A Novel Image Reconstruction Algorithm Based on Compressed Sensing for Electrical Capacitance Tomography

Chen Deyun, Li Zhiqiang, Gao Ming, Wang Lili and Yu Xiaoyang

School of Computer Science and Technology, Harbin University of Science and Technology, Harbin, 150080, China
chendeyun@hrbust.edu.cn

Abstract

According to the image reconstruction accuracy influenced by the "soft field" nature and the limited projection data in electrical capacitance tomography, based on the working principle of the electrical capacitance tomography system, a Novel image reconstruction algorithm based on compressed sensing is proposed in the paper. The method based on ART (algebra reconstruction technique) organically combines the gradient sparse of image and ART, and reduces the norm of image gradient with full-variational method, and improves the accuracy and speed of image reconstruction. Experimental results and simulation data indicate that the imaging accuracy is markedly improved, and the image is closed to the prototype. This new algorithm presents a feasible and effective way to research on image reconstruction algorithm for Electrical Capacitance Tomography System.

Keywords: *electrical capacitance tomography,, image reconstruction, ART algorithm, compressed sensing*

1. Introduction

The flow imaging is a new technology developed rapidly in recent years, which has great developmental potential and wide industrial application prospect. The Electrical Capacitance Tomography (ECT) has many distinct advantages such as low cost, wide application field, simple structure, non-invade, better safety and so on [1]. It is the research focus and the mainstream development in flow imaging technique at present. Because of the inherent nonlinear characteristics of the Electrical capacitance tomography system, the number of independent capacitance measurements (projection data) is very limited which is far less than the number of pixels of the reconstructed image, and the inversion problem does not exist analytical solutions. Because of the nonlinear and "soft field" nature the stability of the solution by the ECT system is bad and there is a serious morbid, so the image reconstruction is more difficulty [2, 3].

The successful application of ECT measurement is largely dependent on the accuracy and speed of the imaging algorithm. Now there are the more commonly used methods for ECT image reconstruction, such as linear back projection algorithm (LBP), regularization method, Landweber iteration method, projection Landweber iteration method, conjugate gradient method (CG) [4-8, 12] and so on. The characteristic of the LBP algorithm has simple and fast reconstruction speed. However, because of the relatively poor image quality, strictly speaking, the algorithm is only a qualitative algorithm. Regularization parameter choice for regularization method has greater impact on image quality, so that it generally adopts the empiric value. Particularly Pre-iteration algorithm (OIOR) which developed on the basis of the Landweber iteration method has the same image quality with repeatedly Landweber iteration when it has the same speed with LBP, but the spatial resolution is still not over the

Landweber method. The projection Landweber iteration method can significantly improve the stability of the iteration and effectively control the noise, but usually require a large number of iterations to achieve satisfactory results for the complex flow pattern, so that its application is limited. Conjugate gradient method is suitable the situation that the coefficient matrix is symmetric positive definite. For the simple flow pattern the method takes short imaging time and fast convergence, but for the complex flow patterns, the results are unsatisfactory. Neural network algorithm used in ECT in essence is a pattern recognition method, and the successful application of the method depends on the rational structure of the neural network structure and completely training samples. Access to complete training sample is more difficult due to the randomness and complexity of the multiphase flow mid-stream change, and there are some difficulties in determining the structure of the network in the actual application process. In this paper, the image reconstruction algorithm based on compressed sensing is introduced. This method based on algebraic reconstruction algorithm (ART), combines the image gradient sparse and the projection data, reduces the image gradient norm by total variation method, has more accurate imaging results. The method provides a new way of thinking for ECT image reconstruction.

2. Basic Principle of ECT System

2.1. Structure ECT System

The principle of a typical 8-electrode system is shown as Figure 1. The ECT system is generally composed of three basic parts, such as capacitance sensor, capacitance data acquisition system, and image reconstruction computer. Its basic principle is to use the multiphase flow media with different dielectric constant to obtain the capacitance value of each electrode from installed in insulated pipeline outer wall of the capacitance sensor. The computer uses these capacitance values which reflect the distribution of the dielectric constant in pipeline to obtain the two-phase flow concentration distribution map on the pipeline cross-section, according to certain algorithms for image reconstruction, namely intuitive access to distribution information of the multiphase fluid phase.

