

## Image Segmentation Using Two-dimensional Extension of Minimum Within-class Variance Criterion

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### Abstract

*Thresholding based on variance analysis of gray levels histogram is a very effective technology for image segmentation. However, its performance is limited in conventional forms. In this paper, a novel method based on two-dimensional extension of within-class variance is proposed to improve segmentation performance. The two-dimensional histogram of the original and local average image is projected to one-dimensional space firstly, and then the minimum within-class variance criterion is constructed for threshold selection. The effectiveness of the proposed method is demonstrated by using examples from the synthetic and real-world images.*

**Keywords:** Image thresholding; Two-dimensional histogram; Minimum within-class variance

### 1. Introduction

Image segmentation is fundamental to many image analysis tasks such as object tracking, character recognition, and document analysis, just to name a few [1]. Thresholding is a simple but effective tool for image segmentation [2]. The purpose of this operation is that objects and background are separated into non-overlapping sets. In many applications of image processing, the use of binary image can decrease the computational cost of the succeeding steps compared to using gray-level images. Since image thresholding is a well-researched field, there exist many algorithms for determining an optimal threshold of the image. A survey of thresholding methods and their applications exists in literature [2]. Of which, the variance-based thresholding technology is a kind of famous method for image segmentation [3-9]. In earlier research, Otsu proposed minimum within-class variance criteria to select the best threshold [3]. For every possible threshold value, the method evaluates the goodness of this value if used as the threshold. This evaluation uses either the heterogeneity of both classes or the homogeneity of every class. By minimizing the criterion function, the means of two classes can be separated as far as possible and the variances in both classes will be as minimal as possible. This method still remains one of the most referenced thresholding methods.

Most of the proposed methods selection thresholds which depend solely on the one-dimensional (1D) gray level histogram of the image. However, more information contained in the image can be utilized to obtain better segmentation. Since 1D histogram of an image, which only represents the gray level distribution of the image, does not take into consideration the spatial correlation between the pixels in the image, the performance of the

proposed methods based on 1D histogram might degrade rapidly when the signal-to-noise ratio (SNR) is decreased. For this reason two-dimensional (2D) thresholding methods which employ point pixel information and the local average gray level of the neighborhood pixels have been proposed. Liu and Li [4] proposed a 2D variance-based thresholding method which performs much better than 1D Otsu's method when images are corrupted by noise.

Although the extension of the 1D thresholding method to the 2D histogram results in much better segmentation, it gives rise to the exponential increment of computation. The computation complexity of Liu and Li's method, using an exhaustive search, is bounded by  $O(L^4)$ , where  $L$  is the number of gray levels of image. Gong *et al.*, [5] proposed a fast algorithm which can reduce the computation complexity to  $O(L^2)$ . But the computation complexity of the method of Otsu (1D method) is only bounded by  $O(L)$ . In this paper, a novel 2D extension of variance-based thresholding method which decreases the computation complexity of 2D thresholding from  $O(L^2)$  to  $O(2L)$  is introduced. In our new method, the gray levels of the pixels and the local average gray level of the neighborhood pixels form a 2D histogram. However, this 2D parameter space is reduced to a 1D histogram through 1D projection summation of 2D histogram, while assigning equal weights to both variables of gray levels. Then, the variance-based thresholding method is applied on the obtained 1D histogram. The experiments on artificial images with different levels of noise and real images demonstrate that the proposed approach can perform segmentation well.

## 2. Conventional Variance-based Thresholding Method

Without losing generality, let  $I$  denote a gray scale image with  $L$  gray levels  $G=[0, 1, \dots, L-1]$  of size  $M \times N$ ,  $f(x,y)$  be the gray value of the pixel located at the point  $(x,y)$  and  $x \in \{1,2,\dots,M\}$ ,  $y \in \{1,2,\dots,N\}$ . The number of pixels with gray level  $i$  is denoted by  $n_i$  and the total number of pixels by  $M \times N$ . The probability of gray level  $i$  appeared in the image is defined as

$$p_i = \frac{n_i}{M \times N}, \quad p_i \geq 0, \quad \sum_{i=0}^{L-1} p_i = 1 \quad (1)$$

