Localization Algorithm based on Positive Semi-definite Programming in Wireless Sensor Networks

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Abstract

In this paper, we propose an algorithm to locate an object with unknown coordinates based on the positive semi-definite programming in the wireless sensor networks, assuming that the squared error of the measured distance follows Gaussian distribution. We first obtain the estimator of the object location based on the maximum likelihood criterion; then considering that the estimator is a non-convex function with respect to the measured distances between the object and the anchors with known coordinates, we transform the non-convex optimization to convex one by the positive semi-definite relaxation; and finally we take the optimal solution of the convex optimization as the estimated value of the object location. Simulations results show that our algorithm is superior to the R-LS algorithm regardless of whether the object is located within the convex hull composed of the anchors.

Keywords: *Wireless Sensor Networks; Localization; Positive Semi-definite Programming; Convex Optimization*

1. Introduction

Wireless sensor network (WSN) is essential an ad hoc network which is composed of a lot of sensor nodes in the monitoring area. WSN could be applied to many domains, such as military, industry, traffic, and has wide applications. Localization is an important function and is one of the core supporting technologies for the WSN. WSN is mainly used to monitoring and tracking, and these two applications in most cases require the location information of the target node. And some route protocols and management mechanisms designed for the WSN also need the nodes locations.

Existing localization principles can be roughly divided into the following four categories: the first is based on the received signal strength (RSS) or energy [1]; the second is based on the signal time of arrival or time difference of arrival [2]; the third is based on the signal angle of arrival [3]; and the final is based on the combination of above aspects [4]. Regardless of any principle, we need to achieve the distance or angle from the target node with unknown coordinates to the anchors with known coordinates, and based on the multilateration or the multangularation, we acquire the maximum likelihood estimator or the least square estimator of the object location; and finally estimate the location using the optimization techniques.

However, the estimator of the object location is usually a non-linear and non-convex function of the measured values of the distances, and it is difficult to directly achieve its global optimal solution using existing optimization methods. To the best of our knowledge, the common concerned solutions could be divided into three categories: first, to make the whole optimization problem into a convexity by abandoning the non-linear constraint part [5]; second, to linear the optimization problems by plane intersect [6], spherical interpolation [7] or spherical intersection **Error! Reference source not found.**; and the third is based on the second order cone relaxation technique [9, 10] or the positive semi-definite cone relaxation [11, 12] to convex the optimization problem. In above three methods, the third method is a commonly used method.

Tseng [10] pointed out that positive semi-definite programming can achieve better performance than the second order cone programming, so the localization algorithm based on the positive semi-definite programming has attracted more researchers. However, optimization problems with different object functions and different distance models are solved with different positive semi-definite relaxations, which correspond to different localization algorithms. Chen [13] proposed a localization algorithm when the square error of the measured distance follows Gaussian distribution and objective function is to minimize the maximum of the all the distance errors. Different with Chen, Wang [14] is to minimize the sum of all the distance errors. Beck [15] and Pinar [16] assume the error of the measured distance follows Gaussian distribution, and the objective function is the same to Wang [14].

In this paper, we also assume that the squared error of the measured distance follows Gaussian distribution and propose an algorithm to locate the object based on the positive semi-definite programming. We first obtain the estimator of the object location based on the maximum likelihood criterion; then considering that the estimator is a non-convex function with respect to the measured distances between the object and the anchors with known coordinates, we transform the non-convex optimization to convex one by the positive semi-definite relaxation; and finally we take the optimal solution of the convex optimization as the estimated value of the object location. We compare our algorithm with the R-LS algorithm when the object is in and not in the convex hull composed of the anchors, and the results show that our algorithm is superior to R-LS algorithm regardless of whether the object is located within the convex hull.

The rests of this paper are organized as follows: Section 2 is the distance model; Section 3 is the proposed semi-definite programming algorithm; Section 4 is the simulations and analysis; and this paper is concluded in Section 5.

2. Distance Model

We assume that in a d dimensions space, there are N sensor nodes which is called as anchors with known coordinates denoted as N column vectors $s_1, \cdots, s_N$, and there is one object node with unknown coordinates denoted as a column vector u. Then the measured distance d_i between anchor i and the object is expressed by **Error! Reference source not found.**:

$$d_i^2 = \| u - s_i \|^2 + n_i, \quad i = 1, \cdots, N \tag{1}$$

where n_i is the measured noise which follows Gaussian distribution with zero mean and variance δ^2.

Based on the maximum likelihood criterion, we could obtain the estimator $\hat{u}$ of the object node.

$$\hat{u} = \arg\min_{u} \sum_{i=1}^{N} (d_i^2 - \| u - s_i \|^2)^2 \tag{2}$$

3. Semi-definite Programming

3.1. Relaxation of the Positive Semi-definite Programming

Obviously, (2) is non-convex and we can hardly obtain its global minimum. So we need to change the expression of (2) and take some relaxation to make it convex.

