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Effect of Noise in Estimation of Fractal Dimension of Digital Images

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Abstract

In present paper the effect of noise on fractal dimension of digital images has been tested. Since fractal dimension is a measure of texture which is property of neighbourhood, it is interesting to check how noise affects the values of fractal dimension. For this purpose, three standard digital images have been used and Gaussian noise, salt and pepper noise and speckle noise have been applied to these images to generate noisy images. The fractal dimension values of actual images and noisy images have been estimated and compared. Various aspects related to the estimation of fractal dimension of digital images are discussed for noisy and non-noisy images. Since variation of noise makes sense for a local window in which fractal dimension is estimated, it is required to observe the noise effect in the window.

Keywords: *Fractal dimension, Fractal image, Global and local fractal dimension, Gaussian noise, Moving window, Salt and pepper noise, Speckle noise*

1. Introduction

Fractal geometry has widely been used in digital image analysis for various purposes including image classification, texture identification, object recognition, image segmentation, and roughness measurement. Among these, texture study with fractals is important which evolved after the major work of Pentland [1] based on application of fractal geometry in image analysis. Pentland [1] presented a way to model and analyse natural objects in form of digital images using fractals. Earlier, the application of fractal geometry was explored by Mandelbrot [2], who coined the word fractal and described its applications in various fields of natural science and in daily life. Since ordinary life deals with the things which are not well shaped, mercurial, irregular and so complex that the conventional geometry, known as Euclidean geometry, cannot define them very well and even sometimes fails. The Euclidean geometry deals with simple structures, e.g., lines, curves, circles, spheres, rectangles and other shapes while the objects experienced in practice are not so simple. This fact led to explore beyond Euclidean geometry and gave birth to non-Euclidean geometry, especially fractal geometry which deals with the objects called fractals in a simple sense.

A fractal is defined to be an object having two properties, viz., self-similarity and a fractal dimension [1, 3]. As the name indicates, self-similarity means the object under consideration is exactly similar to itself while scaled down or scaled up, i.e., if the object is divided into parts by a scale r and the new object obtained is scaled up by a factor r, the original object will be formed. This condition is to be followed to an infinite level by definition, however, in real physical world it is limited to few steps instead of infinite steps [2, 3]. The other property, *i.e.*, fractal dimension is mathematically defined in a number of ways; however the simplest and basic one is called a self-similar dimension [1-6] denoted by D and given by

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$$D = \frac{\log(N_r)}{\log(1/r)} \quad (1)$$

where N_r is the number of self-similar objects when the object is scaled down by ratio r.

Mandelbrot [2] discussed the geometry beyond Euclidean geometry and explored the possibilities where fractal geometry could be applied. As a result, applications of fractal geometry evolved in almost all the fields of science. In the field of image analysis, fractals took place an immediate step and became popular. As a matter of fact, fractals are more popular in analysis of remotely sensed images because these images are rich in irregular objects like water bodies, land covers, erratic costal lines and so on which are to be analysed with the help of fractals [3]. Texture analysis is another field where fractals became popular and the reason behind this fact is the similarity in estimation of texture and fractal features [3, 7-9]. Since texture is a property of neighbourhood which is measured for a local window of pixels, the same procedure is applied to estimate fractal features too. In digital images, for a single pixel there is nothing to say about its fractal behaviour [8]. Therefore textures are identified with fractals in a more familiar way. Besides this, fractals are used for object detection, face recognition, human eye tracking, object tracking in moving images and many more applications of digital images and video processing [10, 11]. One of the important features of fractals is fractal signature [2, 12], which could be used to identify various objects on the basis of their fractal behaviour. This property is unique one and has been utilized for various applications [10].

Fractal analysis is based on the extraction of fractal features which include fractal dimension, self-similarity, scaling behaviour, lacunarity, Hurst exponent and others. These features are useful in various ways and their estimation is the key factor in fractal analysis. Estimation of fractal features from digital images is application dependent and vice-versa. For example, in remote sensing images, fractal dimension may be used to identify various objects and utilized for classification or segmentation purpose whereas in the same image scaling behaviour can also be tested to find the properties of objects at different scales. Thus, estimation of fractal features requires a keen attention, so that accurate methods could be developed accordingly. Fractal dimension, the most popular fractal feature, can be estimated for digital images by considering the pixel values of predefined neighbourhoods as done in numerous available methods [1, 3, 9-16], viz. differential box counting method [17], Triangular prism surface area method (TPSAM) [18, 19], Fourier spectrum based method [2, 3], Variogram method, Robust fractal dimension estimation [3], 2D variation method [20] which deal with image pixels and their orientation at a varying scale. Nevertheless, the effect of noise on estimation of fractal dimension is not discussed along these methods. Therefore, there is a strong requirement to discuss the effect of noise in digital images while estimating their fractal dimension. Since fractal dimension can be estimated on a global basis or a local basis, corresponding effect of noise on both makes proper sense. The global or local fractal dimension is application dependent and useful for various analyses, noise may play significant role in these specified applications.

