An Improved Image Denoising Algorithm based on Shearlet

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Abstract

In allusion to remove Racian noise while lessen the loss of details as low as possible, this paper proposed an filter algorithm which comprehensive utilize Multi-Objective Genetic Algorithm (MOGA) and Shearlet transform based on a Multi-scale Geometric Analysis (MGA) theory. First, it performs a wavelet multi-scale decomposition of image. Then, it builds target function in MOGA by several evaluation methods such as Signal to Noise Ratio (SNR). Third, it uses the MOGA to optimal coefficients of Shearlet wavelet threshold value in different scale and different orientation. Finally, it obtains the composite image by using inverse lifting wavelet transform. Experimental results show tha our proposed new algorithm presented here is more effective in removing Rician noise, and giving better Peak Signal Noise Ratio (PSNR) gains, without manual intervention in comparison with other traditional filters.

Keywords: *Shearlet, MOGA, Rican denoise*

1. Introduction

Digital images are frequently corrupted by Rician noise during image gaining or transmission. That makes noise reduction be one of the key problems in image processing and the basis of image subsequent processing. Preservation of image details and attenuation of noise are the two important of noise reduction but they are contradictory in nature. So, this research emphasis is on the removal of Rician noise while lessening the loss of details as low as possible.

Wavelet transform techniques is a representative of image sparse representation based on the harmonic analysis, it take many factors such as time-frequency localization, multi-scale characteristic and spare representation into consideration of target function. And accordingly, compare with traditional algorithms, it have superior and provide more satisfactory results while preserving image details. However, wavelet transform cannot achieve the optimal spare approach for images contained higher dimension singularity. To overcome the limitation, multi-scale geometric analysis theory is proposed, and based on it, a series of method sprang out, for example, ridgelet [1], curvelet [2], contourlet [3], One of the most successful construction based on this idea are the curvelets of Candes and Donoho, that achieve an (almost) optimal approximation property for 2–D piecewise smooth functions with discontinuities along C^2 curves.

Recently, Labate *etc.*, described a new class of multidimensional representation systems, called Shearlet. One advantage of this approach is that these systems can be constructed using a generalized multi-resolution analysis and implemented efficiently using an appropriate version of the classical cascade algorithm[4-9].

Simple threshold denoising method of Shearlet transform can get good performance, for its multi-scale and multi-direction characteristic, image sparse representation. However, there is a lot to be improved. The simple threshold denoising algorithm does not take energy distribution of different scales and different directions into consideration, eventually result in killing excessively the coefficient, so the detail of image lost. Based on it, Hui Sun etc[10] proposed two sub-swarm exchange particle swarm optimization, it uses particle swarm optimization algorithm of adaptive to find optimal threshold of the highest PSNR.

Based on the achievement in the past, this paper proposed a new image filter arithmetic based on MOGA [11, 12] and Shearlet Transform. It has three characteristics, it changes the simple hard threshold into a soft threshold; it builds target function in MOGA by several evaluation methods such as Signal to Noise Ratio (SNR); it uses the MOGA to optimal coefficients of Shearlet wavelet threshold value in different scale and different orientation.

The remaining paper is organized as follows. Section 2 introduces related theories. Section 3 presents our algorithm, including workflow. Section 4 provides the experimental results of our proposed algorithm. Finally, Section 5 concludes this paper.

2. Related Theories

2.1. Rician Noise

Noised digital image v can be defined as $v_{(i)} = u_{(i)} + n_{(i)}$, here, $u_{(i)}$ is original image pixels, $n_{(i)}$ is noised pixels. When images are computed using the magnitude of a single complex raw data, its distribution can be modeled as a Rician model [13,14].

$$p(m) = \frac{m}{\sigma_n^2} e^{\frac{m^2+A^2}{2\sigma_n^2}} I_0\left(\frac{Am}{\sigma_n^2}\right) \tag{1}$$

Here, σ^2 is the standard deviation (STD) of Gaussian noise, A is the amplitude of the signal without noise, x is the value in the magnitude image and I_0 is the 0th order modified Bessel function. This model is used by the majority of the noise estimation methods.

When SNR is small enough (*i.e.*, SNR=0), the Rician distribution is considerate as a Rayleigh distribution.

$$p(m) = \frac{m}{\sigma_n{}^2} e\left(\frac{m^2}{\sigma_n{}^2}\right) \tag{2}$$

When SNR is high (*i.e.*, SNR>3), the Rician distribution is approximated as a Gaussian distribution.

$$p\ (m) = \frac{1}{2\pi\sigma^2} e\left(-\frac{\left(m^2 - \sqrt{A^2+\sigma_n^2}\right)^2}{2\sigma_n^2}\right) \tag{3}$$

2.2. Shearlet Transform

Labate *etc*., [4-9] proposed Shearlet transform based on wavelet. In dimension n=2， affine system:

$$\Psi_{AB}(\psi) = \left\{\psi_{j,l,k}(x) = \left|\det A\right|^{j/2} \psi\left(B^l A^j x - k\right) : j,l \in \mathbb{Z}, k \in \mathbb{Z}^2\right\} \tag{4}$$

Here, $\psi \in L^2(R^2)$, A, B is 2*2 invertible matrices with $|\det B| = 1$.

