Direction-of-Arrival Estimation Based on Khatri-Rao Product and Redundancy Arrays

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Abstract

Difference co-array can be constructed by using Khatri-Rao (KR) product, which can increase the degrees of freedom (DOF) significantly. Combined with fourth order cumulants, the KR product can be used to constructed fourth order difference co-array. The fourth order difference co-array of a four level nested array contains a uniform linear array (ULA), however, its second order difference co-array has missing holes, which may result in the ambiguity for DOA estimation. And the method based on KR product and fourth order cumulants has two main drawbacks. First it cannot be employed to Gaussian source signals. Second it needs a large number of snapshots. In this paper, a novel approach is proposed to construct a virtual ULA based on KR product and redundancy spacing of arrays for a four level nested array. Unlike the existing method based on KR product and fourth order cumulants, the new method only uses second order statistics. And compared to the method based on KR product and second order statistics, the new method achieves higher resolution. Numerical results are provided to demonstrate the effectiveness and superior performance of the proposed algorithm.

Keywords: *Augmented covariance matrix; Difference co-array; Direction-of-arrival estimation; Khatri-Rao product;Nested array*

1. Introduction

DOA estimation with nonuniform linear array (NLA) has been an active research area in these years mainly due to that it can span larger aperture than uniform linear array (ULA) for the same number of sensors. Moreover, NLA can resolve more sources than sensors in some cases.

There were mainly three ways to detect more sources than sensors in earlier works. The first way is to exploit the minimum redundancy arrays (MRA)[1]. In [2] and [3], it has been shown that by constructing an augmented covariance matrix, the degrees of freedom (DOF) can be almost increased up to $N(N-1)/2$ with N sensors array. However, the augmented covariance matrix is not positive definite for finite number of snapshots. Besides, there is no close form expression for the sensor positions and computer search must be done to find the optimal sensor placement. Using high order cumulants is the second way to increase the DOF. It has been shown that in [4-6], the cumulant based algorithm can increase the DOF significantly. However, these methods are only applicable to non-gaussian sources, and a large number of snapshots are needed to compute the high order cumulants. Lately, by using Khatri-Rao (KR) product, Ma[7] proposed a novel approach which can detect $2N-1$ sources using a ULA with N sensors. After that, much attention has been attracted to the use of Khatri-Rao product. Based on KR product and second order statistics, a new array geometry called nested array was proposed by Pal.P[8], where the DOF was increased up to $\mathrm{O}(N^2)$ using only N elements by constructing the difference co-array. It was shown that the second order difference co-array of a two level nested array is a filled ULA, but has miss holes for more than two stage of nesting. In [9], by exploiting KR product and $2q(q \geq 2)$ order cumulants, Pal.P extended the two level nested array to $2q$ level nested array, which showed that $2q\text{th}(q \geq 2)$ order difference co-array of $2q(q \geq 2)$ level nested array contains a ULA with $\mathrm{O}(N^{2q})$ virtual sensors, where N is the number of the original array. However, a large number of snapshots are needed to be computed high order cumulants. Besides, the method cannot be applicable for Gaussian sources. In this paper, we propose a novel approach to construct a half of the fourth order difference co-array, and by exploiting four level nested array, we can identify $\mathrm{O}(N^4)$ sources using only N sensors. Firstly, we construct the second order difference co-array of a four level nested array using KR product. Then, by making use of the redundancy lag of the covariance matrix, we construct the final virtual array based on the second difference co-array. Our method can be used to detect Gaussian and non-Gaussian signals since it only exploits second order statistics. And compared with the method based on KR product and second order statistics[8], our method shows higher resolution ability.

2. Signal Model and Preliminaries

In this section, we first introduce the signal model, then two existing DOA estimation methods will be presented, which will be exploited in the proposed method thereafter.

2.1. Signal Model

Consider the case in which K narrowband far-field sources are impinging on a NLA of N elements. The received signal of the ith element is

$$x_i(t) = \sum_{k=1}^{K} a_{ik} s_k(t) + v_i(t), i = 1, 2, ..., N \tag{1}$$

where $a_{ik} = \exp(-j\frac{2\pi}{\lambda} d_i \sin\theta_k)$,where λ denotes the wavelength of the signal, d_i denotes the position of the ith sensor, we assume the sensors to be placed on a linear grid, which is an integer multiple of the smallest spacing in the underlying grid. $s_k(t)$ denotes the kth signal and $v_i(t)$ denotes the noise received by the ith sensor.

