# Implementation of Biorthogonal Wavelet Transform Using Discrete Cosine Sequency Filter

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#### Abstract

Biorthogonal wavelet transform has been widely used in the fields of image denoising and image coding. It is usually realized by linear phase filters. All phase digital filter is a new type of linear phase digital filter which has been proposed in recent years and has frequency structure. On the basis of analyzing principle of biorthogonal wavelet transform and discrete cosine sequency filter (DCSF), this paper proposes a new algorithm to implement biorthogonal wavelet transform by using discrete cosine sequency filter. Simulation experiments to typical test images are conducted in MATLAB. Experimental results show that whether using the discrete cosine sequency filter or the biorthogonal wavelet transform, the results of the decomposition and reconstruction are the same. It comes to conclude that the proposed algorithm is valid.

**Keywords:** Biorthogonal Wavelet Transform; Filter Design; Linear Phase Filter; Zero-Phase Filter; Discrete Cosine Transform (DCT); Discrete Cosine Sequency Filter (DCSF)

### 1. Introduction

The linear filter was designed in the last century. Due to its well-developed theories and easy implementation by fast Fourier transform (FFT) using hardware, it has been playing an important role in image filtering. However, its computational complexity is high. It's hard to process signals in real-time because a large quantity of calculation is needed. Though it can effectively suppress Gaussian noise, it can't suppress other noises well such as the pulse-interference. Meanwhile it causes the edge of signal blurred.

Because the linear filter has many disadvantages, some scholars try to improve it. Enlightened by the fact that linear filter is implemented by FFT, people first think of the discrete cosine transform (DCT) [1]. Compared with FFT, DCT has many advantages. Its properties make it easy to extract the correlated characteristics of transformed signal, so it is conducive to the realization of data compression. Because the DCT transform matrix easily reflects the characteristics of image signals and human speech signals, DCT is considered as the quasi-perfect transform [2]. However, DCT also has many disadvantages such as blocking artifacts and nonlinear phase characteristics. Therefore, the DCT filter is also in the process of development. Combining with overlapped block digital filtering, several kinds of all phase digital filters based on the discrete Fourier transform (DFT) and DCT were proposed [3-5]. The discrete cosine sequency filter (DCSF) not only has excellent characteristics of DCT filter, but also has some other advantages which the above linear filters don't have.

As an important form of wavelet analysis, the biorthogonal wavelet transform has a lot of advantages, for example, the wavelet base function can be constructed easily and can be used flexibly. The wavelet filter banks are symmetrical with compact support [6, 7]. Because of many advantages discussed above, biorthogonal wavelet transform has been used widely in the fields of image denoising [8] and image coding [9]. This paper studies the relationship between the discrete cosine sequency filter and biorthogonal wavelet transform, and implements biorthogonal wavelet transform using this kind of linear filter.

Based on analyzing the principle of biorthogonal wavelet transform and discrete cosine sequency filtering, this paper proposes a new algorithm to achieve biorthogonal wavelet transform by using the all phase filter. Because multi-resolution analysis of 2-D images is one important application of biorthogonal wavelet transform, so in order to verify the feasibility of the algorithm and find out the characteristics of this algorithm, the experiments to typical test images are done in MATLAB at the end of the paper. The experimental results can be used to analyze the advantages and disadvantages of the algorithm.

The rest of this paper is organized as follows. Section 2 starts with a brief review of biorthogonal wavelet transform, and then the proposed method is described. The discrete cosine sequency filter is introduced in Section 3. Section 4 presents the algorithm design to implement biorthogonal wavelet transform using the discrete cosine sequency filter. Experimental results on the proposed method to typical images are presented in Section 5. Conclusion and future work are given finally in Section 6.