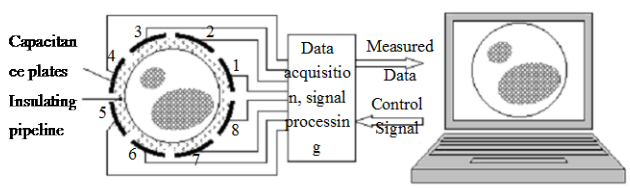


Figure 1. Composed of ECT System

2.2. Measurement Principle

Due to each phases medium of two-phase flow with different dielectric constant, when the change of the two-phase flow mixed fluid equivalent dielectric constant caused by the change of the each phases species concentrations and their distribution, capacitance measurement values will change accordingly. And the size of the capacitor value reflects the size and distribution of the two-phase fluid phase concentration. Multiple capacitance measurement values of each phase distribution of two-phase flow can be obtained by mutual combination between the electrodes of the sensor with multi-electrode array of capacitive sensors. Suitable image reconstruction algorithm for the projection data has been adopted based on the principle of medical CT. It can be reconstructed image which react the two-phase flow phase distribution of the pipeline or a cross-section of the device, enabling the measurement of flow visualization or two-phase flow parameters.

M is the total number of independent electrode pair, which is available from the system with the N-electrode plate.

$$M = C_N^2 = \frac{N \cdot (N-1)}{2} \tag{1}$$

In this paper, the typical 8-electrode capacitance sensor is the research object, which can be obtained 28 independent measured values. They are C_1, C_2, …, C_{28}. The measurement value between the plate capacitance is composed of two parts which are the capacitance formed by the dielectric between the plates in the pipeline part and the capacitance formed by the shield dielectric between the pipeline outside and the shield. Through the rational design of capacitive sensors, it can enable the second part of the capacitance small, and the capacitance can be neglected. Suppose the dielectric constant of continuous phase and discrete phase of the two-phase is ε_1 and ε_2 respectively, and the discrete phase is evenly distributed in the continuous phase. The volume of the two media are V_1 and V_2, and ε is the equivalent dielectric constant of the two-phase flow.

$$\varepsilon = (V_1 / V) \cdot \varepsilon_1 + (V_2 / V) \cdot \varepsilon_2 \tag{2}$$

Where V is the total volume of the two-phase flow, V= $V_1 + V_2$. Thus,

$$\varepsilon = [(V - V_2)/V] \cdot \varepsilon_1 + (V_2 / V) \cdot \varepsilon_2 = \varepsilon_1 + (V_2 / V) \cdot (\varepsilon_2 - \varepsilon_1) \tag{3}$$

β is the discrete phase concentration.

$$\beta = V_2 / V \tag{4}$$

In this way, C which is attainable capacitance measuring is a function of equivalent permittivity ε.

$$C = K \cdot \varepsilon = K \cdot g(V_2 / V) = K \cdot g(\beta) \tag{5}$$

According to electricity principle, ignoring each phase distribution changes of pipeline radial multiphase flow and the impact of the shield, the capacitor C can be expressed for between any two electrodes i and j from 8 electrode array capacitance sensor.

$$C_{i,j} = \iint_D \varepsilon(x, y) \cdot S_{i,j}(x, y, \varepsilon(x, y)) dxdy \tag{6}$$

Where j = 1,2,…,28, D is the pipeline cross-section, $\varepsilon(x, y)$ represents the distribution function of the dielectric in the pipeline cross-section, $S_{i,j}(x, y, \varepsilon(x, y))$ is the sensitivity distribution function of the capacitor $C_{i,j}$, namely the sensitivity of the capacitance $C_{i,j}$ to the media on point (x, y).

If define the gray-scale of media distribution image pixels within the pipeline as:

$$f(x,y)=\frac{(\varepsilon(x,y)-\varepsilon_1)}{(\varepsilon_2-\varepsilon_1)}\ ,\ f(x,y)\in[0,\ 1] \tag{7}$$

The definition of the image gray directly combines media distribution with image gray. More importantly, equation (3) and (4) shows that.

$$f(x,y)=\frac{(\varepsilon(x,y)-\varepsilon_1)}{(\varepsilon_2-\varepsilon_1)}=\frac{V_1}{V}=\beta \tag{8}$$

Where V_1 and V is respectively the discrete phase and multiphase flow volume at the point (x,y). So defined gray f(x,y) exactly react the discrete phase holdup at the point (x,y). So it can calculate the concentration of multiphase flow through the reconstruction of multiphase flow pipeline cross-section image.