If $\psi_{AB}(\psi)$ satisfied Parseval $L^2(R^2)$, then, Those elements of $\psi_{AB}(\psi)$ are called composite wavelets.

Shearlet is an special example of $L^2(R^2)$, for only when

$$A = A_0 = \begin{pmatrix} 4 & 0 \\ 0 & 2 \end{pmatrix}, B = B_0 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix},$$

here $A = A_0$ the anisotropic dilation matrix, $B = B_0$ the shear matrix.

For $\xi = (\xi_1, \xi_2) \in R^2, \xi_1 \neq 0$, when $\psi^{(0)}, \hat{\psi}_1, \hat{\psi}_2$ satisfy

$$\begin{cases} \hat{\psi}^{(0)}(\xi) = \hat{\psi}^{(0)}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_1)\hat{\psi}_2(\xi_2/\xi_1) \\ \hat{\psi}_1, \hat{\psi}_2 \in C^\infty(\hat{\mathbb{R}}), \operatorname{supp}\hat{\psi}_1 \subset [-1/2, -1/16] \cup [1/16, 1/2], \operatorname{supp}\hat{\psi}_2 \subset [-1, 1] \\ \sum_{j \geq 0} \left|\hat{\psi}_1(2^{-2j}\omega)\right|^2 = 1 \ for\ |\omega| \geq 1/8,\ j \geq 0 \\ \sum_{l=-2^j}^{2^j-1} \left|\hat{\psi}_2(2^j\omega - l)\right|^2 = 1 \ for\ |\omega| \leq 1 \end{cases} \tag{5}$$

Then, we get:

$$\sum_{j \geq 0} \sum_{l=-2^j}^{2^j-1} \left|\hat{\psi}^{(0)}(\xi A_0^{-j} B_0^{-l})\right|^2 = \sum_{j \geq 0} \sum_{l=-2^j}^{2^j-1} \left|\hat{\psi}_1(2^{-2j}\xi_1)\right|^2 \left|\hat{\psi}_2(2^j \xi_2/\xi_1 - l)\right|^2 = 1 \tag{6}$$

Then, $\{\hat{\psi}^{(0)}(\xi A_0^{-j} B_0^{-l})\}$ form a tiling of the set $D_0 = \left\{(\xi_1, \xi_2) \in \hat{\mathbb{R}}^2 : |\xi_1| \geq 1/8, |\xi_2/\xi_1| \leq 1\right\}$

From the condition on the support of $\hat{\psi}_1, \hat{\psi}_2$, it is easily deduced that $\hat{\psi}_{j,l,k}$ have frequency support contained in the set.

$$\operatorname{supp}\hat{\psi}_{j,l,k}^{(0)} \subset \left\{(\xi_1, \xi_2) : \xi_1 \in \left[-2^{2j-1}, -2^{2j-4}\right] \cup \left[2^{2j-4}, 2^{2j-1}\right], \left|\xi_2/\xi_1 + l2^{-2j}\right| \leq 2^{-j}\right\},$$

Thus, every element in $\psi_{j,l,k}$ is supported on a pair of trapezoids of approximate size $2^{2j} \times 2^j$, oriented along lines of slope $l2^{-j}$.

For $L^2(D_1)^\vee$, he re D_1 is the vertical cone, when formula (7) was satisfied:

$$\begin{cases} D_1 = \left\{(\xi_1, \xi_2) \in \hat{\mathbb{R}}^2 : |\xi_2| \geq 1/8, |\xi_1/\xi_2| \leq 1\right\} \\ A_1 = \begin{pmatrix} 2 & 0 \\ 0 & 4 \end{pmatrix}, B_1 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \\ \hat{\psi}^{(1)}(\xi) = \hat{\psi}^{(1)}(\xi_1, \xi_2) = \hat{\psi}_1(\xi_2)\hat{\psi}_2(\xi_1/\xi_2) \end{cases} \tag{7}$$

Then, collection $\left\{\psi_{j,l,k}^{(1)}(x)=2^{3j/2}\psi^{(1)}\left(B_1^l A_1^j x-k\right): j\geq 0, -2^j \leq l \leq 2^j-1, k\in\mathbb{Z}^2\right\}$ is a Parseval frame for $L^2(D_1)^{\vee}$.