## 2. Biorthogonal Wavelet Transform

The biorthogonal wavelet transform is composed of the decomposition process and the reconstruction process with two different wavelets  $\psi$  and  $\tilde{\psi}$ .  $\psi$  is used in the decomposition process, and  $\tilde{\psi}$  is used in the reconstruction process.  $\psi$  and  $\tilde{\psi}$  are dual and orthogonal to each other, and this relationship is called biorthogonal. Meanwhile, there are two scale functions  $\phi$  and  $\tilde{\phi}$  in the above processes, these two scale functions are also dual and orthogonal. One is used in the decomposition process, and the other one is used in the reconstruction process. Therefore, there are four filters in biorthogonal wavelet transform. They are the decomposition low-pass filter  $\{h_n\}$ , the decomposition high-pass filter  $\{g_n\}$ , the reconstruction low-pass filter  $\{\tilde{h}_n\}$  and the reconstruction high-pass filter  $\{\tilde{g}_n\}$ . Unlike the orthogonal wavelet transform, the reconstruction filters and the decomposition filters are different.

With filter coefficients  $\{h_n\}$ ,  $\{g_n\}$ ,  $\{\tilde{h}_n\}$ , and  $\{\tilde{g}_n\}$ , fast wavelet transform—Mallat algorithm [6] can be performed. The decomposition and reconstruction processes of  $\{c_{N,k}\}$  using Mallat algorithm is shown in Figure 1.

What we can conclude from Figure 1 is that: the essence of Mallat algorithm is filtering the signal  $\{c_{N,k}\}$  by decomposition filters  $\{h_n\}$  and  $\{g_n\}$ . Then the results are sub-sampled by factor 2. The results of decomposition have two parts. One part is the signal  $\{c_{N-1,k}\}$  generated by low-pass filter  $\{h_n\}$ , which can be seen as the approximation of the original signal. And the other part is the signal  $\{d_{N-1,k}\}$  generated by high-pass filter  $\{g_n\}$ , which can be seen as the detail of the original signal. On the contrary, the reconstruction process takes the reverse process to reconstruct the original signal by reconstruction filters  $\{\tilde{h}_n\}$  and  $\{\tilde{g}_n\}$ .

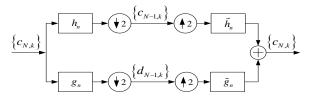


Figure 1. Decomposition and Reconstruction Processes of 1-D Mallat Algorithm

Similar to the case of 1-D, multi-resolution analysis of 2-D signals can be described as follows. Denote  $\{c_{j,k}\}$  and  $\{d^i_{j,k}(i=1,2,3)\}$  as the approximation and detail of 2-D signal f(x,y) at scale j respectively. If the original data can be considered as a 2-D discrete signal after the sampling process, the 2-D discrete wavelet transform to signal  $\{c_{j,k}\}$  can be indicated by Figure 2.

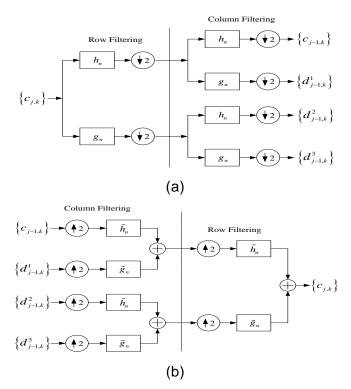


Figure 2. Separable Filter Banks: (a) Decomposition, (b) Reconstruction

After the horizontal and vertical filtering processes, four different frequency bands  $\{c_{j-1,k}\}$ ,  $\{d_{j-1,k}^1\}$ , and  $\{d_{j-1,k}^3\}$  are obtained respectively. Continue to process  $\{c_{j-1,k}\}$  with the method discussed above, we can get a pyramidal decomposition. On the contrary, each reconstruction process is the reverse process of the decomposition process.

In summary: with filter banks  $\{h_n\}$ ,  $\{g_n\}$ ,  $\{\tilde{h}_n\}$ , and  $\{\tilde{g}_n\}$ , wavelet transform can easily be done. According to this idea, this paper studies the relationship between filter

banks  $\{h_n\}$ ,  $\{g_n\}$ ,  $\{\tilde{h}_n\}$ ,  $\{\tilde{g}_n\}$  and the all phase filter in DCT domain, and realizes biorthogonal wavelet transform using the discrete cosine sequency filter.