Using matrix notation, (1) can be rewritten as

$$\boldsymbol{x}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{v}(t) \tag{2}$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), ..., x_N(t)]^T$ is the vector of the received signals, $\boldsymbol{A} = [\boldsymbol{a}(\theta_1), \boldsymbol{a}(\theta_2), ..., \boldsymbol{a}(\theta_K)]$, $\boldsymbol{a}(\theta_k) = [a_{1k}, a_{2k}, ..., a_{Nk}]^T$. $\boldsymbol{A}$ and $\boldsymbol{a}(\theta_k)$ are the array manifold and the steering vector of the array, respectively. $\boldsymbol{s}(t) = [s_1(t), s_2(t), ..., s_K(t)]^T$ denotes the vector of the source signals. $\boldsymbol{v}(t) = [v_1(t), v_2(t), ..., v_N(t)]^T$ denotes the vector of the received noise.

To estimate the incident angles of the signals, we make the following assumptions:

A1): The signals are uncorrelated each other and uncorrelated with the noise.

A2): The noise is assumed to be spatially and temporally white.

A3): The signals are nonstationary, but each source signals are stationary with frame length L, *i.e.*,

$$E\{s_k(t)s_k^H(t)\} = \sigma_{km}^2, \forall t \in [(m-1)L, mL], m = 1,2,...,M, k = 1,2,...,K \tag{3}$$

The covariance matrix of the mth frame received signals can be expressed as

$$\boldsymbol{R}_m = E\{\boldsymbol{x}_m(t)\boldsymbol{x}_m^H(t)\} = \boldsymbol{A}\boldsymbol{\Sigma}_{sm}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I} \tag{4}$$

where $\boldsymbol{\Sigma}_{sm} = diag[\sigma_{1m}^2,...,\sigma_{Km}^2]$, where $\sigma_{km}^2, k = 1,...,K, m = 1,...,M$ denotes the power of the mth frame of the kth signal, and σ_n^2 denotes the power of noise.

2.2. *2q*th Order Nested Array

Nested array, which is proposed by Pal.P *et. al.*, [9], is a new array geometry and can significantly increase the DOF of linear arrays. A '2q-Level' nested array, parameterized by $q, N_1, N_2, ..., N_{2q}$, is defined as one where the sensor positions are given by the set $S_{2q-level} = \bigcup_{i=1}^{2q} S_i$, where

$$\begin{aligned} S_i &= \{nd\prod_{j=1}^{i-1} N_j, n = 1,2,...,N_i - 1\}, i = 1,2,...,2q-1 \\ S_K &= \{nd\prod_{j=1}^{2q-1} N_j, n = 1,2,...,N_{2q}\} \end{aligned} \tag{5}$$

where d denotes the minimum grid. The nested array has $\sum_{i=1}^{2q}(N_i - 1) + 1$ sensors, each level with $N_k - 1$ sensors in the k-th level except N_{2q} sensors in the $2q$th level. It has been shown that the $2q$th order difference co-array of a $2q$ level nested array with N sensors contains a ULA with $\mathrm{O}(N^{2q})$ virtual sensors[9]. However, the second order difference co-array of an array with more than two stage of nesting is not a ULA[8]. In this paper, we consider the problem of the second order difference co-array of four level nested array. A virtual ULA will be constructed from four level nested array based on Khatri-Rao product and redundancy spacing of arrays.

