Image Denosing Method based on Tensor Driven Linear Integral Convolutions

Shenghua Gu[1], Lu Liu[2], Yuhui Zheng[2] and Shunfeng Wang[3]

[1]Division of Science & technology, Nanjing University of Information Science & Technology, Nanjing 210044, China
[2]College of Computer & Software, Nanjing University of Information Science & Technology, Nanjing 210044, China
[3]College of Bin Jiang, Nanjing University of Information Science & Technology, Nanjing 210044, China
mars811_nj@hotmail.com

Abstract

In the past decades, many nonlinear partial differential equation (PDE) based denosing methods have been suggested, among which the curvature-preserving PDE image denosing method is one of the outstanding models. To effectively preserve image edge well, a tensor driven linear integral convolutions based Image Denosing Method is proposed, which employ total variation flow based nonlinear structure tensor to analysis different integral curve. It is a new implementation of our former work [10]. Experimental results show that the new method can achieve better denosing results in a variety of standard test images, and the new approach shows superior performance on edge and curvature preserving face image and texture image.

Keywords: *Image denoising, Structure Tensor, Linear Integral Convolution*

1. Introduction

In the field of digital processing, denosing is a most important research subject, on the one hand, it can be able to suppress noise effectively, and on the other hand, the study of denosing can promote solution of other signal restoration problems due to that it is very closely associated with regularization method and image modeling theory in the terms of theory. A relatively complete review of denosing can be found in [1].

The existing denosing methods can be parted into two groups: (1) Sparse representation [2]; (2) Smoothing denosing [3-10]. The former regards that the signal can be sparse decomposition, and the construction of a sparse dictionary is one of the key problems. The latter views noise as local oscillation signal which can be removed by smoothing method. As the sparse decomposition method generates larger time consumption comparatively, this paper focuses on the research of smoothing denoising method.

Smoothing denosing methods mainly concentrate on two aspects: (1) PDE based methods [3-7]; (2) Non-local mean filter [9]. PDE based method is a local smoothing method, and Nonlocal mean filter is a non-local filtering. Local denosing methods carry out certain weighted averaging operations on the intensity values of a local neighborhood of the pixel under consideration, and have developed fast in the last 20 years, which has been applied to many fields of computer vision successfully. But in recent years, the non-local filter and its variations gradually became the hot research topic. These methods perform some kind of weighted averaging to restore gray value of the pixel being processed, and in this case not only it incorporates intensity values of pixels in a sub-image or entire image, but also employs patch-based technique. Although nonlocal mean filter can yield outstanding results, theirs time consumptions are relatively high. Yet from the standpoint of practical application the local adaptive denosing still has development opportunity.

Nonlinear PDE based method is introduced by the pioneering work in [4], which used scalar-valued decreasing function to control the process of smoothing. In facts, this model is isotropic denosing method. The most representative of anisotropic PDE denosing methods are tensor-driven method such as divergence-based PDE first presented by Weichert in work [5], which replaces the scalar-valued function in [4] by a matrix-valued diffusion tensor that describes the direction of smoothing, computed from the so-called structure tensor. Subsequently, a tensor-driven trace-based PDE model for color image was introduced in [6], which replaces the divergence operator of divergence-based PDE in [5] by the trace operator. Meanwhile a link between [4] and [5] was made in the work. As the trace based PDE can't preserve the curvature structure very well, a refreshing curvature-preserving PDE filtering model was suggested in [7], which has better performance on curvature preserving and was implemented by averaging of different Gaussian-pondered Line Integral convolutions along the integral curves of vector field that obtained from projecting the square root of diffusion tensor into different orientations.

In our former work [10], we propose a improved version of the curvature preserving PDE model, which adopted weighted strategy to design adaptively weight coefficients for PDEs under different vector field. In this paper we proposed a new implement version of the former model. In other words, we implemented the weighted Curvature Preserving PDE denosing by directly using Tensor Driven Line Integral convolutions. The remainder of this paper is as follows: Section2 introduces curvature preserving PDE related models and our new implement in detail. The new approach is tested on a standard test image database in Section3. Section4 concludes the whole work.