# 3. Discrete Cosine Sequency Filter (DCSF)

In 2002, Hou *et al.*, proposed a concept of all phase data space. Based on the DFT/IDFT filtering, a new type of zero-phase filter, called the all phase DFT (APDFT) filter, was designed [3, 4]. The APDFT is a new FIR filter. Theoretical analysis and simulation experiments showed that it has the advantage of the frequency sampling in overall filter performance, and can be used to realize strictly power complementary sub-band filtering.

In order to fully develop potential advantages of the discrete cosine transform (DCT) in the field of signal filtering, a new linear phase digital filter—discrete cosine sequency filter (DCSF) was introduced and a convolution algorithm in time domain for DCSF was proposed [5]. The implementation structure of DCSF is shown in Figure 3.

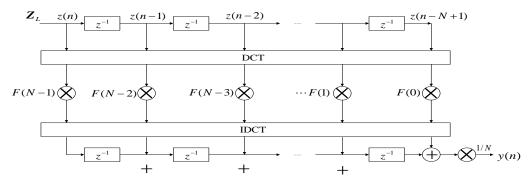


Figure 3. Structure of the Discrete Cosine Sequency Filter

The discrete cosine sequency filter based on DCT can be expressed as [5]:

$$Y_{L} = C^{T} \{F \cdot CZ_{L}\}$$
(1)

where  $Z_L$  is the Lth vector with length of N that can be got by shifting discrete time-domain signal  $\{z(n)\}$ ; F is the DCT transform matrix; C is the response vector of columns whose length is N; represents two vectors multiplied by the corresponding element;  $Y_L$  is the output component. If all  $Y_L$  are connected, the output signal  $\{y(n)\}$  will be obtained.

$$y(n) = Q(n) * z(n), \qquad (2)$$

where

$$\begin{cases}
Q(n) = \frac{1}{N} \sum_{m=n}^{N-1} H_m(m-n) \\
Q(-n) = \frac{1}{N} \sum_{m=0}^{N-1-n} H_m(m+n)
\end{cases}, n = 0, 1, \dots, N-1.$$
(3)

Eq. (2) is the convolution form of discrete cosine sequency filter, it means that the output signal  $\{y(n)\}$  is the convolution of the discrete signal  $\{z(n)\}$  in time domain and the DCSF  $\{Q(n), -N+1 \le n \le N-1\}$ . DCSF is one kind of FIR filters, and its length is 2N-1. This

kind of filter is not only easy to implement, but also can eliminate the block effect of conventional discrete cosine transform filtering.

As for discrete cosine transform (DCT), we have

$$C^{\mathrm{T}}(i,j) = \begin{cases} \frac{1}{\sqrt{N}}, & j = 0, \ i = 0, 1, \dots, N-1\\ \sqrt{\frac{2}{N}} \cos \frac{j(2i+1)\pi}{2N}, & j = 1, 2, \dots N-1, \ i = 0, 1, \dots, N-1. \end{cases}$$
(4)

$$H_i(j) = \sum_{i=0}^{N-1} C^{\mathrm{T}}(i,k)C(k,j)F(k), \quad i, j = 0,1,\dots,N-1.$$
 (5)

When Eq. (5) is applied to Eq. (4),

$$H_i(j) = \frac{1}{N} \left[ F(0) + \sum_{k=1}^{N-1} 2\cos\frac{k(2i+1)\pi}{2N} \cos\frac{k(2j+1)\pi}{2N} F(k) \right]. \tag{6}$$

Because of  $H_i(j) = H_i(i)$ , we get

$$Q(n) = Q(-n), n = 0, 1, \dots, N - 1.$$
 (7)

Therefore, the frequency response of the system is

$$H(e^{j\omega}) = Q(0) + 2\sum_{k=1}^{N-1} Q(k)\cos(k\omega)$$
 (8)

So, the system has strict zero phase characteristics and is an all phase filter.