Suppose that the pixels in the image are divided into two classes  $C_0$  and  $C_1$  by a gray level  $t$ ;  $C_0$  is the set of pixels with levels  $[0, 1, \dots, t]$ , and  $C_1$  is the set of pixels with levels  $[t+1, t+2, \dots, L-1]$ .  $C_0$  and  $C_1$  normally correspond to the object class and the background one, or vice versa. Then the probabilities of the two classes are given by

$$\omega_0 = \sum_{i=0}^t p_i, \quad \omega_1 = \sum_{i=t+1}^{L-1} p_i \quad (2)$$

The mean gray levels of the two classes can be defined as

$$\mu_0 = \sum_{i=0}^t i p_i / \omega_0, \quad \mu_1 = \sum_{i=t+1}^{L-1} i p_i / \omega_1 \quad (3)$$

and corresponding class variances are given by

$$\sigma_0^2 = \sum_{i=0}^t (i - \mu_0)^2 p_i / \omega_0, \quad \sigma_1^2 = \sum_{i=t+1}^{L-1} (i - \mu_1)^2 p_i / \omega_1 \quad (4)$$

the within-class variance in Otsu [3] method is defined by

$$\sigma_w^2 = \omega_0 \sigma_0^2 + \omega_1 \sigma_1^2 \quad (5)$$

the optimal threshold  $t^*$  can be determined by

$$t^* = \arg \min_{t \in G} [\sigma_w^2(t)] \quad (6)$$

The performance of 1D Otsu method might degrade rapidly when the SNR is decreased. For this reason 2D Otsu approaches which employ point pixel information and the local average gray level of the neighborhood pixels have been proposed [4-5].

### 3. The Proposed Method

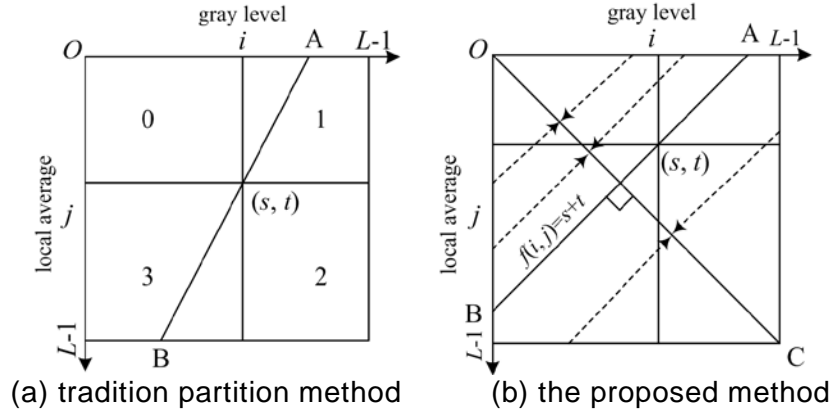
In this section, a 2D extension of within-class variance criterion is presented. The computation complexity of the new method reduces to  $O(2L)$  from  $O(L^2)$  of fast 2D Otsu method, where  $L$  is the number of gray levels in the image. Let  $g(x,y)$  denotes the local average gray value in a  $w \times w$  neighborhood window, *i.e.*,

$$g(x,y) = \left\lfloor \frac{1}{w \times w} \sum_{m=-a}^a \sum_{n=-a}^a f(x+m, y+n) \right\rfloor \quad (7)$$

where  $a = \lfloor w/2 \rfloor$ ,  $\lfloor \bullet \rfloor$  denotes the integer part of the number  $\bullet$ ,  $w < \min(M, N)$ , in general  $w$  is an odd number. Let  $n_{ij}$  be the frequency of pair  $(i,j)$ , where  $f(x,y)=i$  and  $g(x,y)=j$ , then

$$p_{ij} = \frac{n_{ij}}{M \times N}, \quad 0 \leq i < L, \quad 0 \leq j < L \quad (8)$$

is a 2D histogram of image  $I$ . The 2D histogram is a matrix of size  $L \times L$ , which is shown in Figure 1. If we assume that the pair  $(s,t)$  is a threshold vector to be used for thresholding, the  $(s,t)$  divides 2D histogram into four quadrants. The intersection of the orthogonal lines produces the overall thresholding point  $(s,t)$ , as shown in Figure 1(a). These quadrants can be further classified into the diagonal quadrants 0 and 2 and off-diagonal quadrants 1 and 3, respectively, in Figure 1(a). Since two of the quadrants, 1 and 3, contain information about edges and noise alone, they are ignored in the calculation in traditional 2D histogram thresholding methods, these methods only used two of the quadrants for segmentation, *i.e.*, quadrant 0 pixels as background and quadrant 2 pixels as foreground. However, these methods discarded the pixels located in quadrants 1 and 3, which may ignore important information concerning the objects to be segmented. Sahoo *et al.*, [10] recognized this problem firstly, and then proposed a thresholding line in the 2D histogram plane to provide better segmentation. Such a line, straight line AB is shown in Figure 1(a). However, this method is still a costly method, as there are approximately  $4 \times 10^5$  lines that bisect a  $256 \times 256$  histogram [11]. Based on the idea of Sahoo *et al.*'s method, some improved methods have been proposed to overcome the defects of time-consuming [11].