2.3. Directed Augmented Approach

By employing the MRA, the directed augmented approach constructs an augmented covariance array almost to increase the DOF to $N(N-1)/2$ with an N element array. The (i, j) entrance of the covariance matrix of the mth frame received signals is

$$r_{mz} \triangleq r_{m,i-j} = \boldsymbol{R}_{m,ij} = \sum_{k=1}^{K}\sigma_{km}^2 \mathrm{e}^{-j\frac{2\pi}{\lambda}(d_i - d_j)\sin\theta_k} + \sigma_n^2\delta(i-j) \tag{6}$$

It is well known that the covariance matrix of the received signals of a ULA is toeplitz. And there are $N+1$ spatial lags in a covariance matrix of a ULA with N elements. As long as all the spatial lags are measured, a ULA is just constructed. From (6), we know that we can use fewer sensors to measure all elements in the covariance matrix of a ULA. However, as we mention above, there is no close form for the array geometry so that exhaustive computer search must be performed to find the optimal placement for a given number of sensors. The MRA for the sensor number $N<17$ is given in[10].

After finding all the spatial covariance lags for z from 0 to Z, the augmented covariance matrix can be constructed as follows:

$$\boldsymbol{R}_{ma}=\begin{pmatrix} r_{m0} & r_{m1} & \cdots & r_{mZ} \\ r_{m1}^{*} & r_{m0} & \ddots & \vdots \\ \vdots & \ddots & r_{m0} & r_{m1} \\ r_{mZ}^{*} & \cdots & r_{m1}^{*} & r_{m0} \end{pmatrix} \tag{7}$$

And $\boldsymbol{R}_{ma}$ behaves like a covariance matrix of a ULA with $Z+1$ elements, which is half of the difference co-array of the physical array. The subspace methods such as MUSIC can be applied to the augmented covariance matrix to get better performance.

2.4. KR-based Method

KR-based method vectorizes the covariance matrix of the original array to obtain a new vector, which can be treated as received signal vector of a new array. Vectorize $\boldsymbol{R}_m$ to get the following vector:

$$\boldsymbol{y}_m = vec(\boldsymbol{R}_m) = vec(\boldsymbol{A}\boldsymbol{\Sigma}_{sm}\boldsymbol{A}^H + \sigma_n^2\boldsymbol{I}) = (\boldsymbol{A}^* \odot \boldsymbol{A})\boldsymbol{p}_m + \sigma_n^2\vec{\mathbf{1}}_n, m=1,...,M \tag{8}$$

where $\boldsymbol{p}_m=[\sigma_{1m}^2,\sigma_{2m}^2,...,\sigma_{Km}^2]^T$, $\vec{\mathbf{1}}_n=[e_1^T,e_2^T,...,e_N^T]^T$ with e_i being a column vector of all zeros except a 1 at the ith position and the symbol $\odot$ denotes the KR product of two matrix. Compared it with (2), it can be noted that $\boldsymbol{y}_m$ behaves like a received signal of a new array, where $\boldsymbol{A}^* \odot \boldsymbol{A}$ is the array manifold matrix, $\boldsymbol{p}_m$ is the new signal vector and the noise is denoted by $\sigma_n^2\vec{\mathbf{1}}_n$. If the positions of sensors of the original array are in the set $\{\vec{x}_i, 1\le i\le N\}$, the positions of the virtual array can be expressed as $\{\vec{x}_i-\vec{x}_j, 1\le i,j\le N\}$, that is just the sensor location of the difference co-array. Some classic DOA estimation methods can be applied to the co-array to acquire better performance compared to the physical array.

3. The Proposed Method

Here we focus on four level nested array, whose fourth order difference co-array contains a ULA with $2(N_1N_2N_3N_4+N_1N_2N_3)-1$ sensors. Although the second order difference co-array is not a filled ULA, we can use the co-array to construct an augmented covariance matrix, whose aperture length is equivalent to a ULA with $N_1N_2N_3N_4+N_1N_2N_3$ sensors.