2. Tensor Driven Linear Integral Convolutions Denosing methods

Defined over a bounded domain $\Omega \subset R^2$, I and I^0 are respectively the restored Although the trace based PDE method does a good job in denosing with edge preservation, it usually leads to rounding effect due to that the method does not take curvatures into account. For this, Tschumperle[7] proposed the curvature preserving PDE :

$$\frac{\partial I}{\partial t} = tr(DH) + \frac{2}{\pi}\int_{\theta=0}^{\pi} (\nabla I)^T J_{\sqrt{D}\alpha_\theta} d\theta \qquad (2.1)$$

where I represents the image, $tr(\cdot)$ represents the matrix trace, $H = \begin{pmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{pmatrix}$ is Hessian matrix , I_{xx} represents the second partial of I in x direction, $\alpha_\theta = (\cos\theta, \sin\theta)$ is unit direction angle, D is called diffusion tensor which reflects the image directional information, $\sqrt{D}\alpha_\theta = (u, v)$ represents vector field which obtained from projecting the diffusion tensor into orientation α_θ .

In order to improving the curvature preserving PDE, in the former work we adopted weighted strategy to propose a weighted curvature preserving PDE model:

$$\frac{\partial I}{\partial t} = tr(DH) + \frac{\int_{\theta=0}^{\pi} W\left(\sqrt{D}\alpha_\theta\right)(\nabla I)^T J_{\sqrt{D}\alpha_\theta} d\theta}{\int_{\vartheta=0}^{\pi} W\left(\sqrt{D}\alpha_\theta\right)} \qquad (2.2)$$

where $W\left(\sqrt{D}\alpha_\theta\right)$ is the weighted function of vector field $\sqrt{D}\alpha_\theta$. For the detail of designing weighted function, we refer to [10].

In the facts, which equals to weighted averaging of different Line Integral convolutions utilizing local information to adaptively design weight coefficients for different integral curves. Beyond that, the implement of equation (2) often encounter the problem of iterations selection. For the above reason, here we give out the implementation steps for detail using Line Integral convolutions Operations.

To construct the diffusion tensor, we compute the nonlinear structure tensor as follow:

$$\partial_t S_{ij} = div\left(\frac{1}{\sqrt{\sum_{m,n=1}^{2} \left|\nabla s_{m,n}\right|^2}} \nabla S_{ij}\right) \qquad (2.3)$$

where $S = \begin{pmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{pmatrix}$ is called structure tensor. Different from the above tensor driven PDE models, we use the nonlinear structure tensor to design diffusion tensor. The linear structure tensor employed linear technology to smoothing the four channels of structure tensor. When using it to analysis the image local information, the results are not accurate.

When we gain the nonlinear structure tensor, the diffusion tensor can be constructed followed by the work [7, 10]. Then project the square of diffusion tensor into different directions α_θ, and track the related integral curves from different vector field. After this, perform a Line Integral Convolution of image along each integral curve. At the last, compute weighted averaging of different convolutions results using weighted function $W\left(\sqrt{D}\alpha_\theta\right)$.

In equation (3.3) , h is a sensitive parameter. Recently, Coupe et al [14] have pointed that h needed to take into account the size of the patch. Subsequently, they estimated the noise level σ via pseudo-residuals and proposed a automatical tuning of h for 3D image denoising[16]. Here, we define pseudo-residuals as:

$$\varepsilon_i = \left(2I_i^0 - \left(I_{i+(1,0)}^0 + I_{i+(0,1)}^0\right)\right) / \sqrt{6} \qquad (2.4)$$

After this, the standard deviation of noise $\hat{\sigma}$ is computed as fellow:

$$\hat{\sigma} = \sum_{i \in \Omega} \varepsilon_i^2 \Big/ N \qquad (2.5)$$

Finally, we get $h = \beta \hat{\sigma}^2$, where constant β allows to adjust the automatic estimation of h. In [14], it have been suggested that the best value of β is 0.7 for low levels of noise and 1 for high levels of noise.

3. Results and Discussion

In the experimental evaluation, we compare the performance of four image denosing mehtods: the trace based PDE method, curvature preserving PDE method, weighted preserving PDE method (called weighted PDE for short), and our method. All methods were implemented in MATLAB and were test on a commonly-used image database including Lena, Barbara, Boat, House, and Pepper. As a quantitative evaluation for quality we stick to the peak signal-to-noise ratio (PSNR). In all experiments, image intensities were tuned to the range between 0 and 1. All computers times are on a Pentium IV 2.4 GHz.