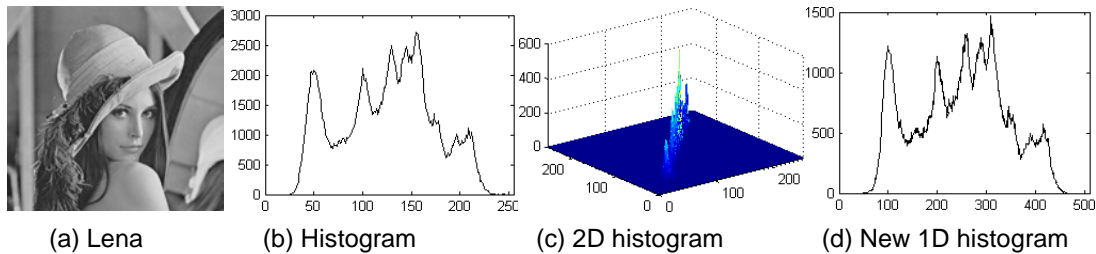
**Figure 1. 2D Gray Level Histogram and its Partition**

Based on the idea of thresholding line to divide the 2D histogram, an easier and effective form for dividing the 2D histogram plane is adopted in this paper. As shown in Figure 1(b), passing through  $(s, t)$ , the line AB is defined to be the line perpendicular to principal diagonal OC of the 2D histogram. And then segment the image using line AB as the optimal thresholding line.

From Figure 1(b), it can be seen that the geometric equation of the line AB is  $f(i, j) = s + t$ , and  $0 \leq f(i, j) \leq 2(L-1)$ . If we consider the straight line OC as a projection axis, let us now consider the function  $f(i, j)$  as a new variable  $r$ , and construct its histogram. Obviously, each bin in this histogram will contain a contribution only from a unique line in the two-dimensional gray level histogram matrix, *i.e.*,

$$p_r = \sum_{r=i+j} p_{ij}, \quad 0 \leq i < L, \quad 0 \leq j < L \quad (9)$$

Hence, the 2D parameter space was reduced to a 1D histogram of the variable  $r$ , obviously  $r \in \{0, 1, \dots, 2(L-1)\}$ . Figure 2 illustrates an example of 1D projection of 2D histogram. In Figure 2, the image is *Lena* with size of  $512 \times 512$ , and the neighborhood window for forming local average image is  $5 \times 5$ .



**Figure 2. An Example of 1D Projection of 2D Histogram**

At the new 1D histogram, let  $z$  is an assumed threshold value. For image segmentation, If we let

$$\pi_0 = \sum_{r=0}^z p_r, \quad \pi_1 = \sum_{r=z+1}^{2(L-1)} p_r \quad (10)$$

and

$$m_0 = \frac{1}{\pi_0} \sum_{r=0}^z r p_r, \quad m_1 = \frac{1}{\pi_1} \sum_{r=z+1}^{2(L-1)} r p_r \quad (11)$$

$$\sigma_0^2 = \frac{1}{\pi_0} \sum_{r=0}^z (r - m_0)^2 p_r, \quad \sigma_1^2 = \frac{1}{\pi_1} \sum_{r=z+1}^{2(L-1)} (r - m_1)^2 p_r \quad (12)$$

then we can construct the within-class variance criterion on the new 1D histogram, *i.e.*,

$$\sigma_w^2 = \pi_0 \sigma_0^2 + \pi_1 \sigma_1^2 \quad (13)$$

When  $\sigma_w^2$  is minimized, the value at the new 1D histogram is considered to be the optimum threshold value. This can be achieved with a cheap computational effort.