Define a new matrix $\boldsymbol{A}_1$, which is constructed from $\boldsymbol{A}^* \odot \boldsymbol{A}$, to be the manifold matrix of the second order difference co-array. $\boldsymbol{A}_1$ is acquired by removing the repeated rows from $\boldsymbol{A}^* \odot \boldsymbol{A}$ and sorting the location of the virtual sensor in ascending order. Here we provide a smoothing way to decrease the variance. For instance, we can average the same rows of $\boldsymbol{A}^* \Box \boldsymbol{A}$, and use the averages in place of the original rows. The same operation is performed to $\boldsymbol{y}_m$ and $\vec{\mathbf{1}}_n$ to get new vector $\boldsymbol{y}_{1m}$ and $\boldsymbol{w}$. Thus we have

$$\boldsymbol{y}_{1m} = \boldsymbol{A}_1 \boldsymbol{p}_m + \sigma_n^2 \boldsymbol{w} \tag{9}$$

The covariance matrix of the vector $\boldsymbol{y}_{1m}$ is

$$\begin{aligned}\tilde{\boldsymbol{R}} &= E\{\boldsymbol{y}_{1m}\boldsymbol{y}_{1m}^H\} = E\{(\boldsymbol{A}_1\boldsymbol{p}_m + \sigma_n^2\boldsymbol{w})(\boldsymbol{A}_1\boldsymbol{p}_m + \sigma_n^2\boldsymbol{w})^{\mathrm{H}}\} \\ &= \boldsymbol{A}_1\boldsymbol{p}_m\boldsymbol{p}_m^H\mathbf{A}_1^H + \sigma_n^2\mathbf{A}_1\boldsymbol{p}_m\boldsymbol{w}^H + \sigma_n^2\boldsymbol{w}\boldsymbol{p}_m^H\mathbf{A}_1^H + \sigma_n^4\boldsymbol{w}\boldsymbol{w}^H\end{aligned} \tag{10}$$

Let

$$\boldsymbol{p}_m\boldsymbol{p}_m^H = \boldsymbol{B} = \tilde{\boldsymbol{B}} + \boldsymbol{\Sigma}_p \tag{11}$$

where $\boldsymbol{\Sigma}_p$ is the diagonal matrix of the matrix B. Substituting (11) into (10), we have

$$\begin{aligned}\tilde{\boldsymbol{R}} &= \boldsymbol{A}_1\boldsymbol{\Sigma}_p\mathbf{A}_1^H + \boldsymbol{A}_1\tilde{\mathbf{B}}\mathbf{A}_1^H + \sigma_n^2\mathbf{A}_1\boldsymbol{p}_m\boldsymbol{w}^H + \sigma_n^2\boldsymbol{w}\boldsymbol{p}_m^H\mathbf{A}_1^H + \sigma_n^4\boldsymbol{w}\boldsymbol{w}^H \\ &= \boldsymbol{A}_1\boldsymbol{\Sigma}_p\mathbf{A}_1^H + \boldsymbol{U}\end{aligned} \tag{12}$$

where $\boldsymbol{U} = \boldsymbol{A}_1\tilde{\mathbf{B}}\mathbf{A}_1^H + \sigma_n^2\mathbf{A}_1\boldsymbol{p}_m\boldsymbol{w}^H + \sigma_n^2\boldsymbol{w}\boldsymbol{p}_m^H\mathbf{A}_1^H + \sigma_n^4\boldsymbol{w}\boldsymbol{w}^H$. Note that the diagonal elements of $\boldsymbol{B}$ are far more than the non-diagonal elements of $\boldsymbol{B}$, we will verify that in Section 4.1. So $\boldsymbol{U}$ can be regard as the noise part. Unfortunately it is color noise.

The (i, j) entrance of the covariance matrix of $\boldsymbol{y}_{1m}$ is

$$\tilde{\boldsymbol{R}}_{ij} = \sum_{k=1}^{K} P_k e^{-j\frac{2\pi}{\lambda}(\tilde{d}_i - \tilde{d}_j)\sin\theta_k} + \boldsymbol{U}_{ij} \tag{13}$$

where P_k denotes the power of the kth signal of $\boldsymbol{p}_m$, $\tilde{d}_i$ and $\tilde{d}_j$ denote the ith and jth sensor of the difference co-array, respectively, and $\boldsymbol{U}_{ij}$ is the noise term of the covariance matrix.