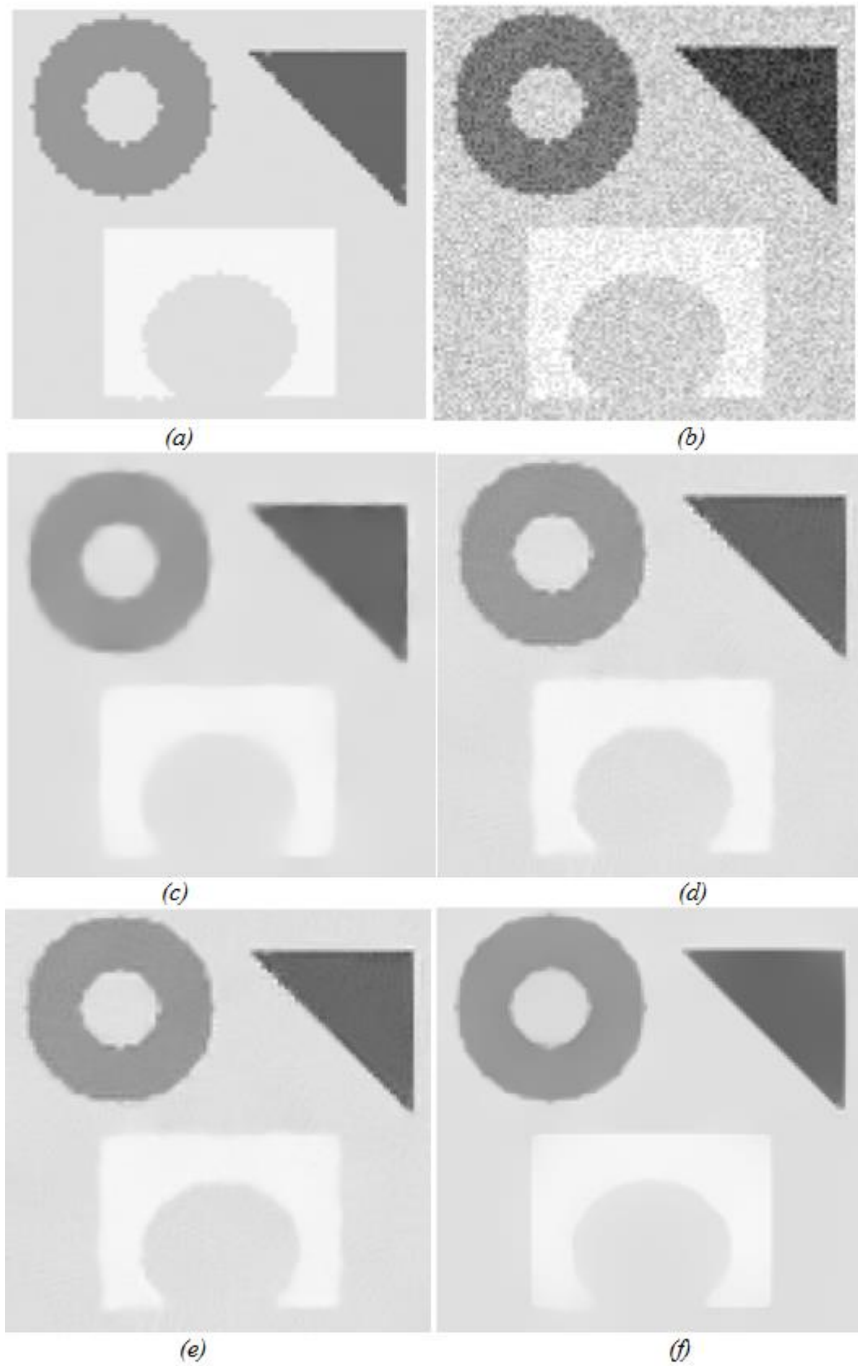

Figure 1. Boat image: (a) original image; (b) noisy image; (c) Trace based PDE method; (d) Curvature Preserving PDE method; (e) Weighted Cuvature Preserving PDE method; (f) Our method