$$z^* = \arg \min_{z \in U} [\sigma_w^2(z)] \quad (14)$$

where  $U = \{0, 1, \dots, 2(L-1)\}$ . From above, if the optimum threshold  $z^*$  is found, it signifies the optimum threshold line  $f(i,j) = i+j = z^*$  in Figure 1(b) is found. So in the new method, the pixels in the  $\Delta OAB$  region would be below the threshold  $z^*$  while the rest would be above it. The result of thresholding an image function  $f(x,y)$  at optimal threshold  $z^*$  is a binary function  $\bar{f}(x,y)$  such that

$$\bar{f}(x,y) = \begin{cases} 0, & \text{if } (f(x,y) + g(x,y)) \leq z^* \\ 1, & \text{if } (f(x,y) + g(x,y)) > z^* \end{cases} \quad (15)$$

where  $f(x,y)$  and  $g(x,y)$  are the intensities at  $(x,y)$  of the original image and the local average image, respectively.

## 4. Performance Evaluation and Experiments

In this section, we present the experimental results obtained via the application of our method to different kinds of images. This section is divided into three parts; in the first subsection, we evaluate the performance results in comparison with of three other methods: Otsu method [3], one method based on two-dimensional Otsu criterion [5] and Li *et al.*, [9] method based on statistical variance analysis theory. The experiments on the real-world images are discussed in the second subsection.

The algorithms are coded in Matlab and run on a 2.80 GHz Pentium(R) Dual-Core CPU personal computer, under Microsoft Window 7 Operating system. In our experiments, when  $w=3$ , the proposed method can obtain good segmentation results. So, unless otherwise specified, the local neighborhood window  $w$  applied to generate 2D histogram is set to 3.

### 4.1. Evaluation of the Performance based on Synthetic Images

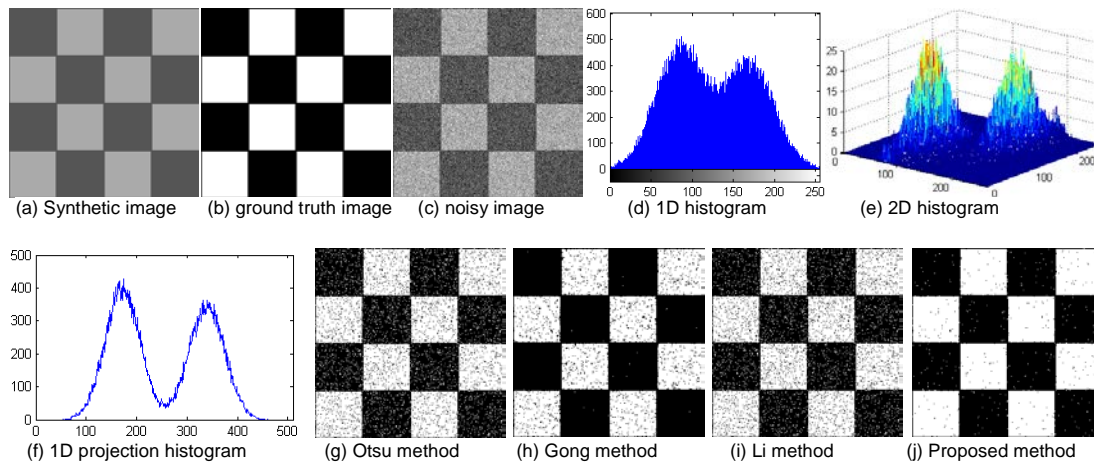
In order to measure the performance of the segmentation, we used the criterion of misclassification error (*ME*) [2]. *ME* is defined in terms of correlation of images with human observation. It is the ration of the number of pixels in the background falsely classified in the foreground and vice versa. *ME* can be simply expressed as

$$ME = 1 - \frac{|B_o \cap B_T| + |F_o \cap F_T|}{|B_o| + |F_o|} \quad (16)$$

where background and foreground are denoted by  $B_o$  and  $F_o$  for the original image, and by  $B_T$  and  $F_T$  for the segmented image, respectively. In the best case of ideal thresholding,  $ME$  is equal to 0 and, in the worst case,  $ME$  value is 1.