Define

$$\tilde{r}_z = \tilde{r}_{i-j} = \sum_{k=1}^{K} P_k e^{-j\frac{2\pi}{\lambda}(\tilde{d}_i - \tilde{d}_j)\sin\theta_k} + \bar{\boldsymbol{U}}_{ij} \tag{14}$$

where $\bar{\boldsymbol{U}}_{ij}$ is the average of $\boldsymbol{U}_{ij}$ satisfied with $i - j = z$. $\tilde{r}_{i-j}$ is approximately equal to the $\tilde{\boldsymbol{R}}_{i-j}$, and we use $\tilde{r}_{i-j}$ to replace $\tilde{\boldsymbol{R}}_{i-j}$. This replacement operation may result estimation error, however, simulation demonstrates that our method shows superior performance. Interestingly when four level nested array is exploited, these autocorrelation lags are identical with those corresponding to a uniform array, which is contained by one half of fourth order difference co-array of the four level nested array. The difference is that the virtual noise is no longer temporally white. From section 2.3, it can be seen that the final virtual array is non-negative part of the fourth order difference co-array, which can be expressed as the set

$$\{\vec{x}_i + \vec{x}_j - \vec{x}_k - \vec{x}_l \,|\, \vec{x}_i + \vec{x}_j - \vec{x}_k - \vec{x}_l \geq 0, 1 \leq i, j, k, l \leq N\} \tag{15}$$

where $\vec{x}_i, 0 \leq i \leq N$ is the position of the physical array. After measuring all the covariance of the spatial lag, we can construct the covariance matrix of the final co-array to use the similar way just like (7). Then the classic method such as MUSIC can be employed to perform DOA estimation. Considering that the virtual noise is not temporally white, the generalized eigenvalue decomposition will be used to obtain the signal subspace and noise subspace.

For a four level nested array, each level with sensor N_1, N_2, N_3 and N_4, we summarize the proposed algorithm as follows:

1) Divide the received signal sequences $\boldsymbol{x}(t)_{t=1}^{T}$ into M segments, and $T = L \times M$.

2) Estimate the covariance matrix $\hat{\boldsymbol{R}}_m$ of each segment, $m = 1,..,M$.

3) Vectorize all the covariance matrices $\hat{\boldsymbol{R}}_m$ to get signal vector $\boldsymbol{y}_m$ in each segment.

4) Use the smoothing method to get $\boldsymbol{A}_1$ from $\boldsymbol{A}^* \odot \boldsymbol{A}$, and perform the same operation to all the $\boldsymbol{y}$ to obtain $\boldsymbol{y}_1$.

5) Compute the covariance matrix $\tilde{\boldsymbol{R}}$ of $\boldsymbol{y}_1$ and all the $\tilde{r}_z$ for $z = 0,1,..., N_1N_2N_3N_4 + N_1N_2N_3$.

6) Construct a new covariance matrix $\tilde{\boldsymbol{R}}_a$ using the similar way just like (7).

7) Apply MUSIC or some other subspace methods to $\tilde{\boldsymbol{R}}_a$ to estimate all the DOAs.

4. Simulation

In this section, we firstly verify that auto-correlation coefficients of the actual signal powers are far smaller than the cross-correlation coefficients of those. Then we demonstrate some numerical examples to show the superior performance of the proposed method. We consider a NLA with 6 sensors. We place them to form a four level and a two level nested array, whose positions are given by the set $\{d, 2d, 3d, 6d, 12d, 24d\}$ and $\{d, 2d, 3d, 4d, 8d, 12d\}$, respectively, where d is selected as half of the wavelength of the signals. We mainly compare the proposed algorithm to the method based on KR product and two level nested array[8]. For short, we call the two methods KR-ACM and KR, respectively. From section 3, we know the proposed method can identify 35 sources whereas the KR method can detect 23 sources. The sources are assumed to be nonstationary, which is generated by Table 2 in[7]. Note that our algorithm and KR algorithm are also applicable stationary, but need to use spatial smoothing[11] to preprocessing. The noise is assumed to be spatially and temporally white.

