

A Spatial Relation Modeling for ‘Between’ ‘Among’ and ‘Surround’ based on F-histogram

Wang Shu-zhong

Shandong College of Information Technology, Weifang, 261061, China
Wshzh2000@sina.com

Abstract

Spatial relationships among image objects play an important role in countless domains of computer. But the research on the expression method of special directional relation ‘between’, ‘among’ and ‘surround’ is immature. In this paper, firstly the characteristics and defects of present basic directional relation models and special directional relation models are analyzed; secondly the basic idea and construction method of F-histogram are introduced; thirdly a new expression method to judge special directional relation ‘between’, ‘among’ and ‘surround’ based on F-histogram is described in detail; finally several calculation examples which can prove the correctness of the method are given.

Keywords: *spatial relation; between; among; surround; F-histogram*

1. Introduction

Spatial relationships among image objects play a vital role in countless domains of computer vision. Freeman tried to draw up an exhaustive list of ‘primitive’ relationships and came up with 13 names (such as ‘above’, ‘near’, ‘between’, ‘inside’, *etc.*) [1]. But he can not distinguish between topological and direction relations, for example ‘inside’ is a topological relation.

Spatial relation is mainly constituted of directional relation, topological relation and distance relation. The fundamental theories of topology relation and distance relation are basically perfect, but the fundamental theory and expression modeling of directional relation have not any unified standard (especially special direction relations, such as ‘between’, ‘among’ and ‘surround’). So the establishment of special directional relation modeling is very essential.

In the article [2], directional relation is divided into the basic directional relation and the special directional relation. The basic directional relation is the directional relation that can be described by basic directional terms (above, below, left, right and so on) or combined directional terms (above-left, loosely above-right and so on). The special directional relation is the directional relation that can not be described by one or two basic directional terms, such as ‘surround’, ‘between’, ‘among’ *etc.* In this paper, we will propose a new modeling based on F-histogram and fuzzy model for the special directional relation such as ‘surround’, ‘between’ and ‘among’.

2. Background

2.1. Existing Formal Models of Directional Relation

Freeman [1] proposed the use of fuzzy relations, because ‘all or nothing’ standard mathematical relations are not suited for models of spatial relationships. His ideas were widely adopted. But 2D objects were often assimilated to very elementary entities such as a point [3] or a rectangle [4]. The procedure is practical, but yields poor models because of loss of a lot of morphological information, such as shape, size and distance information of the object for judging directional relation. Krishnapuram [5] and Miyajima [6] tackled this problem in parallel and came up with similar propositions. They developed the idea that the relative position between two objects can have a representation of its own and can thus be described in terms other than spatial relationships. This model can take into account shape, size and distance information, thus we can get exact results from angle histogram. However it has high computational complexity because of too many point pairs. In [7, 8], Matsakis and Wendling introduced the histogram of forces (F-Histogram). It proved to be a powerful tool, studied for numerous applications. The histogram of forces lends itself, with great flexibility, to fuzzy definitions of directional relations.

However, all these models are used for the basic directional relationships, the expression modeling of the special directional relation is rarely. Spatial relationships such as ‘surround’, ‘among’ and ‘between’ play an important role in the interpretation of a scene, and a few methods of assessing these relationships have been proposed. There are two main approaches for special directional relation. The first one is based on such rules [5]. For example, knowing that the object A is somewhat above, below, to the right and to the left of the object B as well, one could conclude that the object A surrounds the object B . But the directional relation has semantic inverse principle, consequently there is no way to know which one surrounds (or includes) the other. The main weakness of this approach comes from the fact that it is very difficult to express the necessary conditions for a spatial configuration to occur. The second approach derives from Rosenfeld’s visual surroundedness [9]. In [5, 6], it supposes the object A is connected and does not intersect the object B . For any pixel P of the object B , let θ_P be the angle made by the two tangents from P to A . To each element θ of $[0, 2\pi]$, the histogram associates the number of pixels P such that θ_P is equal to θ . Finally, membership values are calculated by fuzzy sets approach. However, the computational complexity is very high because of calculating angle-histogram, and this method can not handle disconnected objects. In [10], F-histogram is used for accessing special directional relation, but these judgment approaches are difficult to embody the directional relations of the every parts of the target object relative to reference object, and therefore these approaches are only suitable to the object with regular shape or small size.