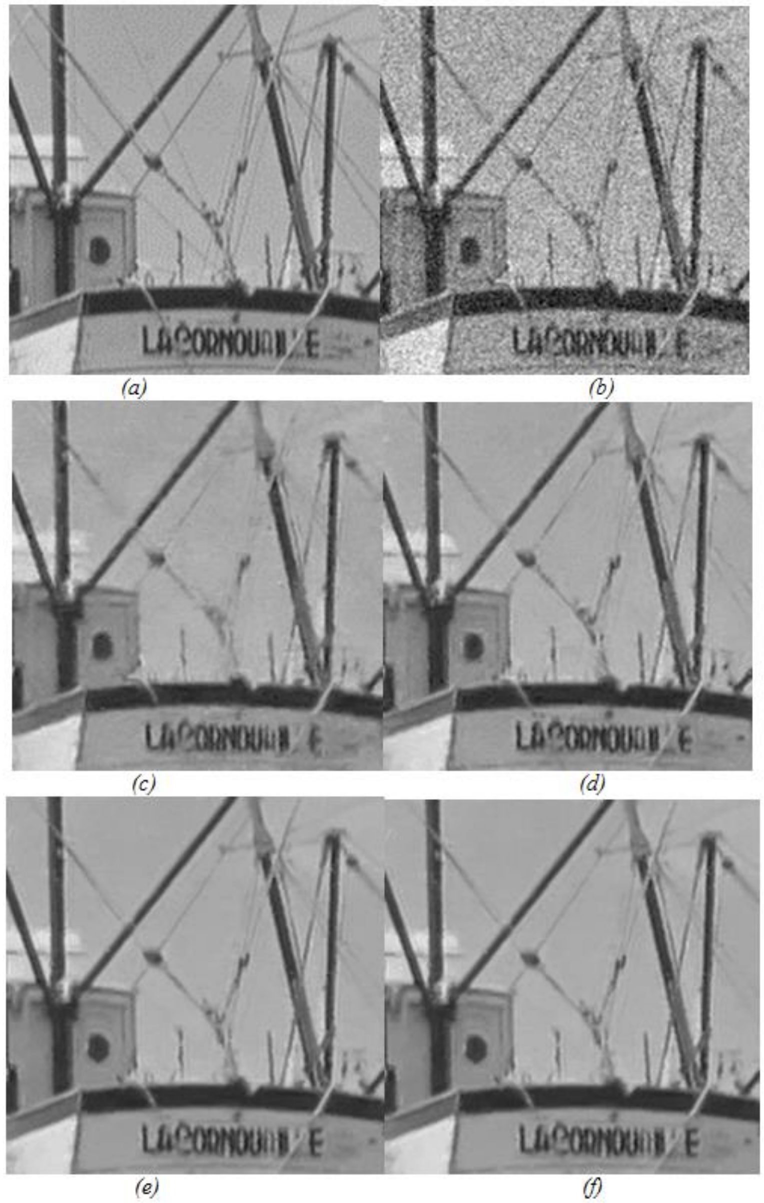

Figure 2. Boat image: (a) original image; (b) noisy image; (c) Trace based PDE method; (d) Curvature Preserving PDE method; (e)Weighted Cuvature Preserving PDE method; (f) Our method

Figure 1 shows the denoising results of the synthetical image using the 4 methods. Figure 2(a) is the noisy image and Figure 1(c)-(f) are the filtering results using the trace based PDE method, curvature preserving PDE method, Weighted Cuvature Preserving PDE method and our method, respectively. In Figure 1(c), the trace based PDE method resulted in better noise removal but it produces a vague image of points. Because the curvature preserving PDE method adds the curvature preserving item, it has the ability of point keeping, but fuzzy phenomenon at the edge and points are relatively obvious in Figure 2(d). Figure 2(e) shows the result of the method w Weighted Cuvature Preserving PDE method, which demonstrates its ability to keep points resulting in good performance in terms of keeping edges. Figure 2(f) shows the result of our method. From this figure, it is seen that our method can keep the edges and corners of the image well.

Figure 2 shows the denosing results of the four methods for boat image. In this image, for detail weak image region, Weight Cuvature Preserving PDE method and our method achieved well result. Compared with the two methods, the Curvature Preserving PDE method and Trace based PDE method yieded over smoothed result near image edge. By comparison of Figure 1(e) and Figure 3(f), we can see that our method preserves the texture structures very well while efficiently removing the noise. Meanwhile, the computational complexity of denosing method is lower than the Curvature Preserving PDE method. See Table 2 for details the texture structures very well while efficiently removing the noise.