4.1. Correlation Coefficients of the Power of the Actual Signals

In this example, ten trials of correlation coefficients of the power of the actual signals are given in Table 1, where ρ_{ij} denotes the correlation coefficient and $Ti, i = 1, 2, ..., 10$ denotes the ith run. The correlation coefficient is defined as

$$\rho_{ij} = \frac{r_{ij}}{\sqrt{r_{ii} \cdot r_{jj}}}, 1 \le i, j \le K$$

where r_{ij} denotes the (i, j) element of the covariance matrix $\boldsymbol{B}$. Obviously $\rho_{ij} = 1$ when $i = j$. Here, we consider three non-stationary signals. The number of snapshots is 2000, which was divided into 20 segments, each with frame length 100. The SNR is 5dB. According to the literature[12], two vectors usually have a small correlation when the correlation coefficient of the two vectors are less than 0.3 and can be regarded as non-correlated when it is less than 0.1. From Table 1, it can be clearly seen that all the correlation coefficients is less than 0.3, and most of them are less than 0.2, which shows that the cross-correlation of the power of the actual signals is far less than the auto-correlation thereof.

Table 1. The Correlation Coefficients of the Power of the Actual Signals. ρ_{ij} Denotes the Correlation Coefficient of the ith and jth Signal Powers, $Ti, i=1,2,...,10$ denotes the ith run

	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
ρ_{12}	0.013	0.012	-0.078	-0.058	0.022	0.067	0.049	0.006	0.047	-0.058
ρ_{13}	0.095	-0.281	-0.077	0.003	0.088	-0.097	0.110	0.208	-0.150	0.001
ρ_{23}	-0.105	0.090	-0.042	-0.222	-0.236	-0.236	-0.143	0.103	-0.186	-0.109

4.2. Spatial Spectra

We now investigate the spatial spectrum of the proposed methods. Figure 1 depicts the spatial spectra of the two methods. Two sources are given by 18 and 20 degrees. The SNR is 0dB. The number of snapshot is 6400, which is divided to 80 segments, with 80 snapshots in each segment. It is clearly to say that the two methods can identify the two sources well, but the proposed method KR-ACM shows sharper peaks than the KR method.

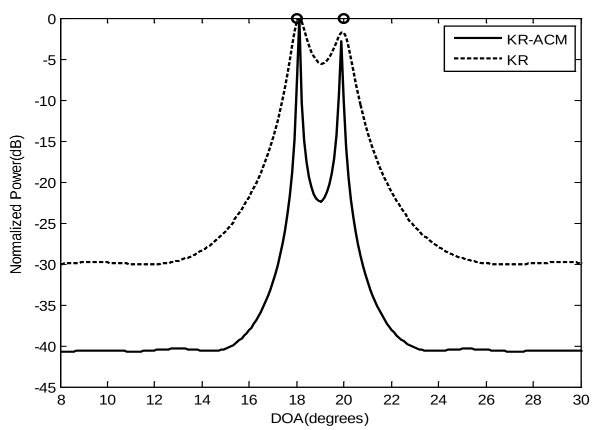

Figure 1. Spatial Spectra of the Two Methods, SNR=0dB, $L = M = 80$, $T = 6400$

4.3. RMSE versus SNR

In this example, a Monte Carlo simulation is carried out to evaluate the RMS angle error performance of the KR and the proposed method, with respect to signal-to-noise ratio (SNR). The RMSE is defined as

$$RMSE = \sqrt{E\{\frac{1}{K}\sum_{k=1}^{K}(\hat{\theta}_k - \theta_k)^2\}}$$

where $\hat{\theta}_i$ and θ_i denotes the ith estimating and real bearing angle, respectively. We consider two sources radiated from -15 and 22 degrees, and the frame length L and the number of frame M are both 100. 1000 Monte Carlo trials are run. The RMSE of the two methods as a function of SNR is given at Figure 2. It can be noted that the proposed method performs better in low SNR, compared to the KR algorithm, which is due to that the KR-ACM method produces more sensors than the KR method. But when SNR is large enough, KR outperforms KR-ACM.