2.2. F-Histogram Model

Matsakis [7] proposes F-Histogram, and F-Histogram is also based on angle histogram, but longitudinal sections of all angles are taken into account, not the point pairs. The F-histogram is built by computing the integral sum of longitudinal sections of different angles. This histogram added metric parameters in initial establishment, considering that different distances impact on spatial relation by setting different

parameters. The computational complexity of F-Histogram is relatively small, and anisotropy is considered.

Specifically, the relative position of a 2D object A with regard to another object B can be represented by a function F^{AB} called the histogram of forces associated with (A,B) via F, or the F-histogram associated with (A,B). For any direction θ , the value $F^{AB}(\theta)$ is the scalar resultant of elementary forces. These forces are exerted by the points of A on those of B, and each tends to move B in direction θ (Figure 1). Actually, the letter F denotes a numerical function. Consider any real number r. If the elementary forces are in inverse ratio to d , where d represents the distance between the points considered, and then F is denoted by F_r . The F_0 -histogram and the F_2 -histogram have very different and very interesting characteristics. The former provides a global view of the situation. It gives equal consideration to both the object's closest and farthest parts, whereas the F_2 -histogram focuses on the closest parts. In this paper, we only use F_0 -histogram for a global view of the situation.

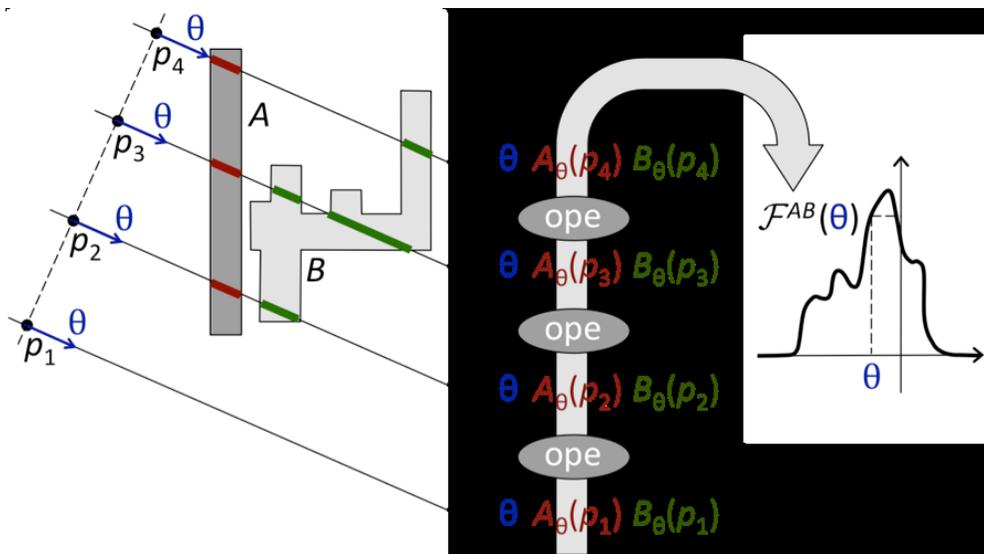


Figure 1. F-histogram

Computational complexity of F-histogram is $O(KN\sqrt{N})$, smaller than angle-histogram. But one main problem of F-histogram is that the amount of computation is still large. Ni [11] reduces the time complexity of F-Histogram from $O(KN\sqrt{N})$ to $O(M\log N)$ by FFT. Now the F-Histogram is becoming a study hot point for its advantages, and many improved methods and applied researches have constantly emerged. In this paper we adopt Ni's method

3. New Spatial Modeling for 'between', 'among' and 'surround'

Firstly, we calculate the F_0 -histograms $F_\theta(\text{Point}_{ij}^A, B)$ of every point Point_{ij}^A to the reference object B, and the Objects may be non-connected. Secondly, we find the largest open interval z_1 and the second largest open interval z_2 of which value is zero. θ_1 and θ_2 respectively are the right borders of z_1 and z_2 , as shown in Figure 2. Of course, we may also find the other open intervals z_1, z_2, \dots, z_q of which value is zero and their

corresponding angles $\theta_1, \theta_2, \dots, \theta_q$. All these angles belong to the open interval $[0, 2\pi]$. Moreover: $z_1 \geq z_2 \geq \dots \geq z_q$. q is the number of the open intervals of which value is zero.

