set the window pixel format and after completing drawing, use function SwapBuffers() to exchange buffers, exchanging the content in the graphic buffer into the current screen buffer. With alternating emergence of display data in the graphic buffer and the current screen buffer, the flickers in the single current screen buffer caused by computing delay can be reduced and the simulation effect can be effectively improved.

6. Experiment Result

Modeling and mapping are two steps of graphics generation in Computer Graphics. The goal of modeling is to establish the mathematical model and to obtain geometric element corresponding to the 3D data. The purpose of mapping is to achieve the display or output of the graphics generated by the computer. According to the above analysis, the flow chart of simulating the realistic ocean wave is shown in Figure 4. According to the flow chart, we use Visual C++ 6.0 and OpenGL as a development tool to simulate the realistic ocean wave, and experiment result is shown in Figure 5.

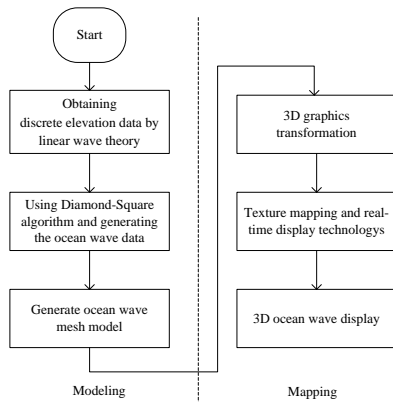


Figure 4. Flow Chart of Simulating Ocean Wave

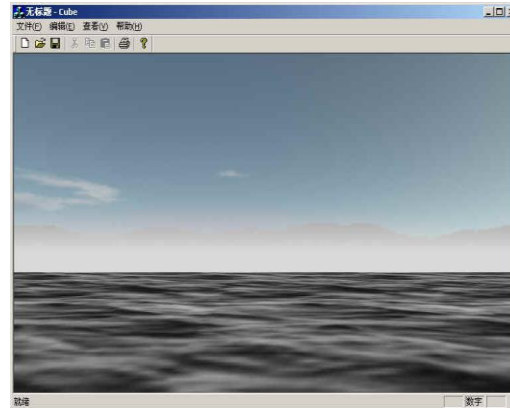


Figure 5. Effect Figure of Simulation

7. Conclusion

Fractal theory has obvious advantages in the simulating the ruleless nature. It has been widely application in many fields. Firstly, based on the linear wave theory, the paper acquires three-dimensional ocean wave data with the multi-directional spectrum superposition method. Secondly, In order to obtain more real-time and random data, based on the acquired simulation data, the paper puts forward the generation algorithm of fractal curved ocean surface and generates curved ocean surface with sense of reality by smoothing the elevation data obtained from the random midpoint displayment method. Lastly, it successfully realizes the dynamic and real-time display of three-dimensional ocean waves through optimizing the three-dimensional display by a series of methods.

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