2. Fractal Dimension Estimation Methods

Since fractal dimension can be estimated in a global or local way, the global fractal dimension represents the fractal dimension of the whole images and represents the overall distribution of image pixels, more specifically, the roughness of the whole image [7]. This value is useful when the whole image is used and in this case fractal dimension represents the fractal signature of the image [2, 12]. Although useful, fractal signature is mostly used when image size is small because for large images, this value does not represent the pixel distribution uniformly. This fact is more obvious for remotely sensed images, which represent various land features in a single image and hence global value refers fractal signature of all the land features instead of individual feature. The local fractal dimension, on the other hand, represents various image features at local level and consequently the fractal signatures of individual objects could be identified. In order to estimate the local fractal dimension, a moving window approach is required to be followed in which the fractal dimension is estimated for this defined window.

The size of window is an important issue to be dealt with, since this window represents the local neighbourhood of pixels which consequently corresponds to the image features and various objects. The effect of local window has been reported by Pant *et al.*, [7] for SAR images along with the discussion of minimum feasible size of local window. Following the same approach a minimum size of 5 can be considered for the estimation of local fractal dimension of digital images which has also been adopted in present study.

2.1. Triangular Prism Surface Area Method (TPSAM)

This method was given by Clarke [18] and is a landmark in estimation of fractal dimension of digital images. In this method, the image pixels are considered as 3D columns with heights equal to their pixel values generating imaginary prism in 3D with four pixels at four corners and their average value in the center. This central pixel is the mean pixel value of corner pixels and actually does not exist. Thus, four triangular prisms in 3D space are generated and the total surface area of the four triangles at upper level is estimated. The base area is also estimated and the process is repeated for different base resolutions, starting from 2×2 pixels in multiple of 2. Corresponding upper surface area and base area is used to estimate the slope of best fit line in a log-log plot and used to calculate fractal dimension D by the formula

$$D = 2.0 - slope \qquad (2)$$

The total upper surface area decreases with an increase in base resolution and hence the *slope* in equation (2) usually estimates to be negative and always be greater than −1, so that the fractal dimension of surface lies between 2.0 and 3.0. There are certain issues in estimation of fractal dimension for images, a description of these issues is available in Pant *et al.*, [7] and Sun *et al.*, [3].

3. Noise Models

Noise may be defined as unwanted components of image, *i.e.*, it is a part of image. Since noise is inevitable in digital images occurring due to various reasons, it cannot be ignored in image processing and analysis. Noise may occur due to natural reasons, *e.g.*, haze, visibility or it may be due to sensor properties, *e.g.*, camera lens, photographic films or any such reasons. Consequently there are various types of noise models defined, however in present study only 3 models have been used, viz. Gaussian noise, salt and pepper noise and speckle noise. These are very common kind of noise models and used frequently in digital image analysis.

3.1. Gaussian Noise

Gaussian noise is the additive noise which is the most frequently occurring noise in digital images. Gaussian noise is in fact a very common noise which is part of almost all the signals since it occurs naturally and defined by the probability density function (PDF) [21-23]

$$p(z)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{z-\mu}{2\sigma^2}\right) \tag{3}$$

for any random variable z with mean μ and standard deviation σ. A zero mean distribution, *i.e.*, μ=0 has been followed in the study.

3.2. Salt and Pepper Noise

Salt and pepper noise is a kind of tailed noise which refers to various processes resulting a degraded image. It appears as sprinkling black and white dots in the image and hence given the name. It is also called impulse noise or bipolar noise and defined by the PDF [21, 22]

$$p(z)=\begin{cases} P_a & \text{if } z=a \\ P_b & \text{if } z=b \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Here a and b are assumed to be minimum and maximum values of pixels, i.e., for gray level images a=0 and b=255 in general. When $b>a$, the gray level b appears as light dot and a appears as black dot in the image.

3.3. Speckle Noise

Speckle noise is a specific noise occurring in coherent light imaging which is signal dependent [24]. It is a non-Gaussian kind of noise and consequently becomes one of the complex noise models. Since coherent light imaging is used in laser and radar imaging, speckle noise is an inherent property of radar images and thus it has a specified area of interest in noise analysis [24]. Contrary to the most common Gaussian noise, speckle noise is multiplicative in nature and given for any image I as $I_s=I+N\times I$ where I_s is speckled image N is uniform noise characterized by the PDF [21, 22]

$$p(z)=\begin{cases} \dfrac{1}{b-a} & \text{if } a\le z\le b \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where a and b are real numbers and represent the bounds of data. The uniform distribution with a=0 and b=1 has been considered for present study.