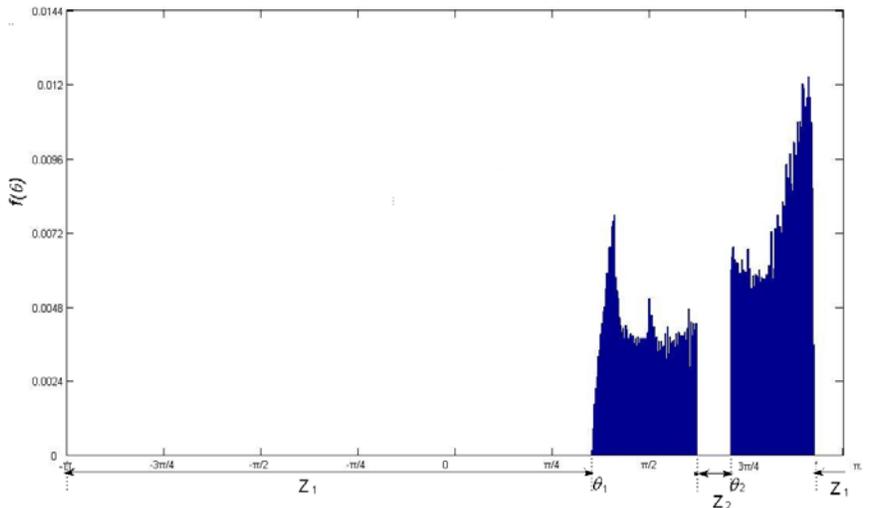


Figure 2. The other Open Intervals and Angles

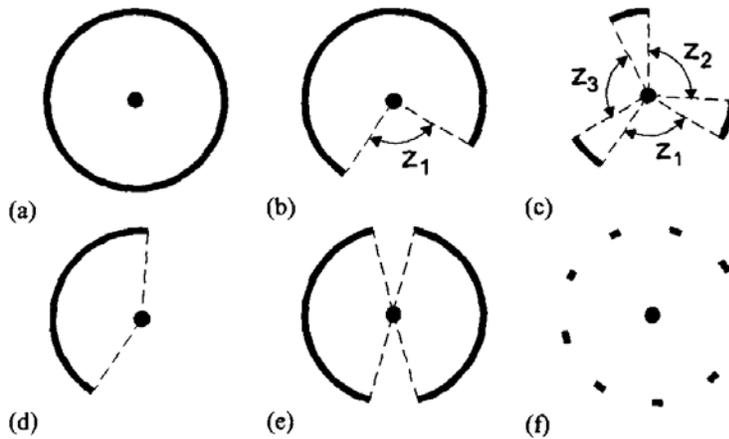


Figure 3. Degree to 'surround'

'Surround' is one of the important special directional relations. The degree of 'surround' depends on the largest open interval z_1 . We can see in Figure 3. The higher z_1 , the lower degree of 'surround' is (see Figure. 3(b)(d)). So in this paper we calculate the membership values $\mu_{surround}^{Point_{ij}^A}$ of 'surround' between every point $Point_{ij}^A$ of the object A and the object B by formula (1) based on visual surroundedness theory, and then calculate the weighted average $\mu_{surround}$ of $\mu_{surround}^{Point_{ij}^A}$ as the membership value of the judgment "object B surround object A" by formula (2). Let M and N be the x axis and y axis coordinates of image matrix. $M \times N$ is the number of the points of object A .

$$\mu_{surround}^{Point_{ij}^A}(z_{ij1}) = \begin{cases} \cos^2 \frac{z_{ij1}}{2} & \text{if } 0 \leq z_{ij1} \leq \pi \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$$\mu_{surround} = \sum_{i=1}^M \sum_{j=1}^N \mu_{surround}^{Point_{ij}^A}(z_{ij1}) \times (1 / M \times N) \quad (2)$$