Let $t_i = d_i^2 - \| u - s_i \|^2$, then (2) is equivalent to:

$$\begin{aligned} &\hat{u} = \arg\min_{u,t_i} \sum_{i=1}^{N} t_i^2 \\ &s.t. \quad t_i = d_i^2 - \| u - s_i \|^2 \end{aligned} \tag{3}$$

Continue to let $y_i \geq t_i^2$, then (3) could be rewritten to:

$$\begin{aligned} &\hat{u} = \arg\min_{u,t_i,y_i} \sum_{i=1}^{N} y_i \\ &s.t. \quad \begin{cases} t_i = d_i^2 - \| u - s_i \|^2 \\ y_i \geq t_i^2 \end{cases} \end{aligned} \tag{4}$$

Now, we define a column vector $\bar{u} = [u^T, 1]^T$, which contains $d+1$ elements, and let $U = \bar{u}\,\bar{u}^{\mathrm{T}}$; we also define $(d+1)\times(d+1)$ matrix $S_i = \begin{bmatrix} I_{d\times d} & -s_i \\ -s_i^T & s_i^T s_i \end{bmatrix}, i = 1, \cdots, N$, and then the equation constraint in (4) could be reformulated as:

$$t_i = d_i^2 - trace(S_i U) \tag{5}$$

Substitute (5) into (4), we will get:

$$\begin{aligned} &\hat{u} = \arg\min_{U,t_i,y_i} \sum_{i=1}^{N} y_i \\ &s.t. \quad \begin{cases} t_i = d_i^2 - trace(S_i U) \\ y_i \geq t_i^2 \\ U = \bar{u}\,\bar{u}^{\mathrm{T}} \\ i = 1, \cdots, N \end{cases} \end{aligned} \tag{6}$$

Obviously, U is a $(d+1)\times(d+1)$ semi-definite matrix. Using semi-definite relaxation, i.e. substituting $U \geq 0$ for $U = \bar{u}\,\bar{u}^{\mathrm{T}}$, and letting $Y_i = \begin{bmatrix} y_i & t_i \\ t_i & 1 \end{bmatrix}$, (6) is changed to:

$$\hat{u} = \arg\min_{U, t_i, Y_i} \sum_{i=1}^{N} Y_i(1,1)$$
$$s.t. \begin{cases} t_i = d_i^2 - trace(S_i U) \\ Y_i \geq 0 \\ U \geq 0 \\ Y_i(2,2) = 1 \\ U(d+1, d+1) = 1 \\ t_i = Y_i(1,2) \\ i = 1, \cdots, N \end{cases} \tag{7}$$

Observing (7), we could easily find that it is a positive semi-definite programming problem, so we could solve it using SeDuMi [18]. It is worthy to notice that although we could obtain the global minimum of (7), it is not the global optimal solution of (6) due to the relaxation of the constraint in (6).

3.2. SeDuMi Formatting

In this subsection, we will demonstrate how to transform (7) into the standard format that the SeDuMi could dealt with in two dimensions space.

SeDuMi is a Matlab toolbox provided by a third party and can bee used to solve optimization problems including linear programming, quadratic programming and semi-definite programming.

The standard primal form for the semi-definite programming is:

$$\min c^T x$$
$$s.t. \begin{cases} Ax = b \\ x \in K \end{cases} \tag{8}$$

And its dual form is

$$\max b^T x$$
$$s.t. \quad c - A^T y \in K \tag{9}$$

Where K is the set of semi-definite cone.

Usually, we invoke the SeDuMi to solve the semi-definite programming problem by its dual form, which means that we should transform (7) to confirm to the dual form.

In the two dimensions space, U is a 3×3 symmetric matrix, and $U(3,3)=1$, so U could be expressed by $\begin{bmatrix} U^1 & U^2 & U^3 \\ U^2 & U^4 & U^5 \\ U^3 & U^4 & 1 \end{bmatrix}$, which contains 5 variants; Similarly, Y_i could be expressed by $\begin{bmatrix} Y_i^1 & Y_i^2 \\ Y_i^2 & 1 \end{bmatrix}$, which contains 2 variants. So we conclude that there are $5+3N$ variants in (7), which forms a column vector X denoted as $X=[U^1,\cdots,U^5,t_1,\cdots,t_N,Y_1^1,Y_1^2,\cdots,Y_N^1,Y_N^2]^T$.

We let $b=[\underbrace{0,\cdots,0}_{5+N},\underbrace{-1,0,-1,0,\cdots,-1,0}_{2N}]$, and then the objective function of (7) could be expressed by $b\times X$.