4. Data Set Used

In present study three standard digital images have been used. The images are Lena, Cameraman and Peppers, which are 256×256 pixels in size and used in grey level format only. These images are named as Im1, Im2 and Im3 respectively and shown in Figure 1 (a), (b) and (c) respectively. The data set extends for noisy images. Since noise could be added to different extents, various noisy images could be generated accordingly. The parameters affecting noise amount also vary according to the noise model. Three kinds of noises have been considered with different parameters and for each parameter, one noisy image is generated. Each kind of noise with three different input parameters have been used which provides three noisy images per set, thus for each image, nine noisy images are obtained. The varying parameter for Gaussian noise is variance which is chosen as 0.001, 0.005 and 0.01 for noise generation. The varying parameter for salt and pepper noise, *i.e.*, noise density is considered as 0.01, 0.03 and 0.05. Similarly, the parameter for speckle noise, *i.e.*, variance is chosen to be 0.005, 0.01 and 0.04. These three values for each noise are not arbitrary and considered after a careful selection process. Various values of these varying parameters are considered and applied on original images, *i.e.*, Im1, Im2 and Im3 and observations are made so that the noise effect are visible in the images. The lower value produces more noise and these bounds are selected such that noise does not become extreme. This fact can be visualized in generated noisy images, which are shown in Figures 2, 3 and 4 (a)-(c) and their details along with the names and noise parameters are given in Table 1. Only Im1 is displayed for the sake of convenience, however, the effect of noise is similar in each of the images.

Table 1. Noise Parameters Applied to the Images

Noise	Gaussian Noise (μ=0) σ^2			Salt and Pepper Noise Density			Speckle Noise σ^2		
Image	0.001	0.005	0.01	0.01	0.03	0.05	0.005	0.01	0.04
Im1	Im1G1	Im1G2	Im1G3	Im1SP1	Im1SP2	Im1SP3	Im1S1	Im1S2	Im1S3
Im2	Im2G1	Im2G2	Im2G3	Im2SP1	Im2SP2	Im2SP3	Im2S1	Im2S2	Im2S3
Im3	Im3G1	Im3G2	Im3G3	Im3SP1	Im3SP2	Im3SP3	Im3S1	Im3S2	Im3S3

5. Methodology

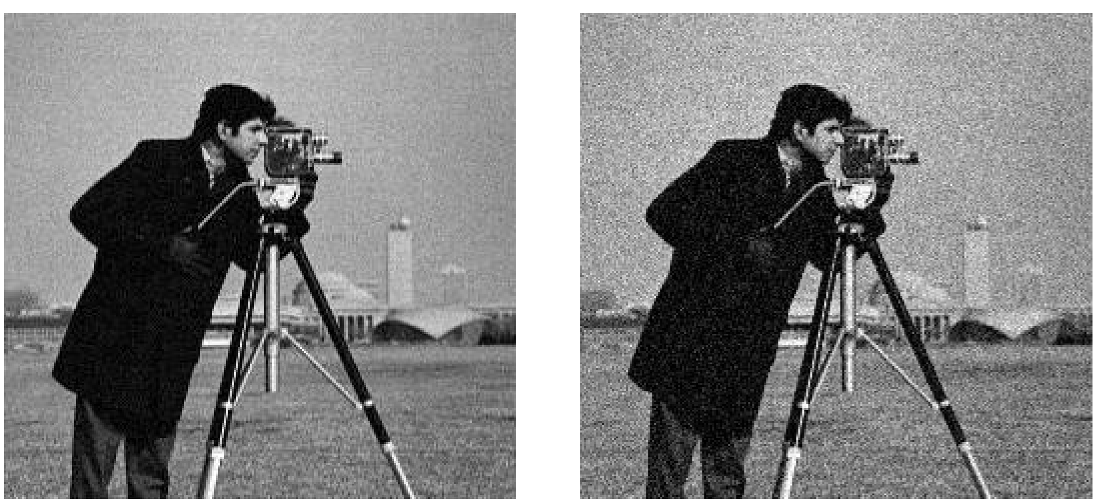
The images Im1, Im2 and Im3 have been used in first step for estimation of global fractal dimension using TPSAM. In second step, local fractal dimension for window size 5, 7, 9 and 11 has been estimated for each of the images, viz., Im1, Im2 and Im3. The values have been stored for further processing for each image and each window size. The image is drawn with pixel values, which are in a specified range, however, when a moving window approach is followed for estimation of fractal dimension in a defined pixel neighbourhood, new images could be generated with these local fractal dimension values. These images drawn with local fractal dimension values are named as fractal images which contain fractal dimension value instead of pixel values. Since, local fractal dimension is estimated in a defined pixel window, each pixel of fractal image is a representation of the properties of this moving window, which is pixel orientation measured as roughness in terms of fractal dimension.

In third step, using the noise models of Section 3, noise is added to each image. Estimation of fractal dimension values is done in noisy image again in global and local way. The global values of fractal dimension for noisy images have been estimated and compared with the value of non-noisy images. The local fractal dimension for noisy images for discussed window size is estimated and compared with the values of non-noisy images. The comparison of fractal dimension values for noisy and non-noisy images is finally discussed.

(a) (b)

(c)

Figure 1. (a) Cameraman: Im1, (b) Lena: Im2, (c) Peppers: Im3



(a) (b)

(c)

Figure 2. Gaussian Noisy Images Generated from Im1

(a)

(b)

(c)

Figure 3. Salt and Pepper Noisy Images Generated from Im1


(a)


(b)


(c)

Figure 4. Speckle Noisy Images Generated from Im1

6. Results and Discussions

The global fractal dimension values of images Im1, Im2 and Im3 are listed in table 2. Since the fractal dimension represents the roughness of image, the roughness value for Im3 is less as compared to Im1 and Im2 since fractal dimension of Im3 is less than fractal dimension of Im1 and Im2. The Table 2 also shows that the value of fractal dimension for Im1 is less than that of Im2 indicating that the roughness of Im2 is higher than that for Im1.