We used a synthetic image with different degrees of noise measured by the standard deviation  $\sigma$  of added Gaussian noise. Figure 3(a) is the synthetic image with  $256 \times 256$  pixels and two intensities (85 and 170). The ground truth of synthetic image is shown in Figure 3(b). Figure 3(c) is an image added Gaussian noise on original synthetic image, whose mean is 0 and variance  $\sigma^2$  is 900, *i.e.*, the standard deviation  $\sigma$  is 30. Figures 3(d) and 3(e) show the 1D and 2D histogram of the noisy image, respectively. Figure 3(f) shows the 1D projection of the 2D histogram. Figures 3(g-i) show the segmented results of noisy image by Otsu method, Gong *et al.*, method, Li *et al.*, method and the proposed method. Table 1 lists the results in terms of thresholds, number of misclassified pixels,  $ME$  and running time obtained by applying each thresholding method to the noisy image as shown in Figure 3(b).

From Figure 3, it can be seen that the separability of foreground and background at 1D projection histogram is better than the original 1D gray level histogram. From the segmented results in Figure 3, we also can see that the result obtained by the proposed method is better than those obtained by the reference methods. From Table 1, it can be seen that the threshold obtained by the proposed method is 254 (corresponding to 586 misclassified pixels and its  $ME$  equals to 0.0089), whereas that by the Otsu method is 127 (corresponding to 5093 misclassified pixels and its  $ME$  equals to 0.0777), Gogn *et al.*'s, fast 2D Otsu method is [154,131] (corresponding to 2152 misclassified pixels and its  $ME$  equals to 0.0328), and Li *et al.*'s, statistical variance analysis method is 129 (corresponding to 5048 misclassified pixels and its  $ME$  equals to 0.0770). Among these thresholding methods, the best result was provided by the proposed method. As can be seen from the thresholded images in Figure 3 and Table 1, one can conclude that the proposed method can provide better results than conventional 1D variance-based thresholding method and Gong *et al.*'s, fast 2D Otsu method, and the result obtained by Gong *et al.*'s, method is better than those by 1D methods.



**Figure 3. A Synthetic Noisy Image and its Segmented Results by Different Methods**

**Table 1. Thresholded Results for the Synthetic Noisy Image of Different Methods**

Method	Otsu method	Gong method	Li method	Proposed method
Threshold	127	[154,131]	129	254
Misclassified pixels	5093	2152	5048	586
<i>ME</i>	0.0777	0.0328	0.0770	0.0089
Running time (ms)	31	97	20	46

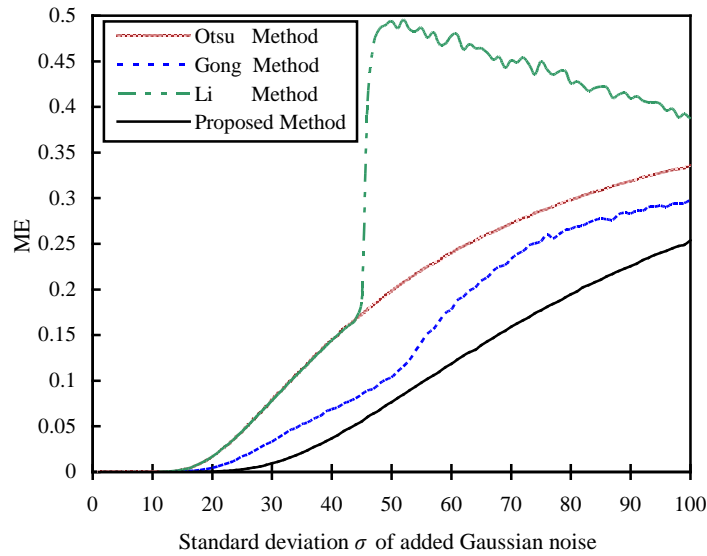
In order to more fully describe the performance of the proposed method, the experiments on synthetic images added different degrees noise are implemented. The comparison of the results *ME* provided by the proposed method and the reference methods, based on the segmentation of the synthetic images, with different degrees of noise (Gaussian noise with 0 mean and variance  $\sigma^2$ ), is presented on Figure 4. Since the noise is randomly added, we run the simulation 20 times to get mean *ME* for each magnitude (represented by the standard deviation  $\sigma$  of Gaussian noise, and the standard deviation  $\sigma$  is gradually increased). From Figure 4, we can clearly see that the proposed method can achieve better performance and lower *ME* at all magnitude of the noise.