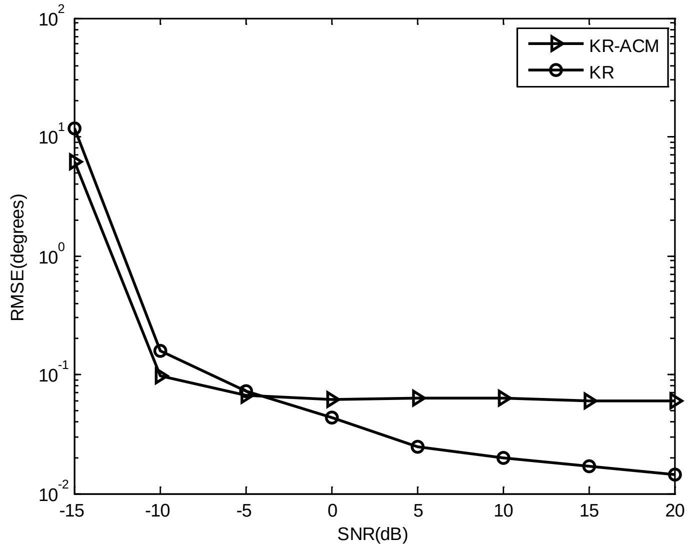

Figure 2. RMSE versus SNR of the KR and the Proposed Method

4.4. RMSE versus Snapshot

From the above examples, we can note that thousands of snapshots were used to estimate the DOAs. In the experiment, we investigate that how many snapshots almost are needed in

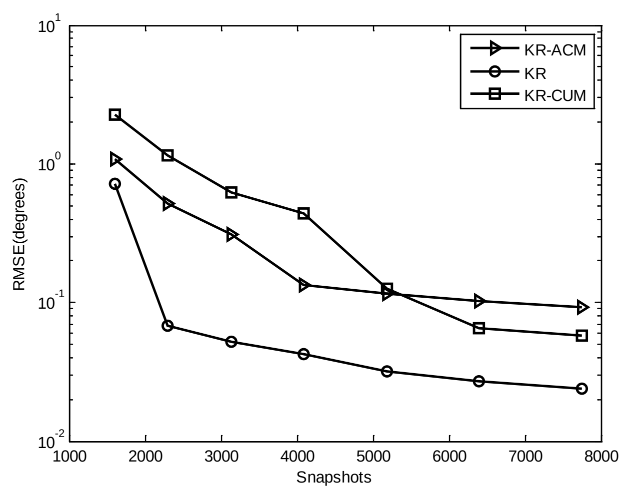

Figure 3. RMSE versus the Number of Snapshots of the KR, KR-CUM and the Proposed Method, SNR=10dB

the KR-ACM method. We compare the KR-ACM method to the method in [9], which is based on KR product and fourth order cumulants with four level nested array. We denote it KR-CUM for short. The RMSE of the KR, KR-CUM and KR-ACM with respect to snapshot is depicted in Figure 3, where the two sources also are given at 5 and 15 degrees, and SNR is 10dB. 1000 Monte Carlo trials are carried out. The performance of all the three methods improves significantly with the increasing of the snapshots, and the KR method shows a lower RMSE than the two other methods. However, fewer snapshots are needed for the KR-ACM method to acquire a steady variance, compared to the KR-CUM method.

4.5. Resolution versus SNR

In the second example, we can note that the KR-ACM method showed higher resolution than the KR algorithm. We now show it specifically. We consider two closely space signals placed at 10 and 12 degrees. By the definition in [13], the two sources are identified in a trial if both the $|\hat{\theta}_1 - \theta_1|$ and $|\hat{\theta}_2 - \theta_2|$ are smaller than $|\theta_1 - \theta_2|/2$, where θ_1 and θ_2 denote the true DOAs, and $\hat{\theta}_1$ and $\hat{\theta}_2$ denote the estimating DOAs. We depict the probability of resolution versus SNR for T=10000 snapshots, averaged over 1000 Monte Carlo trials in Figure 4. It is clearly to note that the KR-ACM method outperforms the KR method.