2.3. Mathematical Principle of Image Reconstruction

The According to the micro-element theory which can calculate sensitive field distribution information with higher accuracy approximately, the mathematical model can be obtained as follows.

$$P=W\cdot F \tag{9}$$

Where F= $[f_1, f_2,\ldots, f_m]^T$ is grey vector, and it represents the gray of m element pixels within the entire pipeline. $f_i\in[0,\ 1]$ is the grey of micro-unit i. P=$[p_1, p_2,\ldots, p_n]^T$ (p_j = C_{rj} (j=1…n))is the projection vector, and it represents n projections ($n=28$ in the 8-electrode system). W ($W(j,i)=S_{rj}(i)\cdot\delta_i$) is the weight coefficient matrix of the grey vector related to sensitive field distribution within pipeline.

The matrix equations which are shown as image reconstruction model (9) expands the form of linear equation.

$$\begin{aligned} p_1 &= w_{11}f_1+w_{12}f_2+\cdots+w_{1m}f_m \\ p_1 &= w_{21}f_1+w_{22}f_2+\cdots+w_{2m}f_m \\ p_N &= w_{N1}f_1+w_{N2}f_2+\cdots+w_{Nm}f_m \end{aligned} \tag{10}$$

Where N=28 is the number of projections. m=100 is the number of pixel.

According to equation (10), image reconstruction problems are transformed to linear equations solving problems. ART algorithm achieves image reconstruction which is not through directly solving the matrix equation, but through the iterative solution of equations (10). In ART algorithm grey vector $F=[f_1, f_2,\ldots, f_m]^T$ which constituted by E pixels can be regarded as a point of m-dimensional and each projection equation in equations (10) can be regarded as a hyperplane. When equations (10) have unique solution, all these hyperplane intersect at one point. This point is the solution of equations, and the solution process of the corresponding local equations comes down to iterative solve the intersection of the hyperplane. The iterative solving processes for algorithm are as follows.

(1) Initial value. Given the image grey initial estimate $F^{[0]}$ ($F^{[0]} = [f_1^{[0]}, f_2^{[0]}, \cdots, f_m^{[0]}]^T$) of the reconstructed image as F[0].

(2) The initial value F is the vector of the m-dimensional space, and $F^{[1]}$ can be obtained when the initial value is projected to the hyperplane which is determined by the first projection equation of the equations (10). Complete iteration through the following ways.

$$F^{[1]} = F^{[0]} - \frac{(w_1^T F^{[0]} - p_1)}{w_1^T w_1} w_1 \tag{11}$$

Where vector ($w_1 = [w_{11}, w_{12}, \cdots, w_{1m}]^T$) is the coefficient (weight coefficient) of the first projection equation in equations (10).

(3) $F^{[2]}$ can be obtained when projection value $F^{[1]}$ is projected to the hyperplane which is determined by the second projection equation of equations (10). This process is repeated. $F^{[i]}$ is the projection value on the i-th hyperplane from $F^{[i-1]}$ which is on the i-1th hyperplane.

$$F^{[i]} = F^{[i-1]} - \frac{(w_i^T F^{[i-1]} - p_i)}{w_i^T w_i} w_i \tag{12}$$

Where $w_i = [w_{i1}, w_{i2}, \cdots, w_{im}]^T$. We can get the first round of the projection results $F^{[N]}$ when the hyperplane determined by the last projection equation of the equations (10) in the projection process.

(4) Begin the second round projection iteration when $F^{[N]}$ is the initial iteration value, namely, $F^{[N]}$ is projected to the first hyperplane again, and then repeat the third step projection iterative process to obtain the second round results $F^{[2N]}$. Cycle repeated, so the vector sequence $F^{[N]}$, $F^{[2N]}$, $F^{[3N]}$, …, can be obtained. If the unique solution exists, the sequence finally converges to one point. The point is the grey estimation of reconstructed image.