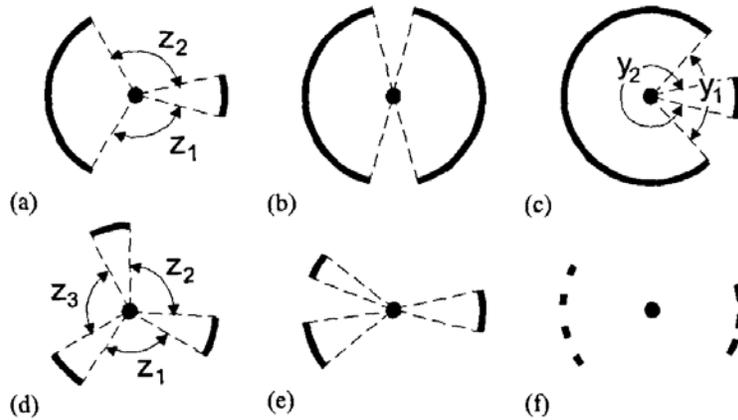


Figure 4. Degree to ‘between’

‘Between’ is a ternary relation of the special directional relation. For example, A is between B_1 and B_2 . We consider that the relationship should not apply when q is 1, and that the highest values should be reached when q is 2. We can see in Figure 4. However, z_1 and z_2 should be close to each other (Figure 4(a)). There is nothing preventing z_1 and z_2 should be high (Figure 4(b)). The situation is not as ideal when q is greater than 2. So we do not consider z_2 .(Figure 4(d)(e)(f)). So in this paper a simple approach firstly calculate the membership value $\mu_{between}^{Point_{ij}^A}$ of the judgment “ $Point_{ij}^A$ between B_1 and B_2 ” by formula (3), and then calculate the weighted average $\mu_{between}$ of $\mu_{between}^{Point_{ij}^A}$ as the membership value of ‘between’ by formula (4). But in many cases, this approach is not correct. For example, the white object is between the two gray objects in figure 5(b), but the judgment by above approach is not correct. But in figure 5(a) the result is correct. As shown in figure 5(b), the parts of the white object are somewhat above or below the gray object, not between, so we should consider the weakening effect of symmetrical parts. According to this idea, we use formula (5) and formula (6) instead of formula (3) and formula (4).

$$\mu_{between}^{Point_{ij}^A}(z_{ij1}, z_{ij2}) = \frac{z_{ij2}}{z_{ij1}} \quad (3)$$

$$\mu_{between} = \sum_{i=1}^M \sum_{j=1}^N \mu_{between}^{Point_{ij}^A}(z_{ij1}, z_{ij2}) \times (1 / M \times N) \quad (4)$$

$$\mu_{between}^{Point^A}(z_{ij1}, z_{ij2}) = \begin{cases} \frac{z_{ij2}}{z_{ij1}} & \text{if } \theta_{ij1} < \theta_{ij2} \\ 2 - \frac{z_{ij2}}{z_{ij1}} & \text{otherwise} \end{cases} \quad (5)$$

$$\mu_{between} = \begin{cases} \mu_{between} & \text{if } \mu_{between} < 1 \\ 2 - \mu_{between} & \text{otherwise} \end{cases} \quad (6)$$