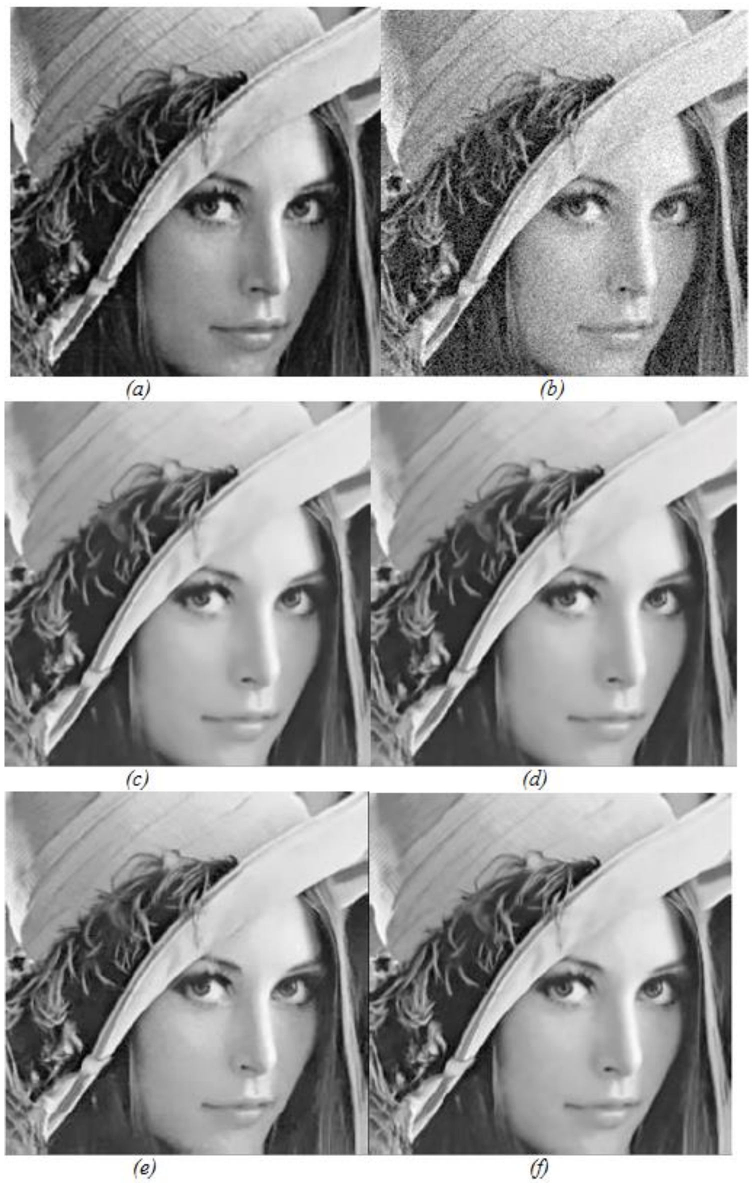

Figure 3. Lena image: (a) original image; (b) noisy image; (c)Trace based PDE method; (d) Curvature Preserving PDE method; (e) Weighted Cuvature Preserving PDE method; (f) Our method

Figure 3 shows the filtering results of our methods on the Lena image. From the filtering results, we can see the human face is relatively smooth using trace based PDE method and curvature preserving PDE method, while the Weighted Cuvature Preserving PDE method shows staircase effect. Compared with Figure 6(e), detailed information such as Lena's lashes and the stripes on the hat is well preserved in Figure 6(f), demonstrating the better filtering performance of our method.