Table 2. Fractal Dimension (Global) Value for Images Im1, Im2 and Im3

Image	*D* (Global)
Im1	2.2815
Im2	2.2997
Im3	2.2439

The fractal dimension for noisy images has been shown in Table 3. In all the images, *i.e.*, Im1, Im2 and Im3 as well as in corresponding noisy images, the global value shows how much distributed the pixels are. The global value in Gaussian noisy images is slightly more than that of non-noisy images, as represented in Table 3. As the variance increases, the noise increases and consequently images become rougher which is clear for the global value of fractal dimension for noisy images. Tables 2 and 3 show that the value increases from 2.2815 to 2.3103 for non-noisy to Gaussian noise image generated by a variance of 0.001. The noise as well as fractal dimension increases in Im1G2 and Im1G3 where variance is 0.005 and 0.01 and fractal dimension is 2.3344 and 2.3549 respectively. A similar explanation can be given for Im2 and Im3 when Gaussian noise is added to them. It is observed from Table 3 that the global fractal dimension for Im2 increases from 2.2997 to 2.3221 which further increases to 2.3541 and 2.3814 for non-noisy images, Gaussian noised image with variance 0.001, 0.005 and 0.01 respectively. For Im3, these values are 2.2439, 2.2799, 2.3201 and 2.3513 respectively.

Table 3. Fractal Dimension (global) for Noisy Images (G1, G2, G3 and others are According To Table 1)

	D (Global)								
	Gaussian Noise			Salt and Pepper Noise			Speckle Noise		
	G1	G2	G3	SP1	SP2	SP3	S1	S2	S3
Im1	2.3103	2.3344	2.3549	2.2994	2.3213	2.3377	2.3039	2.3128	2.3215
Im2	2.3221	2.3541	2.3814	2.3003	2.3400	2.3641	2.3260	2.3432	2.3814
Im3	2.2799	2.3201	2.3513	2.2658	2.3038	2.3244	2.2840	2.3018	2.3730

A similar analysis shows that the salt and pepper noise affects the images in a same way as Gaussian noise does. The fractal dimension value increases with increase in noise which is 2.2994, 2.3213 and 2.3377 for Im1 for noise density 0.01, 0.03 and 0.05 respectively. For Im1, however, global value of fractal dimension, as shown in Table 2 is 2.2815. The effect can also be observed for Im2 and Im3 for salt and pepper noise. Finally, a comparison can be made for speckle noise for all the images using both the tables. It can be concluded from the data available in Table 2 and Table 3 that noise affects the values of fractal dimension and in general increases its value. In form of image surface, it could be deducted that noise increases the roughness of surface, *i.e.*, the pixel values show an enhancement in 3D space due to which fractal dimension increases.

The local fractal dimension estimated with local window size 5, 7, 9 and 11 is estimated for each of the images, *i.e.*, Im1, Im2 and Im3 and the average values are shown in Table 4. The values in Table 4 depict that the average value of local fractal dimension decreases with an increase in local window size. For Im1, the value decreases from 2.2453 to 2.2272 for window size 5 and 11 respectively. Similar pattern is followed by other two images. The fractal images generated for corresponding local windows are shown in Figures 5-7 (a)-(d). It is observed from the fractal images that with an increase in local window size, the fractal images become blurred as few image features are eliminated within moving window.

The process of estimation of local fractal dimension with different local window sizes is repeated for the noisy images sequentially. The average values of local fractal dimension for Gaussian noisy images, *i.e.*, Im1G1, Im1G2 to Im3G3 for window size 5, 7, 9 and 11 are arranged in Table 5. The data of table 5 show that average fractal dimension increases with increase in noise, however the values show a decay when local window size increases. The effect of noise is also visible in fractal images generated from noisy images which are shown in Figure 8 (a)-(d). The local fractal dimension is also estimated for noisy images generated by Im1 with salt and pepper noise and speckle noise each and the average values for these images is displayed in Table 6 and Table 7 respectively. The average value of local fractal dimension for local window 5, 7, 9 and 11 for each of the noisy image mentioned in Table 1 is estimated. The fractal images are shown only for Im1 with each kind of noise in minimum extent, the effect is similar in other images, *i.e.*, Im2 and Im3. The effect of salt and pepper noise on Im1 is displayed in Figure 9 (a)-(d) and that of speckle noise is shown in Figure 10 (a)-(d).