3. Proposed Algorithm

3.1. Threshold Rule

Threshold rule is the most important problem in image denoising of transform domain, and the hard-threshold and the soft-threshold approach are two options. Donoho[18] proposed a threshold rule:

$$\delta=\sigma\sqrt{2\ln(N)} \tag{8}$$

here, N pixels number of image, σ is noise level.

Research shows that Donoho threshold is optimal threshold limit, is not the optimal threshold. Considered this, Donoho etc [15] proposed a improved threshold rules:

$$\delta_k=\sigma\sqrt{2\ln(N)}*2^{(k-K)/2} \quad ,k=0,1,\cdots,K \tag{9}$$

As many researchers point out [10, 13], Formula (8) did not considered energies of sub-wavelets in different direction while in same scale, and this imperfection will lead to coefficients be stifled too much.

Considering the variability of image content, and Shearlet transformation of multi-scale and multi-direction characteristic, a novel threshold selection rule is proposed based on Shearlet transform multi-scale and multi-direction, its rule is as following.

Comprehensive considering complexity of image, multi-scale and multi-direction characteristic of Shearlet transform, this paper proposed an adaptive threshold rule.

$$\delta_{k,j}=Sigmoid\left(\sigma\sqrt{2\ln(N)}*2^{(k-K)/2}\right), \quad k=0,1,\cdots,K \tag{10}$$

$$Sigmoid=\frac{1}{1+e^{-v}} \tag{11}$$

Here, Sigmoid function is adopted to build our rules. The Sigmoid curve is a mathematical concept which has been widely used to model the natural life cycle of many things for its derivative is continuous and with higher accuracy. K is scale level. j is the jth direction under kth scale level.

3.2. Target Function

We build target function in MOGA by Noise Ratio (SNR), Mean Square Errors (MSE).

Signal to Noise Ratio (SNR) can be defined as

$$SNR=10\lg\frac{\sum_{i=0}^{M}\sum_{j=0}^{N}A(i,j)^2}{\sum_{i=0}^{M}\sum_{j=0}^{N}(A(i,j)-O(i,j))^2} \quad (12)$$

Here, O is original image with size of M×N pixels, A is filtered image of noised image, (i,j) is coordinates of pixels.

3.3. Proposed Model

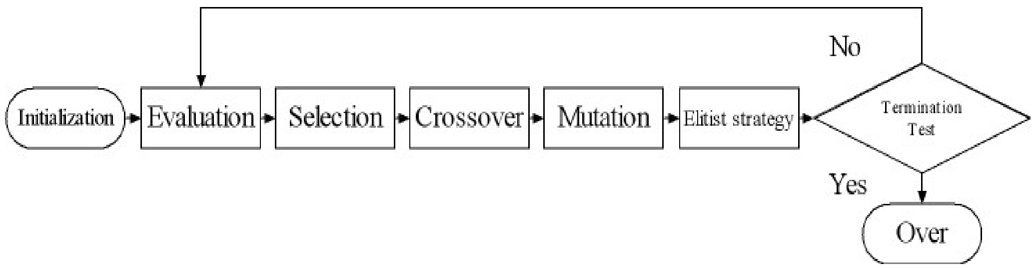


Figure 1. Workflow of Proposed Algorithm

The most critical problem which lies in our optimal filtering performance study is that, under optimization criterion, how to determine coefficients $v,\delta_{k,j}$ not only considering energy of sub-wavelets in different scale but also in different direction.

Here, we proposed our algorithm which adopt MOGA algorithm to determine coefficients $v,\delta_{k,j}$ of each sub-wavelet in different scale and direction of Shearlet transform, intend to get optimal filtering performance.

Our algorithm works as following:

Step 1. (*Initialization*): Generate an initial population containing N_{pop} strings where N_{pop} is the number of strings in each population. These strings contains weight coefficients of SNR, MSE, weight coefficients $\delta_{k,j}$ of Shearlet sub-wavelets, *v* of S function and other parameters in MOGA we needs;

Step 2. (*Ev*aluation):

[1].Use Shearlet Transform to decompose target image;

[2].Sub-wavelets multiply by a weight coefficients $\delta_{k,j}$;

[3].Filtering sub-wavelets by threshold rule;

[4].Reconstruct image by filtered sub-wavelets;

[5].Update a tentative setoff Pareto optimal solution.

Step 3. (*Selection*): Calculate the fitness value of each string using the random weights in (3).Select a pair of strings from the current population according to the following selection probability.

Step 4. (*Crossover*): For each selected pair, apply a crossover operation to generate two new strings. N_{pop} new strings are generated by the crossover operation.