Table 1. PSNRs of four PDE Image Denosing methods for standard test images with standard deviation 20

Image	boat	Babara	House	Lena	Pepper
Trace based PDE	30.39	29.40	31.49	30.82	29.11
Curvature Preserving PDE	31.42	30.92	32.76	31.12	30.03
Weighted PDE	32.25	31.43	33.27	32.04	31.27
Our method	32.70	31.88	33.71	32.86	32.20

Table 2. Computation times of four PDE Image Denosing Methods(s)

Image	Boat	Babara	House	Lena	Pepper
Trace based PDE	17.52	7.269	17.72	17.29	17.21
Curvature Preserving PDE	18.18	8.260	18.26	18.20	18.51
Weighted PDE	18.66	8.432	18.77	18.64	19.07
Our method	18.80	8.659	18.86	18.65	18.82

Table 1 and Table 2 show the comparisions of the PSNR and computational complexity results of the four image denosing methods. Table 1 indicates that for the denoising result of the four methods, the PSNR values of our method are bigger, followed by former work, *i.e.*, weighted Cuvature Preserving PDE method. From Table 2, we can see that although the denoising results of weighted Cuvature Preserving PDE method are outstanding, the computational complexity of this method is higher than the new implement. The new method is relatively higher capability with lower cost.

4. Conclusions

Based on our form work which proposed a weighted curvature-preserving PDE for image denosing, in this paper we suggested a different implementation using Linear Integral Convolutions. The new tensor driven Linear Integral Convolutions denoising method keep better to the edge and curvature geometric structure of the image while filtering. Its time consumption is relatively lower and has a certain degree enhancement to local image. It is a denosing method with higher cost performance.

Acknowledgements

The authors wish to thank National Natural Science Foundation of China(61173072, 61003209), the Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions, Natural Science Foundation of Jiangsu Province(BK2011824, BK2011825).

References

[1] V. Katkovnik, A. Foi, K. Egiazarian and J. Astola, "From local kernel to nonlocal multiple-model image denoising", International Journal of Computer Vision, vol. 86, no. 1, **(2010)**, pp.1-32.
[2] M. Protter, I. Yavneh and M. Elad, "On MMSE and MAP De-noising under Sparse Representation Modeling over a Unitary Dictionary", IEEE Transactions on Signal Processing, vol. 59, no. 8, **(2011)**, pp. 3526-3535.
[3] P. Rodriguez and B. Wohlberg, "Efficient minimization method for a generalized total variation functional", IEEE Transactions on Image Processing, vol. 18, no. 2, **(2009)**, pp. 322-332.
[4] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion", IEEE Transactions on Pattern Recognition and Machine Intelligence, vol. 12, no. 7, **(1990)**, pp. 629-639.
[5] J. Weickert and C. Schnorr, "A theoretical framework for con-vex regularizers in PDE-based computation of image motion", International Journal of Computer Vision, vol. 45, no. 3, **(2001)**, pp. 245-264.
[6] D. Tschumperle and R. Deriche, "Vector-valued image regularization with PDE's: a common framework for direrent applications", IEEE Transactions on Pattern Recognition and Machine Intelligence, vol. 27, no. 4, **(2005)**, pp. 506-517.
[7] D. Tschumperle, "Fast anisotropic smoothing of multi-valued images using curvature-preserving PDE's", International Journal of Computer Vision, vol. 68, no. 1, **(2006)**, pp. 65-82.
[8] M. R. Hajiaboli, M. O. Ahmad and C. Y. Wang, "An Edge-Adapting Laplacian Kernel For Nonlinear Diffusion Filters", IEEE Transactions on Image Processing, vol. 21, no. 4, **(2012)**, pp.1561-1572.
[9] A. Buades, B. Coll and J. M. Morel, "A non-local algorithm for image denoising", In: Proceedings of the IEEE International Conference on Computer Vision and Pattern Recognition, IEEE Press, New York, **(2005)**, pp. 60-65.
[10] Y. H. Zheng and X. P. Zhao, "Image Filtering Using Weighted Curvature Preserving PDE, IEEE Press,New York, **(2005)**, pp. 2536-2539.

Authors

Shenghua Gu, Lecturer at the School of Computer and Software, Nanjing University of Information Science and Technology. His research interest covers image processing, pattern recognition Corresponding author of this paper.

Lu Liu, student at the School of Computer and Software, Nanjing University of Information Science and Technology. Her research interest covers image processing.

Yuhui Zheng, Lecturer at the School of Computer and Software, Nanjing University of Information Science and Technology. His research interest covers image processing, pattern recognition, and remote sensing image restoration.

Shunfeng Wang, Professor at the School of BinJiang, Nanjing University of Information Science and Technology. His research interest covers image processing, pattern recognition.