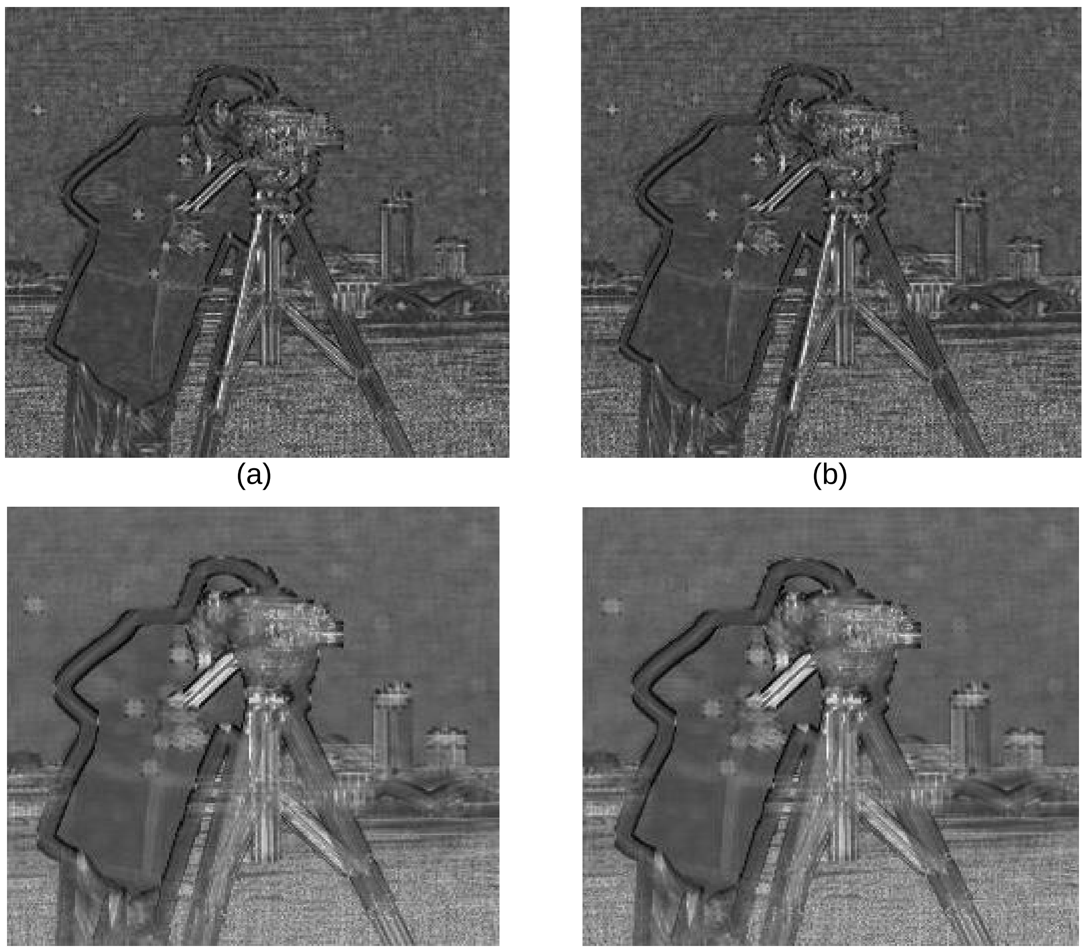


Figure 5. Fractal Images Generated from Im1 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11

Table 4. Average Fractal Dimension (local) for Images Im1, Im2 and Im3 for varying Local Window

	D (Local)			
Image	w=5	7	9	11
Im1	2.2453	2.2447	2.2280	2.2272
Im2	2.2550	2.2550	2.2330	2.2334
Im3	2.2604	2.2602	2.2294	2.2292