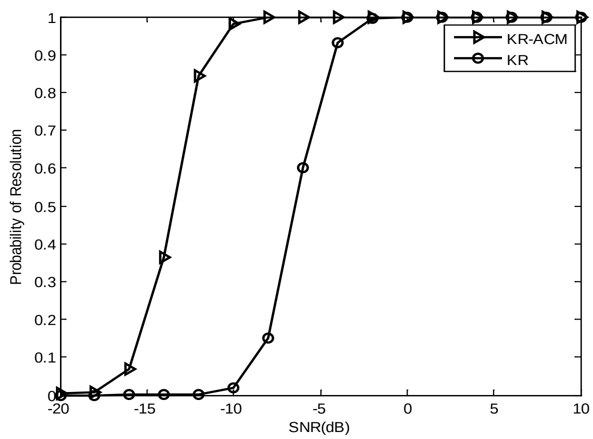

Figure 4. Resolution versus SNR of the KR and the Proposed Method,
$T = 100 \times 100 = 10000$

5. Conclusions

In this paper, a novel approach is proposed to construct half of the fourth order difference co-array. We make use of only the second order statistics of the observed signals and KR product, hence Our approach is applicable to Gaussian signals. And by exploiting four level nested array, we can increase the DOF to $O(N^4)$ only using N sensors. Our method shows higher resolution compared to the standard KR method using two level nested array. In the future area, we will extend to the proposed method to wideband case, and investigate the effectiveness of the proposed method in a real world.

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References

[1] A. Moffet, "Minimum-redundancy linear arrays", Antennas and Propagation, IEEE Transactions on, vol. 16, no. 2,**(1968)**, pp. 172-175.
[2] S. U. Pillai, Y. Bar-Ness and F. Haber, "A new approach to array geometry for improved spatial spectrum estimation", Proceedings of the IEEE, vol. 73, no. 10, **(1985)**, pp. 1522-1524.
[3] S. Pillai and F. Haber, "Statistical analysis of a high resolution spatial spectrum estimator utilizing an augmented covariance matrix", Acoustics, Speech and Signal Processing, IEEE Transactions, vol. 35, no. 11, **(1987)**, pp. 1517-1523.
[4] M. C. Dogan and J. M. Mendel, "Applications of cumulants to array processing", aperture extension and array calibration, Signal Processing, IEEE Transactions on, vol. 43, no. 5,**(1995)**, pp. 1200-1216.
[5] P. Chevalier, L. Albera, A. Ferréol and P. Comon, "On the virtual array concept for higher order array processing. Signal Processing", IEEE Transactions on, vol. 53, no. 4, **(2005)**, pp. 1254-1271.
[6] P. Chevalier, A. Ferréol and L. Albera, "High-Resolution Direction Finding From Higher Order Statistics: The 2q-MUSIC Algorithm", Signal Processing, IEEE Transactions on, vol. 54, no. 8,**(2006)**, pp. 2986-2997.
[7] W. K. Ma, T. H. Hsieh and C. Y. Chi, "DOA estimation of quasi-stationary signals with less sensors than sources and unknown spatial noise covariance: a Khatri–Rao subspace approach", Signal Processing, IEEE Transactions on, vol. 58, no. 4, **(2010)**, pp. 2168-2180.
[8] P. Pal and P. Vaidyanathan, "Nested arrays: a novel approach to array processing with enhanced degrees of freedom", Signal Processing, IEEE Transactions on, vol. 58, no. 8, **(2010)**, pp. 4167-4181.
[9] P. Pal and P. Vaidyanathan, "Multiple Level Nested Array: An efficient geometry for 2qth order cumulant based array processing", Signal Processing, IEEE Transactions on, vol. 60, no. 3, **(2011)**, pp. 1253-1269.
[10] D. Pearson, S. U. Pillai and Y. Lee, "An algorithm for near-optimal placement of sensor elements", Information Theory, IEEE Transactions on, vol. 36, no. 6, **(1990)**, pp. 1280-1284.
[11] P. Stoica and A. B. Gershman, "Maximum-likelihood DOA estimation by data-supported grid search", Signal Processing Letters, IEEE, vol. 6, no.10, **(1999)**, pp. 273-275.
[12] J. Cohen, "Statistical power analysis for the behavioral sciences", Lawrence Erlbaum, NJ, **(1988)**.
[13] T. J. Shan, M. Wax and T. Kailath, "On spatial smoothing for direction-of-arrival estimation of coherent signals", Acoustics, Speech and Signal Processing, IEEE Transactions, vol. 33, no. 4, **(1985)**, pp. 806-811.

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