In the experiment, it made the following filter adjustment in the iterative process when some pixel grey of the iteration results is over the rang [0, 1],

$$f_i = \begin{cases} h1 & if \quad f_i > h1 \\ f_i & if \quad h0 \le f_i \le h1 \\ h0 & if \quad f_i < h0 \end{cases} \tag{13}$$

Where $h0$ is the lower limit of grey, slightly less than 0, and it takes -0.1 in the experiment. $h1$ is the allowable upper limit of the pixel grey, slightly larger than 1, and it takes 1.1 in the experiment. During the imaging process, setting the threshold is V. If the pixel grey level is not less than the threshold V, the pixel grey scale is 1, otherwise the grey scale is 0.

$$g_i = \begin{cases} 1 & if \quad f_i \geq V \\ 0 & if \quad f_i < V \end{cases} \tag{14}$$

Where g_i is the display grey scale of the pixel i.

3. Image Reconstruction Algorithm based on Compressed Sensing

3.1. Compressed Sensing Principle

According to compressive sensing principle, if the signal is compressible or sparse in a transform domain, the high-dimensional from an observation matrix which is not related to the transform-based signal can be projected to a low dimensional space. By solving an optimization problem, reconstruct the original signal with the high probability from a spot of projection, and it can prove that the projection contains enough information on the reconstructed signal. During the image reconstruction process of the oil-water two-phase flow on ECT, it requires high grey values of continuous phase (water) distribution, far higher than the requirement of the flow pattern identification image reconstruction. During the image reconstruction, the continuous phase information is concentrated in some areas within the pipeline, and other regions can be expressed by 0 in the coefficient matrix. Moreover it has the corresponding ECT image reconstruction algorithm. Therefore, the image reconstruct with compressed sensing principle. Signal or image reconstruction with compressed sensing principle must be the following three conditions.

1. The original or image is represented sparsely in a transform domain;
2. Each column of the random projection matrix and the basis function of sparse transformation are irrelevant;
3. Corresponding reconstruction algorithm.

Image has been reconstructed by the compression sensing principle known measurement matrix $\Phi \in \mathrm{R}^{M \times N} (M << N)$ and linear measurement $y \in R^M$ of an unknown signal x in the matrix.

$$y = \Phi x \tag{15}$$

The above equation can also be regards as the linear projection which has been obtained by the original signal x on the Φ, considering x has been reconstructed by y. The dimension of x is higher than that of y dimension, so the above equation will has infinitely many solutions, namely the problem is not suitable to reconstruct the original signal. If the original signal x is K sparse and y and Φ meet certain conditions, the signal x can be accurate reconstruction though solving the problem of optimal l0 norm by the measured value y [3].

$$\hat{x} = argmin\|x\|_0 \quad s.t. \quad \Phi x = y \tag{16}$$

Where $\|x\|_0$ is the l0 norm of vector, and it shows the number of nonzero elements in the vector x. If the sparse representation $\hat{x}$ of f has already been obtained, further accurately reconstruct the original signal $\tilde{f}$ by the transformation-based Ψ by the following formula.

$$\tilde{f} = \Psi \hat{x} \tag{17}$$

The above equation is the core of compressed sensing. The compressed sensing principle solves optimization problems though small amount of linear measurement. The compressed representation y of the signal f has been obtained directly.

3.2. Minimum Full-variational Method

Image reconstruction is an important part of compressed sensing. There are many kinds of algorithm to obtain the suboptimal solution based on compressed sensing principle, mainly including minimum l1 norm method, matching tracing series algorithm, iterative threshold method, and minimum full-variational method to treat specially the problem of two-dimensional image and so on.

4. Author Name(s) and Affiliation(s)

Author names and affiliations are to be centered beneath the title and printed in Times New Roman 12-point, non-boldface type. Multiple authors may be shown in a two or three-column format, with their affiliations below their respective names. Affiliations are centered below each author name, italicized, not bold. Include e-mail addresses if possible. Follow the author information by two blank lines before main text.

For the minimum full-variational method, when a certain point xij on an image whose size is N*N, define the operator as follows.

$$D_{ij}x = \begin{pmatrix} D_{h1ij}x \\ D_{V1ij}x \end{pmatrix} \tag{18}$$

Where $D_{ij}x$ show certain discrete gradient of the image x. The full-variational fraction has been obtained as follows.

$$TV(x) = \sum_{ij} \sqrt{(D_{h1ij}x)^2 + (D_{V1ij}x)^2} = \sum_{ij} \left\| D_{ij}x \right\|_2 \tag{19}$$

The above equation can be transformed into the minimum full-varational problem based on the reconstruction errors for the two-dimensional image.

$$\min \mathrm{TV(x)} \quad \text{s.t.} \quad \left\| \Phi \mathrm{x} - \mathrm{y} \right\| < \varepsilon \tag{20}$$

3.3. ART Image Reconstruction Algorithm based on Compressed Sensing

The solution steps of the ART algorithm iterative process are as follows which is based on compressed sensing during ECT image reconstruction process.