For the first equation in (7), it could be expressed by:

$$\begin{aligned}&\begin{bmatrix} d_1^2-(s_1^1)^2-(s_1^2)^2 \\ \vdots \\ d_N^2-(s_N^1)^2-(s_N^2)^2 \end{bmatrix}_{N\times1} \\ &-\begin{bmatrix}1_{N\times1} & 0_{N\times1} & -2_{N\times1} & 1_{N\times1} & -2_{N\times1} & I_{N\times N} & 0_{N\times2N}\end{bmatrix}\times X \\ &=0_{N\times1}\end{aligned} \tag{10}$$

So, we could obviously let :

$$A_1=\begin{bmatrix}1_{N\times1} & 0_{N\times1} & -2_{N\times1} & 1_{N\times1} & -2_{N\times1} & I_{N\times N} & 0_{N\times2N}\end{bmatrix} \tag{11}$$

and

$$c_1=\begin{bmatrix} d_1^2-(s_1^1)^2-(s_1^2)^2 \\ \vdots \\ d_N^2-(s_N^1)^2-(s_N^2)^2 \end{bmatrix}_{N\times1} \tag{12}$$

For the second equation in (7), it could be expressed by:

$$0_{N\times1}-\left[\underbrace{0,\cdots,0}_{i+4},-1,\underbrace{0,\cdots,0}_{N-1},-1,\underbrace{0,\cdots,0}_{N-i}\right]X=0_{N\times1} \tag{13}$$

So we could let:

$$A_2=[\underbrace{0,\cdots,0}_{i+4},-1,\underbrace{0,\cdots,0}_{N-1},-1,\underbrace{0,\cdots,0}_{N-i}]. \tag{14}$$

and

$$c_2=0_{N\times1} \tag{15}$$

For the first matrix inequation in (7), it could be expressed by:

$$Y_i = Y_i^1 B_1 + Y_i^2 B_2 + B_3 \tag{16}$$

where $B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Obviously, we could obviously let:

$$A_{Yi} = \left[0_{4\times(5+N)}, \underbrace{0,\cdots,0}_{2\times(i-1)}, -vec(B_1), -vec(B_2), \underbrace{0,\cdots,0}_{2\times(N-i)} \right] \tag{17}$$

and

$$c_{Yi} = vec(B_3) \tag{18}$$

For the second matrix inequation in (7), it could be expressed by:

$$U = U^1 B_4 + U^2 B_5 + U^3 B_6 + U^4 B_7 + U^5 B_8 + B_9 \tag{19}$$

Where $B_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B_5 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $B_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $B_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$,

$B_8 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $B_9 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Obviously, we could let :

$$A_U = \left[0_{9\times5}, -vec(B_4), -vec(B_5), -vec(B_6), -vec(B_7), -vec(B_8), \underbrace{0,\cdots,0}_{2N} \right] \tag{20}$$

and

$$c_U = vec(B_9) \tag{21}$$

Finally, we let :

$$C = (c_1; c_2; c_{Y1}; \cdots; c_{YN}; C_U) \tag{22}$$

and

$$At = (A_1; A_2; A_{Y1}; \cdots; A_{YN}; A_U) \tag{23}$$

and (7) is transformed to the dual form and could be solved by SeDuMi now.

4. Simulations and Analysis

We assume that in the 2D space, there are four anchors with coordinates are (0, 0), (0, 10), (10, 10) and（10, 0）respectively and one object node, which is shown in Figure 2.

In order to evaluate the performance of our algorithm, we compare it with the R-LS algorithm **Error! Reference source not found.**.

According to the R-LS algorithm, the objective location is formulated as:

$$\min_{u} \sum_{i=1}^{N} (d_i - \| u - s_i \|)^2 \tag{24}$$

Based on a serial of transformation and SDP relaxation, (24) is changed to:

$$\begin{gathered} \min_{X,G} \sum_{i=1}^{N} (G_{ii} - 2d_i G_{N+1,i} + d_i^2) \\ s.t. \begin{cases} G_{ii} = Trace(C_i U) \\ G \geq 0 \\ U \geq 0 \\ G_{N+1,N+1} = U_{3,3} = 1 \end{cases} \end{gathered} \tag{25}$$

Where $U = \begin{bmatrix} u \\ 1 \end{bmatrix} \begin{bmatrix} u^T & 1 \end{bmatrix}$ and $C_i = \begin{bmatrix} I & -s_i \\ -s_i^T & \|s_i\|^2 \end{bmatrix}$.

Figure 2 and Figure 3 show the estimation location distributions of the two algorithms when the variance of the measure noise is 1 meter and object node locating at [5, 5]. From these two figures, we could roughly understand the performance of the two algorithms. We can see that all the estimation locations are around the true location of the two algorithms.