#### 4.2. Experiments on Real Images

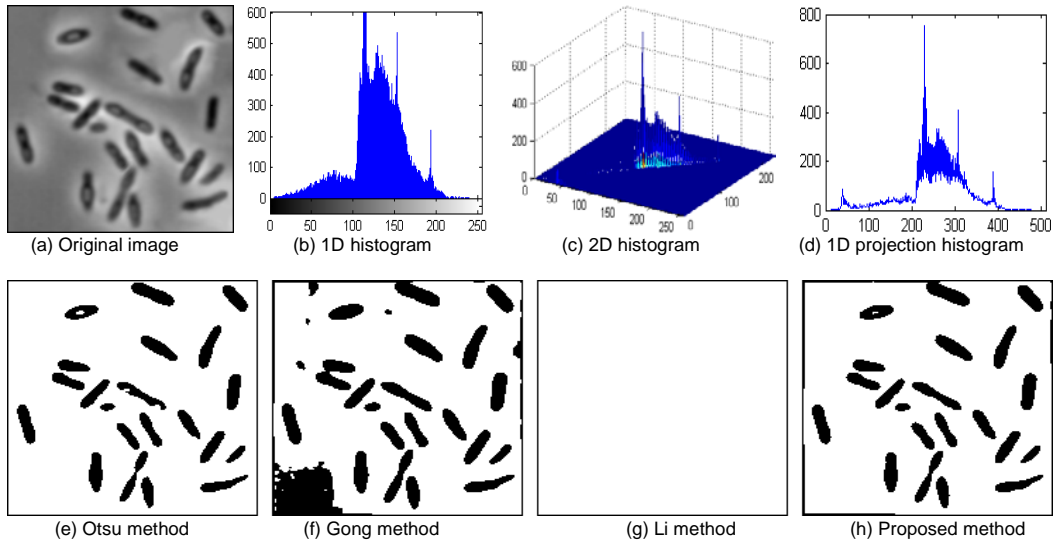
We have applied the proposed method to a variety of real images. Due to the page limit, only some images are shown here. Figure 5-7 give the thresholding experiments.

Figure 5 shows the *Bacteria* image and its segmented results. Figure 5(a) is the original *Bacteria* image with size of 178×178. Figures 5(b-d) show the 1D gray levels histogram, 2D gray levels histogram, and the 1D projection histogram of the 2D histogram of *Bacteria* image, respectively. Figures 5(e-h) show the thresholded image of original *Bacteria* image. From the obtained thresholded results, it can be seen that the result by the proposed method is better than those obtained by other methods.

Figures 6-7 give two thresholding experiments on two noisy images. Figures 6-7(a) are the two original images, one is the *Blood1* image with size of 265×272 and the other is the *Eight* image with size of 242×308. Figures 6-7(b) are the images with Gaussian noise and Figures 6-7(c) are their 1D histograms. Figures 6-7(d) show the 1D projection histogram of the noisy images. The image shows in Figure 6(b) is added Gaussian noisy with zero mean and standard deviation 30 and the image show in Figure 7(b) is added Gaussian noisy with zero mean and standard deviation 40. Figures 6-7(e-h) are the segmented results by the methods that are compared with each other. From Figures 6-7, we can see that the experiments on two noisy image show the proposed method performs much better than other methods.



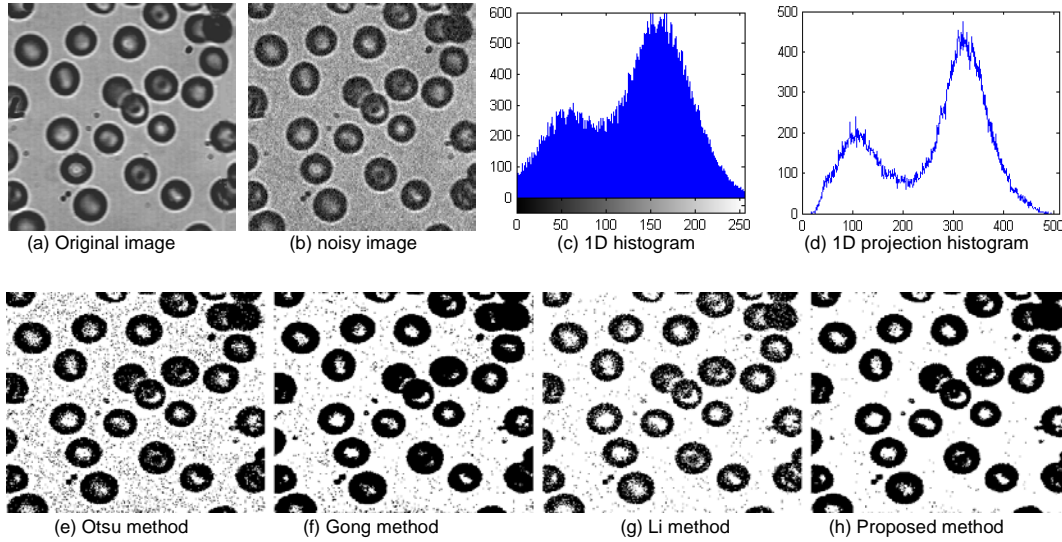
**Figure 4. The Relation between *ME* and the Magnitude of Added Gaussian Noise**