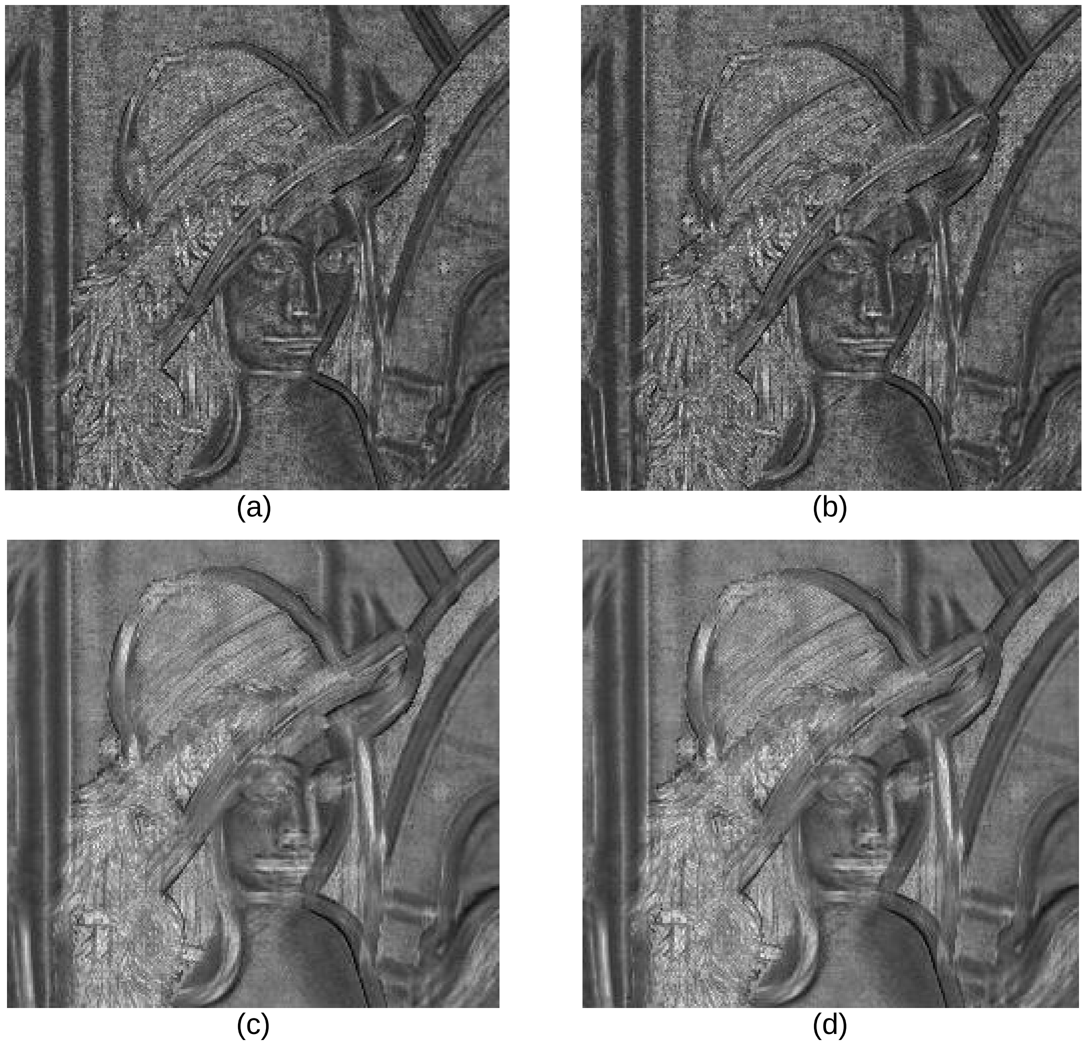
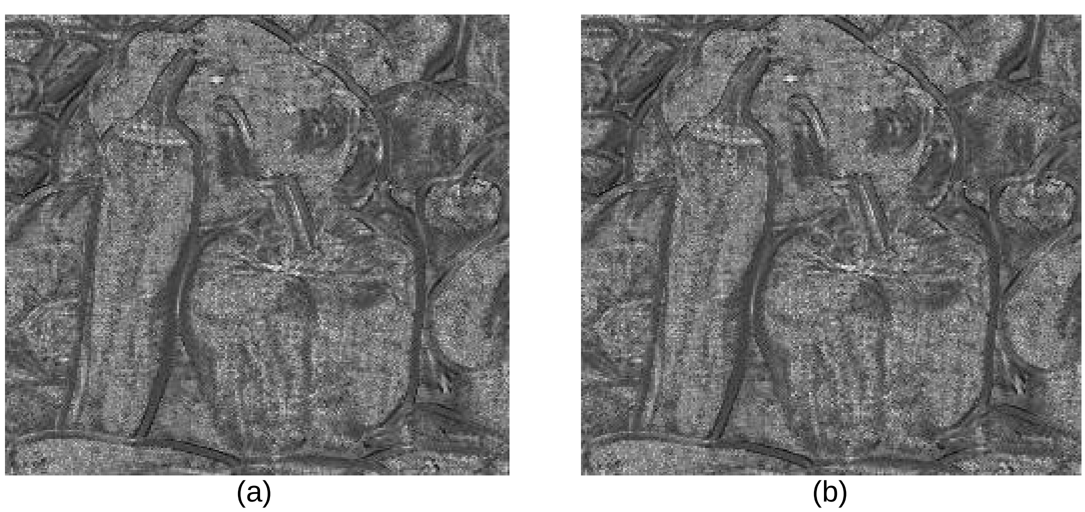


Figure 6. Fractal Images Generated from Im2 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11




(c)


(d)

Figure 7. Fractal Images Generated from Im3 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11

Table 5. Average Value of Fractal Dimension (local) for Gaussian Noisy Images Generated from Im1, Im2 and Im3 for Varying Local Window

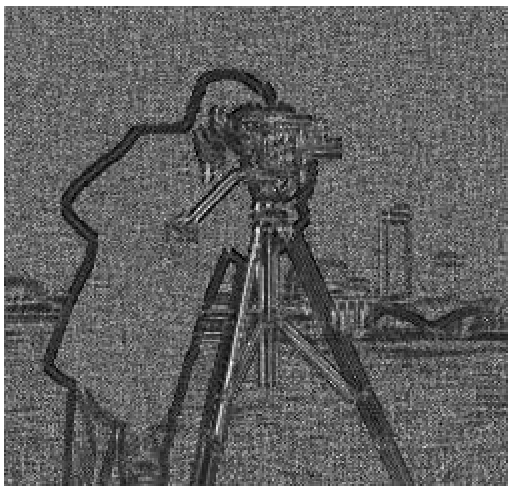
		D		
Image	w=5	7	9	11
Im1G1	2.4236	2.4231	2.3908	2.3901
Im1G2	2.4669	2.4664	2.4413	2.4407
Im1G3	2.4825	2.4822	2.4609	2.4604
Im2G1	2.3820	2.3814	2.3442	2.3438
Im2G2	2.4479	2.4476	2.4183	2.4180
Im2G3	2.4719	2.4716	2.4474	2.4471
Im3G1	2.3684	2.3686	2.3237	2.3238
Im3G2	2.4370	2.4371	2.4006	2.4006
Im3G3	2.4637	2.4637	2.4325	2.4325