1) First obtain the full factorial W and the projection data P, and endow the initial estimated value F0（ $\mathrm{F0} = \left[\mathrm{f}_1^{[0]}, \mathrm{f}_2^{[0]}, ..., \mathrm{f}_\mathrm{m}^{[0]} \right]^\mathrm{T}$ ） of the image grey vector;

2) Iterate with relaxation method;

$$f_j^{(k+1)} = f_j^{(k)} + \lambda \frac{p_i - \Sigma_{n=1}^N w_{in} f_n^{(k)}}{\Sigma_{n=1}^N w_{in}^2} \tag{21}$$

Where the value range of λ is 0< λ <2.

The above equation is equivalent to $F^{[i]} = F^{[i-1]} - \frac{\left(w_1^T F^{[i-1]} - p_i\right)}{w_i^T w_i} w_i$ during the ECT. Where the vector $w_1 = \left[w_{11}, w_{12}, ..., w_{1m}\right]^T$ shows the coefficient of the i-th projection equation, and it also entitles the weight coefficient.

3) The projection value F0 enters in the above equation, the value of F1 has been contained. Circulating the process, the projection value $F^{[i]}$ which is the projection value on the i-th hyperplane from $F^{[i-1]}$ which is on the i-1th hyperplane can be obtained.

4) Adjust $F^{[i]}$ with the full-variational method of compression sensing principle.

$$F^{[i]}{}' = F^{[i]} - a \frac{\partial TV\left(F^{[i]}\right)}{\partial F^{[i]}} \tag{22}$$

Where a shows step factor and the projection angle affects its value.

5) The value of $F^{[i]}{}'$ assigns to $F^{[i]}$, and the value of i+1 assigns to i(i<100). Return the third step and iterate, and re-calculate the projection results.

6) It begins to iterate when $F^{[N]}$ is the initial value of the second round projection iteration. $F^{[N]}$ is projected to the first hyperplane, repeat steps 1-5 and solve the projection value of the second round. Cycle repeated, so the vector sequence $F^{[N]}$, $F^{[2N]}$, $F^{[3N]}$, can be obtained. If the unique solution exists, the sequence finally converges to one point. The point is the grey estimation of reconstructed image.

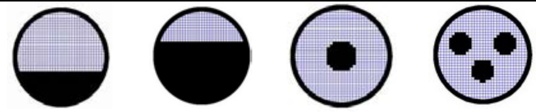
During the oil-water two-phase flow, the pixel grey value is water holdup of this part because the dielectric constant of water is relatively large. The algorithm is optimized by compressed sensing minimum full-variational problem. Calculate the weight factor of the i-th equation with real time fast mesh traversal algorithm during each iteration. Reduce the required memory space, accelerate the reconstruction pace, and reconstruct the images closed to the original flow pattern with less data.

4. Experimental Results and Analysis

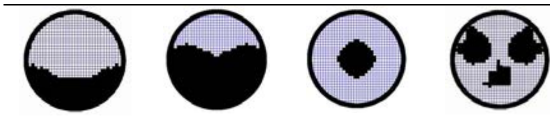
In order to verify the validity of the algorithm, simulation experiments use the 8-electrode system. The pipeline cross-section has been divided into 1024 pixels with 32×32 grids when imaging, and there are 856 imaging units on the effective area of the pipeline cross-section. MATLAB simulation is used on a computer with Pentium dual-core 2.0 CPU and 2G memory. The algorithm described in this paper is used to image reconstruction with the numerical simulation method, and is compared with the traditional ART algorithm. Due to the circulation and core flow are the same substantially, this paper only select two kinds of flow patterns to image reconstruction which are the core flow and laminar flow. The image reconstruction results are as shown in Table 1(Black area is water, and white area is oil).