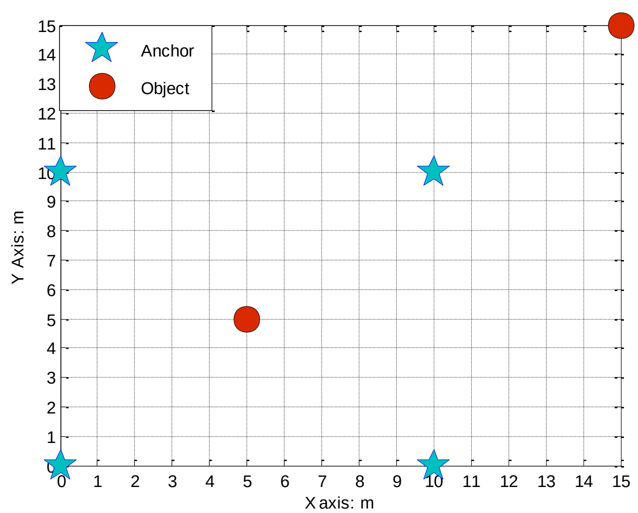

Figure 1. Placement of Anchors and Objects

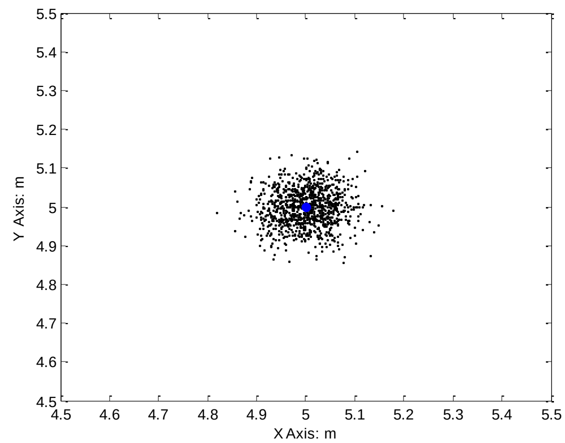

Figure 2. Estimation Location Distribution of Our Algorithm

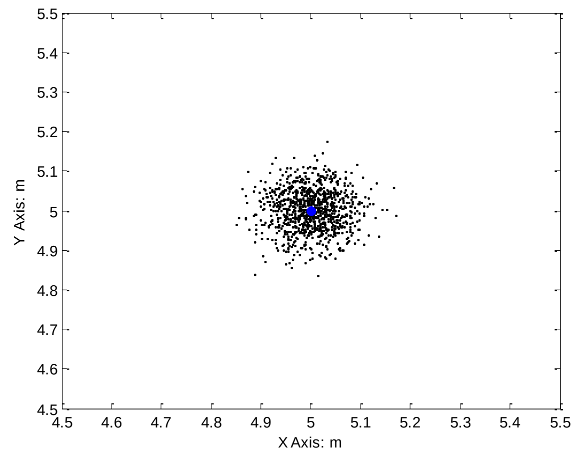

Figure 3. Estimation Location Distribution of R-LS Algorithm

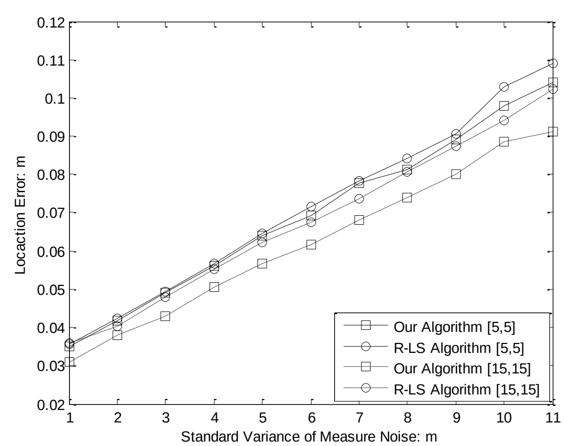

Figure 4. Location Estimation Error Under Different Measure Noise

Figure 4 shows the location error when the object node locating at [5, 5] which is within the convex hull comprised by the four anchors and [15, 15] which is out of the convex hull, and the variance of the measure noise varies from 0.5m to 1.5m. From the figure we could see that the performance of the two algorithms is better when object node is out of the convex hull than within the convex hull and our algorithm is superior to R-LS regardless of whether the object is located within the convex hull.

5. Conclusions

In this paper, we assume that the squared error of the measured distance follows Gaussian distribution and propose an algorithm to locate the object based on the positive semi-definite programming. We first obtain the estimator of the object location based on the maximum likelihood criterion; then considering that the estimator is a non-convex function with respect to the measured distances between the object and the anchors with known coordinates, we transform the non-convex optimization to convex one by the positive semi-definite relaxation; and finally we take the optimal solution of the convex optimization as the estimated value of the object location. We compare our algorithm with the R-LS algorithm when the object is in and not in the convex hull composed of the anchors, and the results show that our algorithm is superior to R-LS algorithm regardless of whether the object is located within the convex hull.

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