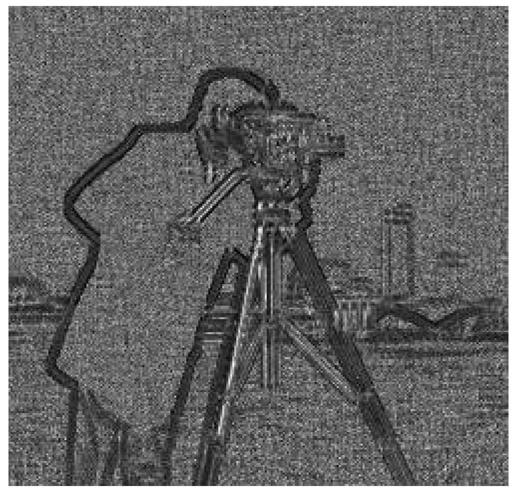
Table 6. Average Value of Fractal Dimension (Local) for Salt and Pepper Noisy Images Generated from Im1, Im2 and Im3 for Varying Local Window

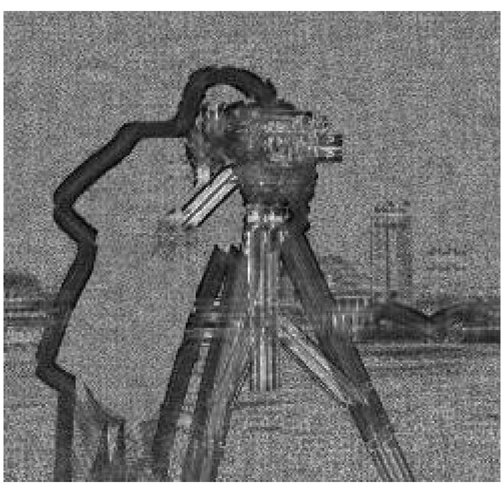
		D		
Image	w=5	7	9	11
Im1SP1	2.3192	2.3188	2.3121	2.3115
Im1SP2	2.4381	2.4375	2.4223	2.4215
Im1SP3	2.5130	2.5121	2.4789	2.4780
Im2SP1	2.3116	2.3113	2.2919	2.2918
Im2SP2	2.4072	2.4066	2.3773	2.3768
Im2SP3	2.4672	2.4664	2.4252	2.4248
Im3SP1	2.3152	2.3150	2.2849	2.2849
Im3SP2	2.3961	2.3964	2.3598	2.3601
Im3SP3	2.4539	2.4537	2.4055	2.4055

Table 7. Average Value of Fractal Dimension (Local) for Speckle Noisy Images Generated from Im1, Im2 and Im3 for Varying Local Window

	D			
Image	w=5	7	9	11
Im1S1	2.3859	2.3848	2.3640	2.3627
Im1S2	2.4039	2.4027	2.3829	2.3816
Im1S3	2.4418	2.4407	2.4226	2.4216
Im2S1	2.3886	2.3881	2.3556	2.3554
Im2S2	2.4126	2.4124	2.3838	2.3835
Im2S3	2.4623	2.4619	2.4403	2.4402
Im3S1	2.3613	2.3611	2.3211	2.3210
Im3S2	2.3888	2.3884	2.3514	2.3512
Im3S3	2.4395	2.4391	2.4113	2.4110


(a)


(b)


(c)

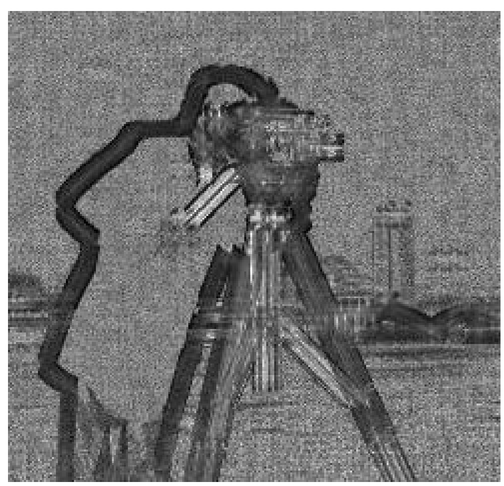

(d)

Figure 8. Fractal Images Generated from Im1G1 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11