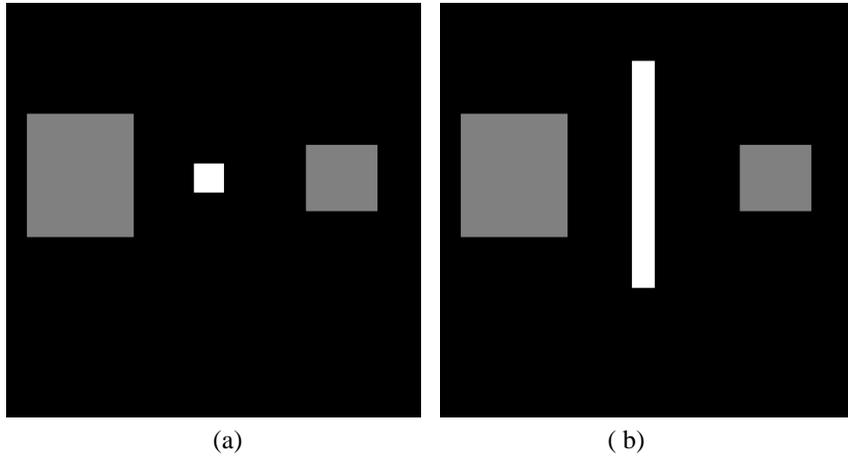


Figure 5. Synthetic Images 1

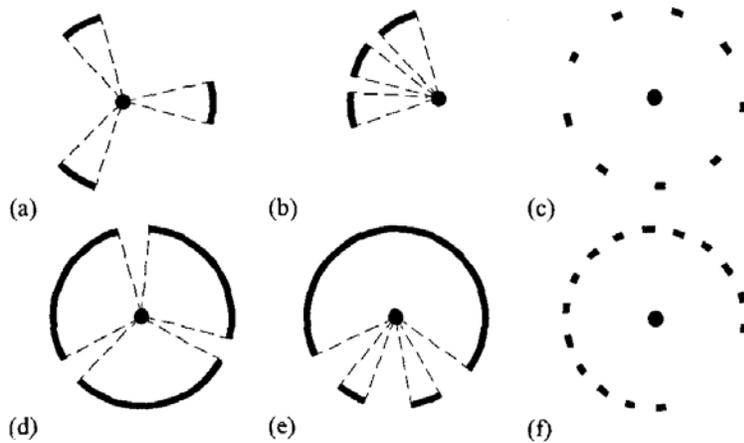


Figure 6. Degree to 'among'

'Among' is a multiple relation of the special directional relation. For example, A is among B_1, B_2, B_3 and B_4 . When q is lower than 3, we consider that the relationship should not apply. When q is greater than or equal to 3, we expect the open intervals can be evenly distributed around the center point. The best case is illustrated by Figure 6(a)(d). The worst cases are illustrated by Figure 6(b)(e). Hence in this paper a simple

approach firstly calculate the membership value $\mu_{Among}^{Point_{ij}^A}$ of the judgment “ $Point_{ij}^A$ is among $B_1, B_2, \dots,$ and B_q ” by formula (7), and then calculate the weighted average μ_{among} of $\mu_{among}^{Point_{ij}^A}$ as the membership value of ‘among’ by formula (8).

$$\mu_{among}^{Point_{ij}^A}(z_{ij}) = \min\left(\frac{\sum_{m=1}^q z_{ijm} - z_{ij1}}{\sum_{m=1}^q z_{ijm} - (\sum_{m=1}^q z_{ijm}) / q}\right) \quad (7)$$

$$\mu_{among} = \sum_{i=1}^M \sum_{j=1}^N \mu_{among}^{Point_{ij}^A}(z_{ij}) \times (1 / M \times N) \quad (8)$$

4. Experiments

Firstly, we respectively use three methods (K [6], SFH [10], FH (our method)) to judge the special directional relation ‘surround’ and ‘between’ of twelve images in figure 7. The results are shown in table 1.

We analyze the judgment results of ‘surround’ firstly. The judgments of K, SFH and FH to Figure 7(a) and Figure 7(b) are basically the same, but the judgments of SFH to Figure 7(c), Figure 7(d), Figure 7(e) and Figure 7(f) are not consistent with human cognition. That is because F-histogram of two objects is difficult to reflect the direction ‘surround’ of every part of the target object. The judgments of K and FH are consistent with human cognition, but the computational time of FH is much shorter than K. So FH method is more suitable to judging the special direction relation ‘surround’ than K and SFH methods.

The judgment results of ‘between’ are analyzed secondly. The judgments of K, SFH and FH to Figure 7(g), Figure 7(h) and Figure 7(i) are basically the same, but the judgment of K to Figure 7(i) is not consistent with human cognition obviously. That is because K method can not consider the weakening effect of symmetrical parts. The judgment values of FH to Figure 7(j), Figure 7(k) and Figure 7(l) are greater than SFH, because SFH method can consider the weakening effect of symmetrical parts but not reflect the direction ‘between’ of every part of the target object. Since FH method can consider weakening effect of symmetrical parts and reflect the direction ‘between’ of every part of the target object, FH method can get the judgment results which are more consistent with human cognition than other methods.