Defined the vector  $\mathbf{Q}_{1/2} = [Q(0), Q(1), \dots, Q(N-1)]^T$ , the DCSF can be obtained with Eq. (7) and  $\mathbf{Q}_{1/2}$ . The solving process of  $\mathbf{Q}_{1/2}$  is shown as follows.

The elements of matrix R and matrix S can be defined as:

$$R(m,n) = \begin{cases} \frac{1}{2}, & n = 0, \ m = 0, 1, \dots, N - 1, \\ \cos \frac{mn\pi}{N}, & n = 1, 2, \dots, N - 1, \ m = 0, 1, \dots, N - 1. \end{cases}$$
(9)

$$S = \begin{bmatrix} N & 2 & 0 & 2 & 0 & \cdots & 0 & 2 & 0 \\ 0 & N-1 & 2 & 0 & 2 & \cdots & 2 & 0 & 1 \\ 0 & 0 & N-2 & 2 & 0 & \cdots & 0 & 2 & 0 \\ 0 & 0 & 0 & N-3 & 2 & \cdots & 2 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 1 \end{bmatrix}.$$

$$(10)$$

Eq. (3) and Eq. (6) can be used to prove that:

$$Q_{1/2} = \frac{1}{N^2} SRF = VF, \qquad (11)$$

where the elements of matrix V = SR can be expressed as

$$V(m,n) = \begin{cases} \frac{N-m}{N^2}, & m = 0, 1, \dots, N-1, n = 0, \\ \frac{1}{N^2} \left[ (N-m)\cos\frac{mn\pi}{N} - \csc\frac{n\pi}{N}\sin\frac{mn\pi}{N} \right], & m = 0, 1, \dots, N-1, n = 1, 2, \dots, N-1. \end{cases}$$
(12)

As discussed above, the discrete cosine sequency filter with length of 2N-1 can be obtained by Eqs. (7), (11), and (12).

# 4. Algorithm Design of Biorthogonal Wavelet Transform Using Discrete Cosine Sequency Filter

#### 4.1. Filter Coefficients Solver

The transfer function of DCSF in DCT domain can be obtained with Eq. (8):

$$H(z) = Q(0) + \sum_{k=1}^{N-1} (Q(k)z^{k} + Q(-k)z^{-k}).$$
 (13)

Obviously,  $Q_{1/2}$  is corresponding to the coefficients of each decomposition and reconstruction filter. Because of the strict zero-phase characteristic, we know Q(k) = Q(-k). It means that coefficients of the biorthogonal wavelet transform must meet the requirement of symmetry

$$h_{2k-n} = h_n, \ g_{2k-n} = g_n, \ \tilde{h}_{2k-n} = \tilde{h}_n, \ \tilde{g}_{2k-n} = \tilde{g}_n.$$
 (14)

In different wavelet transforms,  $Q_{1/2}$  is corresponding to different filters  $\{h_n\}$ ,  $\{g_n\}$ ,  $\{\tilde{h}_n\}$ , and  $\{\tilde{g}_n\}$ , the details are described as follows:

Decomposition filter: low-frequency  $Q_L(k) = h_k$ , input signal x(n); high-frequency  $Q_H(k) = g_{k+1}$ , output signal x(n+1).

Reconstruction filter: low-frequency  $Q_L(k) = \tilde{h}_k$ , input signal x(n); high-frequency  $Q_H(k) = \tilde{g}_{k+1}$ , output signal x(n+1).

Having transfer function of the system, the method for solving the coefficients of each filter is as follows:

- (1) Firstly, the filter order is defined as N, in other words, in corresponding filters  $\{h_n\}$ ,  $\{\tilde{h}_n\}$ ,  $\{g_n\}$ ,  $\{\tilde{g}_n\}$ ,  $N = \max(n) + 1$ ;
  - (2) If  $Q_{1/2}$  is known, the filter parameter F can be obtained by Eq. (11).