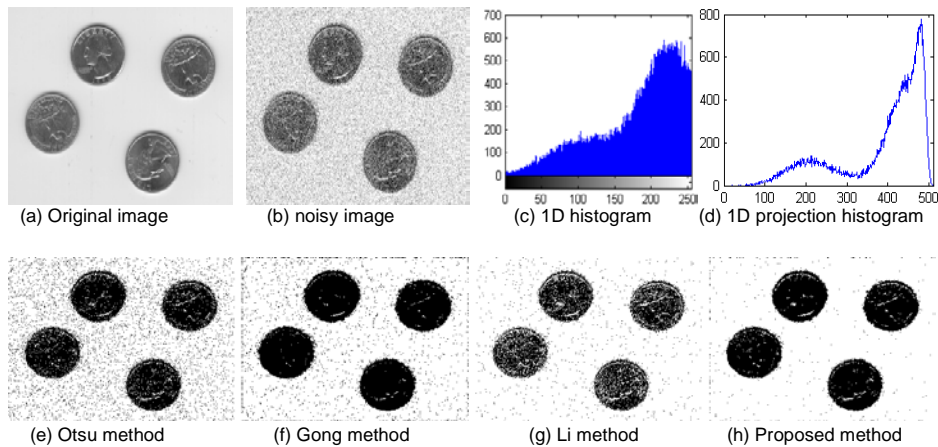
**Figure 5. Bacteria Image and its Segmented Results**

Table 2 shows the thresholds found by both methods on test images. From Figures 5, 6 and 7, it can be seen that the separability of the different part of test images at 1D projection histogram is better than the original 1D gray level histogram. From Table 2, we can see that the thresholds found by the proposed method are closer to the ideal valley point of the 1D projection histogram. From the above examples, we can conclude that the proposed method is robust for the test images. The thresholds found by the proposed method can generate better binary images than those obtained by using other methods. Therefore, the proposed method is better than other methods.





**Figure 6. Blood1 Image and its Segmented Results**



**Figure 7. Eight Image and its Segmented Results**

**Table 2. Thresholds for Test Images by Different Thresholding Methods**

Image	Otsu method	Gong method	Li method	Proposed method
<i>Bacteria</i>	96	[119,101]	0	202
<i>Blood1</i>	114	[124,123]	90	224
<i>Eight</i>	165	[188,171]	134	325

Table 3 shows the time required by every method on test image. The computing time was measured in millisecond. From Table 3, we can see that the time required by 1D method, *i.e.*, Otsu method and Li *et al.*, method, is lower than 2D histogram thresholding methods. Since the computational complexity of the 1D histogram thresholding method is  $O(L)$ , where  $L$  is the number of the gray levels of image. The computational complexity of Gong *et al.*, method is  $O(L^2)$ , therefore, the computational cost is the highest. The computational complexity of the proposed method is  $O(2L)$ ,

therefore, the time required is higher than 1D methods and lower than Gong *et al.*, method.

**Table 3. The Computational Cost (ms) of each Methods on Test Images**

Image	Otsu method	Gong method	Li method	Proposed method
<i>Bacteria</i>	35	124	22	57
<i>Blood1</i>	37	137	23	65
<i>Eight</i>	34	133	23	67

## 5. Conclusions

An automatic thresholding of gray level image using minimum within-class variance criterion based on 1D projection of 2D histogram is proposed. The proposed method is based on the 2D gray level histogram of images, which considers both the local information as well as the pixel intensity. The experiments prove that the obtained threshold can be applied to perform image thresholding. The thresholding results are compared with those from conventional variance-based methods. The experimental results prove that the proposed method outperforms conventional variance-based methods. The proposed approach can have wide application in computer vision and image processing.

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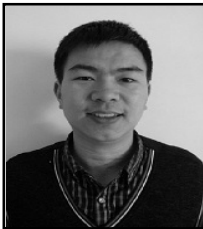
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