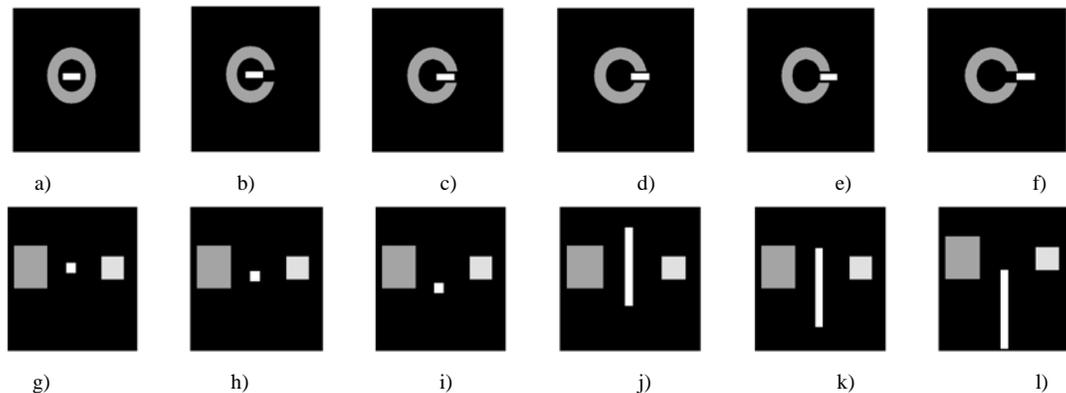


Figure 7. Synthetic Images 2

Table 1. Results of the Spatial Relation ‘surround’ and ‘between’

$\mu_{surround}$ (%)	a	b	c	d	e	f	$\mu_{between}$ (%)	g	h	i	j	k	l
K	100	93	70	41	19	0	K	99	76	51	54	51	29
SFH	100	98	94	88	79	0	SFH	98	71	45	95	55	22
FH	100	93	71	40	18	0	FH	100	75	48	98	64	40

Secondly, we use our method (FH) to judge the special directional relation ‘among’ and of four images in Figure 8. The results are shown in Table 2.

We analyze the judgment results of ‘among’ now. The judgments of FH to Figures 7(a)(b)(c)(d) are basically consistent with human cognition. So we can say that FH method is suitable to judging the special direction relation ‘among’.

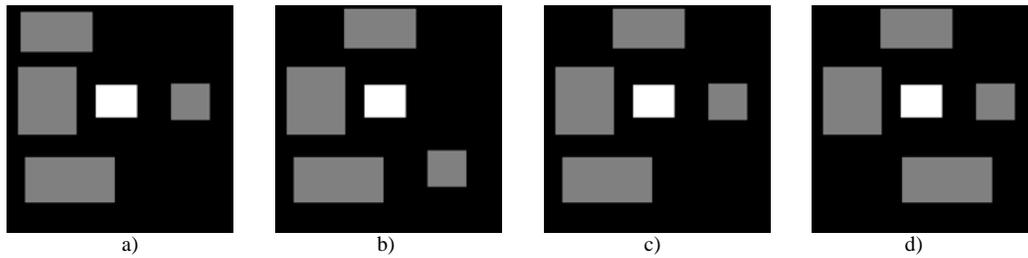


Figure 8. Synthetic Images 3

Table 2. Results of the Spatial Relation ‘among’

μ_{among} (%)	a	b	c	d
FH	38	71	82	91

5. Conclusion

In this paper, a new spatial relation modeling for ‘surround’, ‘between’ and ‘among’ based on F-histogram is firstly introduced. We have shown that the F-histogram can be employed to design fuzzy models of special directional relation like ‘surround’, ‘between’ and ‘among’. Compared to other existing methods, our method has the following advantages: 1) our method is computationally much less expensive; 2) it is able to handle non-connected objects and groups of objects; 3) the experimental results of our method show that this model is in better harmony with human perception.

The next step to research include: 1) to build unified representation modeling of directional relations based on F-histogram; 2) to represent spatial relations extended from the exact images to fuzzy images; 3) to represent spatial relations from 2D images to 3D images.

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Author



Wang Shu-zhong, received the Master Degree in Engineering from Hefei University of Technology in 2004. His current research interests on Graphics and image technology, and Software Engineering.