It is necessary to declare that because of strict zero phase condition Q(k) = Q(-k),  $\{h_n\}$ ,  $\{\tilde{h}_n\}$  and  $\{g_n\}$ ,  $\{\tilde{g}_n\}$  are symmetrical about the even index, so not all of the biorthogonal wavelet transforms can be realized by the discrete cosine sequency filter. For example, as for some biorthogonal spline wavelets, because  $\{h_n\}$ ,  $\{\tilde{h}_n\}$  and  $\{g_n\}$ ,  $\{\tilde{g}_n\}$  are symmetrical about the odd index, they can not be realized by this method.

#### 4.2. Process of Input Signal through the Filter

With the coefficients of biorthogonal wavelet filters, the coefficients of discrete cosine sequency filters can be obtained. So, the biorthogonal wavelet transform can be implemented when the input signal passes the filter. The specific process is as follows:

- Step 1. Extending the input signal x(n) in bi-direction.
- Step 2. To get the output of x(n), the *i*th shift is performed, where  $i \in [0, N-1]$ ; denote x(n) and its preceding N-1 values as  $\{a\}$ , where N is the filter order.
- Step 3. The  $\{a\}$  is transformed with DCT, and then multiplied by F. The  $\{b\}$  can be obtained with the IDCT.
  - Step 4. Using the N-i elements from  $\{b\}$  as the output signal  $y^{(i)}(n)$ .
- Step 5. Finally,  $N y^{(i)}(n)$  can be obtained, and using its average value to represent the output x(n).
  - Step 6. Repeating Step 2 ~ Step 5 until the entire sequence is obtained.

#### 4.3. Example of Algorithm Process

The coefficients of some biorthogonal wavelet transform can easily be found in [10]. This paper selects the biorthogonal wavelet "CDF 9/7" ("bior4.4" in MATLAB) as an example to explain the process of transformation.

#### (1) Calculating each F

The low frequency of decomposition filter  $Q_{1/2}$  corresponds to  $\{h_n\}$ , and the N=4.

With Eq. (11), we can get:

$$F_{\rm L} = [1.41421356237310, 1.25916616277351, 1.58585408385287, 0.30315526787847, -0.29889568183347].$$
 (16)

(2) The other three cases follow the same principle, and results are shown in Table 1-Table 4.

Table 1. Low-pass Decomposition Filter Parameters of the Discrete Cosine Sequency Filter

| Decomposition filter | Low frequency $Q_{1/2}$ corresponds to $\{h_n\}$                                       |                           |
|----------------------|--|---------------------------|
| index n              | $\left[oldsymbol{\mathcal{Q}}_{\scriptscriptstyle 1/2} ight]_{\scriptscriptstyle m L}$ | Decomposition $F_{\rm L}$ |
| 0                    | 0.85269867900889   | 1.41421356237310          |
| 1 (-1)               | 0.37740285561283   | 1.25916616277351          |
| 2 (-2)               | -0.11062440441844  | 1.58585408385287          |
| 3 (-3)               | -0.02384946501956  | 0.30315526787847          |
| 4 (-4)               | 0.03782845550726   | -0.29889568183347         |

Table 2. High-pass Decomposition Filter Parameters of the Discrete Cosine Sequency Filter

| Decomposition filter | High frequency $Q_{1/2}$  | $g_n$ corresponds to $\{g_n\}$ |  |
|----------------------|---|--------------------------------|--|
| index n              | $\left[oldsymbol{\mathcal{Q}}_{\scriptscriptstyle 1/2} ight]_{\scriptscriptstyle  m H}$ | Decomposition $F_{\rm H}$      |  |
| 1                    | -0.78848561640558   | -0.00000000000141              |  |
| 2 (0)                | 0.41809227322162  | 0.00837542194761               |  |
| 3 (-1)               | 0.04068941760916  | -1.50402664285113              |  |
| 4 (-2)               | -0.06453888262870   | -1.65829124471740              |  |
| 5 (-3)               | 0   | 0                              |  |