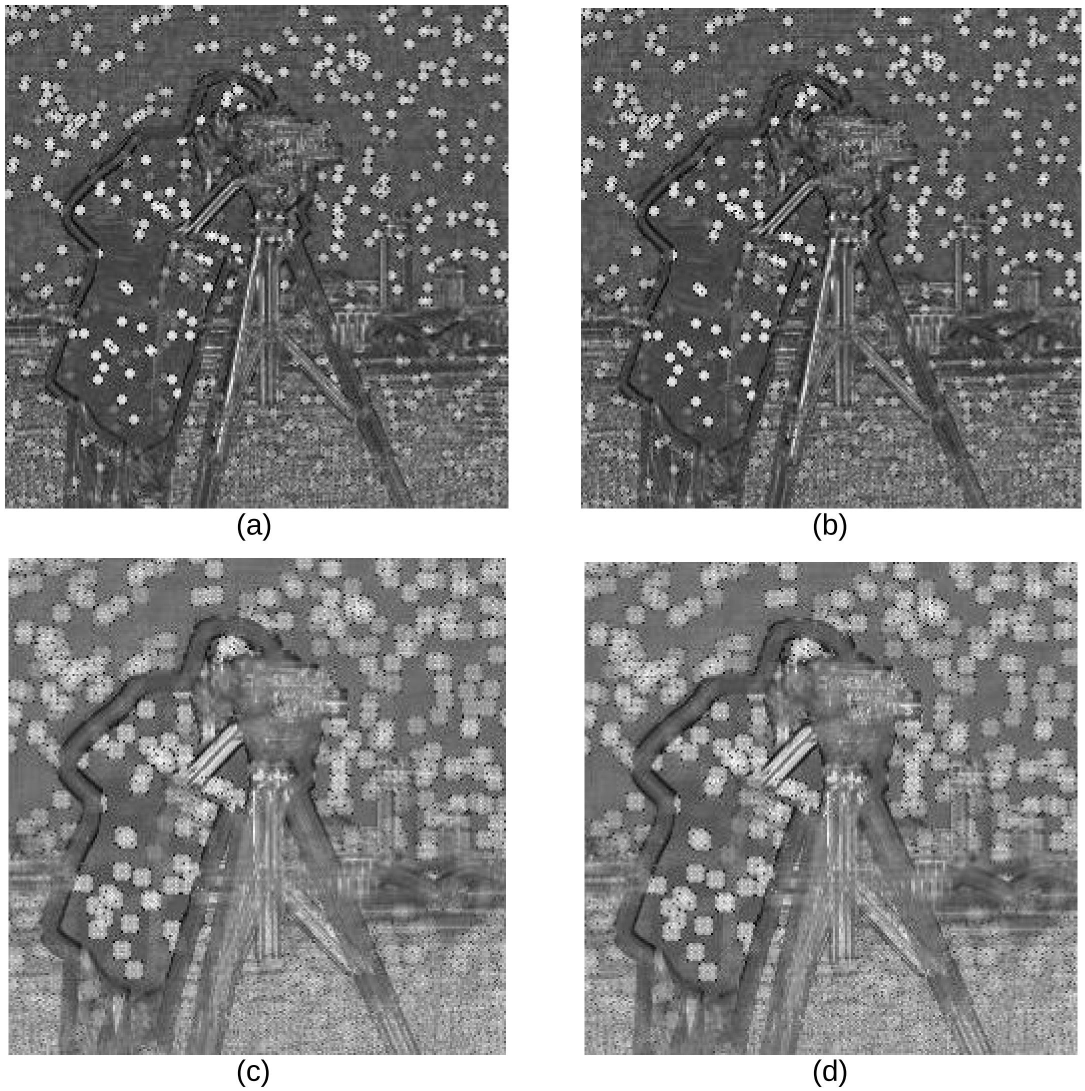


Figure 9. Fractal Images Generated from Im1SP1 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11

(a) (b)

(c) (d)

Figure 10. Fractal Images Generated from Im1S1 for Local Window (a) 5, (b) 7, (c) 9 and (d) 11

On the basis of above analysis, it can be concluded that in noisy images, fractal dimension increases in general. This increment could be treated as an offset, *i.e.*, in non-noisy images, the value of fractal dimension is lower than that of noisy images which differs by this offset value. If the increment in fractal dimension is estimated due to noise effect, it is possible to find the actual value of fractal dimension without removing the noise from the image. This value is sought to be estimated exactly in terms of numeric data and needs to be explored further. This concept can be generalized for various methods used to estimate the fractal dimension.

7. Conclusions

The present study dealt with the effect of various kinds of noise in estimation of fractal dimension of digital images. There are various methods available for estimation of fractal dimension of digital images, but the effect of noise on the estimation methods or fractal dimension values was not reported earlier. This study is important because, the real images of daily life are exuberant in noise and thus dealing with noise is an important challenge for the exact processing of images. One way is to remove the noise first and then go for the processing so that noise effect could be nullified. However, every time this preprocessing is not possible and sometimes it becomes tedious too. It would be a nice idea to work directly on the noisy images provided it is known *apriori* that image contents are noisy and the processing algorithms work on noisy images too. In this way, present study showed that the noise affects the fractal dimension and fractal analysis can also be performed on noisy images. Since, noise creates the pixels to be distorted and in general affects the pixel orientation when considered in 3D space, the fractal dimension of noisy image comes to be slightly higher as compared to non-noisy images. This fact is verified in present study using numeric data as well as by fractal images. The achievement of present study could be highlighted by this fact that noise increases the value of fractal dimension and thus working on noisy images directly for estimation of fractal dimension leads to set an offset value showing the presence of noise in images. In this case the noise removal process could be skipped for estimation of actual value of fractal dimension.

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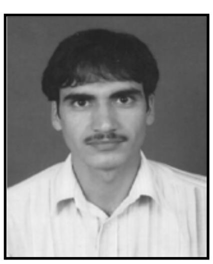

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