Step 5. (*Mutation*): For each bit value of the strings generated by the crossover operation, apply a mutation operation with a prespecified mutation probability.

Step 6. (*Elitist strategy*): Randomly remove N_{elite} strings from the set of N_{pop} strings generated by previous operations, and replace them with N_{elite} strings randomly selected from tentative set of Pareto optimal solutions.

Step 7. (*Termination Test*): If one stopping condition in following is satisfied, go to Step8; if not, return Step2.

- Maximum iterations exceeded;
- The optimal target value is achieved.

Step 8. (*Algorithm termination*): Exit optimal algorithm.

4. Experimental Results and Analysis

4.1. Evaluation index

Peak Signal to Noise Ratio (PSNR) is defined as:

$$PSNR=10\lg\frac{255^2}{\frac{1}{M\times N}\sum_{i=0}^{M}\sum_{j=0}^{N}(A(i,j)\text{-}O(i,j))^2} \qquad (13)$$

Here, O is original image with size of M×N pixels, A is filtered image of noised image，(i,j) is coordinates of pixels.

4.2. Experimental Results

In order to verify the validity of the algorithm, this paper designed two kinds of experimental methods to verify its effectiveness. One is use objective data such as PNSR to objective analyzed its performance; another is make us able to obverse filtering performance directly by naked eyes [16-18].

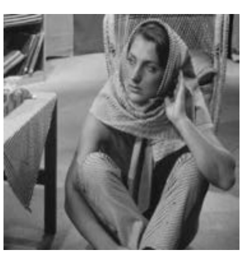
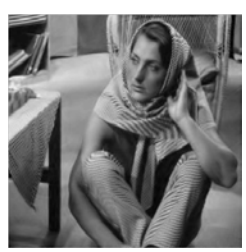
			
(a).Lena	(b). $\sigma=0.05$	(c).Filter (b) by Shearlet	(d). Filter (b) by proposed algorithm
			
(e).Barbara	(f). $\sigma=0.15$	(g). Filter (f) by Shearlet	(h). Filter (f) by proposed algorithm

Figure 2. Experiment Images in Different Noise Level and Different Algorithm

In this experiment, we use the new denosing algorithm to the standard images lena, Baraba in difference noise level and listed results in Table 1. As we have seen from Table 1, PSNRs of proposed algorithm (Shearlet-MOGA) is higher than PSNRs of classical Shearlet algorithm and its performance will be better with noise level increased.

Table 1. Filtering Results to Lena, Baraba

image	σ (%)	PSNR	
		Shearlet	Shearlet-MOGA
Lena	10	34.37	35.13
	20	31.80	33.12
	30	29.23	30.09
Baraba	10	33.17	33.11
	20	29.42	29.30
	30	26.33	27.54

Figure.2 (a) is the original Lena. Figure 2 (b) is Lena noised by 5% Rician noise. Filtered Figure 2 (b) by classical Shearlet algorithm, we got Figure 2 (c). Figure 2 (d) is the output of that we did filtering work to Figure 2 (b) by proposed algorithm.

Figure 2 (e) is the original Baraba. Figure 2 (f) is Baraba noised by 15% Rician noise. Filtered Figure 2 (f) by classical Shearlet algorithm, we got Figure 2 (g). Figure 2 (h) is the output of that we did filtering work to Figure 2 (f) by proposed algorithm.

Through classical Shearlet algorithm and proposed algorithm's filtering performance comparisons, our proposed algorithm could effectively remove the noise from degraded image of Rician noise with unknown intensity level and protect the image detail better at the same time. To MRI image we pay particular attention, experiments image show that our algorithm has excellent performance in background. After strict analysis, we concluded that our algorithm retained the consistent component of low frequency in frequency domain by low-pass filtering, and background.

5. Conclusions

In allusion to remove Rician noise while lessen the loss of details as low as possible, this paper proposed a new image filter arithmetic based on MOGA and classical Shearlet Transform. It builds target function in MOGA by several evaluation methods such as Signal to Noise Ratio (SNR); and uses MOGA to find optimal coefficients of Shearlet wavelet threshold value in different scale and different orientation. Computer simulations and their results are given to verify the efficiency of this algorithm. That our algorithm has excellent performance.

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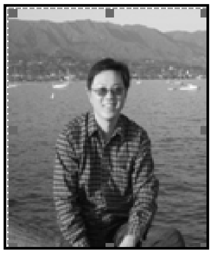

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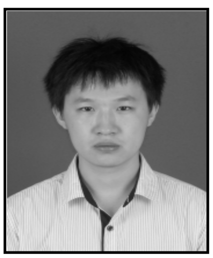

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