Table 3. Low-pass Reconstruction Filter Parameters of the Discrete Cosine Sequency Filter

| Reconstruction filter | Low frequency $Q_{1/2}$ corresponds to $\{\tilde{h}_n\}$                               |                            |
|-----------------------|--|----------------------------|
| index n               | $\left[oldsymbol{\mathcal{Q}}_{\scriptscriptstyle 1/2} ight]_{\scriptscriptstyle m L}$ | Reconstruction $F_{\rm L}$ |
| 0                     | 0.78848561640558   | 1.41421356237310           |
| 1 (-1)                | 0.41809227322162   | 1.68454350122106           |
| 2 (-2)                | -0.04068941760916  | 0.34420737402113           |
| 3 (-3)                | -0.06453888262870  | -0.28902197199296          |
| 4 (-4)                | 0  | 0                          |

Table 4. High-pass Reconstruction Filter Parameters of the Discrete Cosine Sequency Filter

| Reconstruction | High frequency $Q_{1/2}$ corresponds to $\{\tilde{g}_n\}$                                  |                            |
|----------------|--|----------------------------|
| filter index n | $\left[oldsymbol{\mathcal{Q}}_{\scriptscriptstyle 1/2} ight]_{\scriptscriptstyle 	ext{H}}$ | Reconstruction $F_{\rm H}$ |
| 1              | -0.85269867900889  | 0                          |
| 2 (0)          | 0.37740285561283   | 0.01217226618587           |
| 3 (-1)         | 0.11062440441844   | -1.27678977716429          |
| 4 (-2)         | -0.02384946501956  | -1.49708977245604          |
| 5 (-3)         | -0.03782845550726  | -1.50178611161002          |

#### 5. Experimental Results

The principle and implementation of the biorthogonal wavelet transform with discrete cosine sequency filter have been described above. As a verification of the algorithm, in the following, typical test images are used as input signal to simulate the process of wavelet transform in computer by MATLAB. This process is also called the separable multiresolution analysis of 2-D images [11].

The multi-level decomposition and reconstruction are not the focus of this paper, so we use only one layer wavelet transform to illustrate the feasibility of the proposed algorithm. The test image is the MATLAB image "woman2.mat" whose size is  $128 \times 128$ . The biorthogonal wavelet with filter length  $N = \tilde{N} = 4$  is adopted in the simulation.

Two methods, the discrete cosine sequency filter and the biorthogonal wavelet transform, are used to process the image "woman2". The structure of the former is shown in Figure 3. And the structure of the latter is shown in Figure 2. The results of the simulation are shown in Figure 4 and Figure 5.

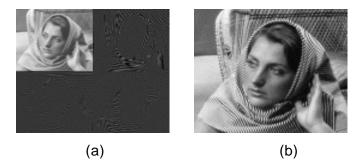


Figure 4. Implementation of Decomposition and Reconstruction using Discrete Cosine Sequency Filter: (a) Decomposition, (b) Reconstruction

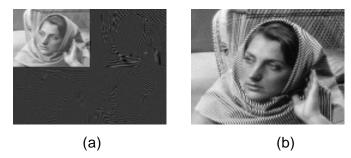


Figure 5. The Decomposition and Reconstruction Results using the Biorthogonal Wavelet: (a) Decomposition, (b) Reconstruction

The discrete cosine sequency filter and the direct wavelet transform method have similar results. The mean squared error (MSE) of the reconstruction and decomposition are zero. The discrete cosine sequency filters are perfect reconstruction filter banks.

#### 6. Conclusion

On the basis of the biorthogonal wavelet transform and the discrete cosine sequency filter, this paper proposes a new algorithm to implement biorthogonal wavelet transform using the all phase digital filter. The experiments to typical test images are done in MATLAB. Experimental results show that whether using the all phase cosine sequency filter or the biorthogonal wavelet transform, the results of the decomposition and reconstruction are the same. So we can draw a conclusion that for biorthogonal wavelets whose filter coefficients are symmetrical about even index, their wavelet transforms can be implemented using the discrete cosine sequency filter.

This paper only focuses on the implementation of biorthogonal wavelet transform using discrete cosine sequency filter. With the decomposition or reconstruction filter parameters designed in this paper, more applications in digital image processing, such as image coding, image denoising, and image edge detection can be realized easily. These issues are left for future research.

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