Table 1. Comparison of Reconstructed Images

Original flow (a)	Original flow (b)	Original flow (c)	Original flow (d)



ART algorithm



ART based on compressed sensing

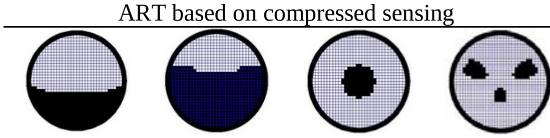


Table 2. Images Error (%)

Original flow	(a)	(b)	(c)	(d)
ART algorithm	20.16	34.57	45.45	84.61
ART based on compressed sensing	17.74	34.81	18.18	43.59

The image reconstruction is shown in Table 1, and the error is shown in Table 2. From Table 1 and Table 2, for core flow and stratified flow. It can be seen that the image reconstruction accuracy error of the core flow is higher than that of the laminar flow, and the ART iterative algorithm based on compressed sensing makes the peak signal to noise ratio to maintain grow well, so the image reconstructed by this algorithm is better than that by the ART algorithm. The accuracy and speed of the reconstructed image has improved. So it proves the validity of the improved algorithm.

Acknowledgements

This work is supported by National Natural Science Foundation of China (60572153, 60972127), Higher Education Doctoral Fund (200802140001), Natural Science Foundation of Heilongjiang Province (QC2012C059), Chunhui Plan of Ministry of Education (Z2007-1-15013), and Science and Technology Project of Department of Education in Heilongjiang Province (11541040, 12511097, 12531094).

References

[1] S. Guangming and L. Danhua, “Advances in Theory and Application of Compressed Sensing”, Acta Electronica Sinica, vol. 37, no. 5, **(2009)**, pp. 1071-1073.

[2] C. Deyun, L. Xiaolong and S. Lijuan, “Concentration Measurement of Oil-Water Two-Phase Flow in Electrical Capacitance Tomography System”, Journal of Test and Measurement Technology, vol. 20, no. 1, **(2006)**, pp. 32-36.

[3] T. Loser, R. Wajman and D. Mewes”, Electrical capacitrical tomography image reconstruction along electrical field lines”, Measurement Science and Technology, no. 12, **(2001)**, pp. 1083-1091.

[4] C. G. Xie and M. S. Beck Plaskowski, “8-electode capacitance system for two-component flow identification”, Part 1: Tomographic flow imaging, IEE Proc A, vol. 136, no. 4, **(1989)**, pp. 173-190.

[5] W. Warsito and L. S. Fan, "Measurement of real-time flow structures in gas-liquid and gas-liquid-solid flow systems using electrical capacitance tomography(ECT)", Chemical Engineering Science, vol. 56, no. 6, **(2011)**, pp. 6455-6462.
[6] W. Q. Yang, "Modeling of capacitance tomography sensors", IEE Proc. Sci. Meas. Technol, vol. 144, **(1997)**, pp. 203-220.
[7] K. L. Ostrowski, S. P. Luke and R. A. Williams, "Simulation of the performance of electrical capacitance tomography for muasurement of dense phase pneumatic conveying", Chemical Engineering Journal, vol. 6, **(1997)**, pp. 8197-8205.
[8] S. Liu, W. Q. Yang and H. Wang, "Investigation of square fluidized beds using capacitance tomography: preliminary results", Measurement Science and Technology, vol. 12, no. 8, **(2002)**, pp. 1120-1125.
[9] W. Q. Yang and L. P. Peng, "Image reconstruction algorithms for electrical capacitance tomography", Meas Sci Technol, vol. 14, **(2003)**.
[10] C. Deyun, Y. Congjing and Z. Guibin, "A Novel Method of Plotting Sensor Sensitivity Field and Image Reconstruction Algorithm for Electrical Capacitance Tomography System", Chinese Journal of Scientific Instrument, vol. 26, no. 3, **(2005)**, pp. 229-234.
[11] C. De-Yun, C. Yu and W. Li-Li, "A novel Gauss-Newton image reconstruction algorithm for Electrical Capacitance Tomography System", Acta Electronica Sinica, vol. 37, no. 4, **(2009)**, pp. 739-743.
[12] S. Liu, L. Fu and W. Q. Yang, "Prior-online iteration for image reconstruction with electrical capacitance tomography", IEE Proc-sci.Meas.Technol, vol. 151, no. 3, **(2004)**, pp. 195-200.

Author

Chen Deyun received PhD from Harbin University of Science and Technology in 2006.He is now a professor in Harbin University of Science and Technology. His research interests are detection and imaging technology, image processing, measurement technology and signal processing.
E-mail: chendeyun@hrbust.edu.cn.