

## Tracking of Moving Objects with 2DPCA-GMM Method and Kalman Filtering

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### **Abstract**

*A new method, 2DPCA-GMM of tracking and segmentation in the dynamic environment of objects is proposed in this paper. The method attempts to link the Gaussian mixture model, (GMM) with the method of two dimensional principal component analysis (2DPCA) and apply Kalman Filtering (KF) for tracking. In this context, the aim of the paper is to tackle tracking of moving object based on 2DPCA-GMM together with Kalman prediction of the position and size of object along the image's sequence. The obtained results successfully illustrate the tracking of a single moving object as well as multiple moving objects based on segmentation generated by 2DPCA-GMM.*

**Keywords:** *pixels, Gaussian mixture model, 2DPCA, noise process, segmentation, tracking, Kalman Filter*

### **1. Introduction**

Identifying object tracking is challenging in several aspects within the field of computer vision. In video surveillance, it assists understanding the movement patterns of moving object to uncover suspicious events. For instance, considering the relation between the background and foreground in the appearance of scenes, it is difficult to separate objects in a dynamic scene environment. Since the moving objects vary with illumination changes and noise. There exists an extensive literature addressing this topic. N. Friedman and S. Russell proposed a Gaussian mixture model (GMM) for the background subtraction [1]. C. Stauffer and W. Grimson developed an algorithm for foreground segmentation based on the Gaussian mixture model [2, 3]. C. Stauffer, W. Grimson, R. Romano and L. Lee used tracking information in multicamera calibration, rough site modeling, object detection, object classification, activity detection and classification [4]. T. Bouwmans, F. ElBaf and B. Vachon surveyed the background modeling using mixture of Gaussians for object detection [5]. F. Zbu and K. Fujimura demonstrated a face tracking method using GMM and EM algorithm [6]. T. Ko, S. Soatto, and D. Estrin developed a background modeling and subtraction scheme that analyzes the temporal variation of intensity or color distributions [6]. I. Gómez, D. Olivieri, X. Vila, and S. Orozco proposed a method for foreground, background segmentation and created a feature vector for discriminating and tracking several people in the scene [7]. On the other hand, the binding ellipsis, due to S. Cheung and C. Kamath used frame-differencing as it produces minimal false foreground trails behind objects [8]. J. Yang, D. Zhang and A. Frang, examined the arbitrary effects of generalized illumination formula of the covariance matrix where lighting condition was considered [9]. Y. Yong and W. Ya-fei used Gaussian mixture model with spatial local correlation and Markov Chain [10]. F. Zbu and K. Fujimura

demonstrated a face tracking method using GMM and EM algorithm [11]. P. KaewTraKulPong and R. Bowden proposed a method to detect moving shadows using Gaussian mixture model. The shadow detection also reduces the effect of small repetitive motions in the background scene [12]. Y. Sheikh and M. Shah Introduce single distribution and kernel density [13]. B. Antoni, M. Vijay and V. Nuno introduced a generalization of the Stauffer-Grimson background subtraction algorithm [14]. R. Chan and J. Lin introduce mixture Kalman filtering for on-line estimation and prediction in conditional dynamic linear model [15]. A. Yilmaz, O. Javed, and M. Shah Surveyed detection of object and tracking [16]. K. Quast and A. Kaup presented Auto GMM-SAMT video surveillance system and used the mean shift algorithm to track the contour of objects of changing shape without the help of any predefined shape model [17]. K. Panta proposed novel data association schemes for the probability hypothesis density [18]. H. Ahmad, M. Jedra, and N. Zahid used 2DPCA method in face recognition that performs more superior than PCA [19]. R. Hassanpour, A. Shahbahrami and S. Wong proposed Gaussian mixture model and considered the higher level a priori knowledge of spatial and temporal relationship between the pixels in a video sequence [20]. G. Rajkumar, K.Srinivasarho and P.Sribivasa proposed a new image segmentation algorithm used the finite mixture of doubly truncated bivariate Gaussian distribution by integrating with the hierarchical clustering [21].

In our present work, the traditional Gaussian mixture model (GMM) to segment the foreground from the background labeling for each sequence frame is considered. Though, the GMM enables the background model to be updated using sufficient statistics and all the estimated parameters to be obtained efficiently. However, the GMM is limited when the foreground is not available or if it has changed under critical situations, like illumination changes or it has been removed from the scene. The 2DPCA-GMM method tackles these limitations. Accordingly, the 2DPCA-GMM generates more precise and clear features of the shape in the scenes as it provides good classification of the object. The tracking of moving object is then based on combining KF and 2DPCA-GMM.

The paper is organized as follows: Section 2 deals with adaptive background subtraction algorithm based on Gaussian mixture model. Section 3 summarizes 2DPCA algorithm. The linkage between 2DPCA and the adaptive model, 2DPCA-GMM is explained in Section 4. The Kalman filtering methods are introduced in the Section 5, followed by applications of the algorithm with their experimental results presented with comments in Section 6.

## 2. Adaptive Background Subtraction Algorithm

In the adaptive mixture of Gaussian we consider each pixel as an independent statistical process, incorporates all pixel observations. The mixture model does not distinguish components that correspond to background from those associated with foreground objects and records the observed intensity at each pixel over the previous N frames. In our work, each pixel processes is considered sequentially at time series. Following C. Stauffer and W. Grimson [3], we used the same number K of components ( $K = 3, 5$ ). We consider the values of a particular pixel,  $\phi_0(x_0, y_0)$  over time t. We define the history of this particular pixel as  $\{X_1, \dots, X_t\} = \{I(\phi_{j-1}, j): 1 \leq j \leq t\}$ , where  $I(\cdot)$  is the image sequence. The probability of pixel observation data  $X_t$  is given by:

$$p(X_t) = \sum_{k=1}^K W_{k,t} * \eta(X_t, \mu_{k,t}, \Sigma_{k,t}) \quad (1)$$

Where  $W_{k,t}$  is the weight of the  $i^{\text{th}}$  component at time  $t$ ,  $\mu_{k,t}$  is the mean and  $\Sigma_{k,t}$  is covariance mixture of  $k^{\text{th}}$  component at time  $t$  respectively and  $\eta$  is a Gaussian probability density defined as follows:

$$\eta(X_t, \mu_{k,t}, \Sigma_{k,t}) = C \cdot \exp\left\{(X_t - \mu_{k,t})^T \Sigma_{k,t}^{-1} (X_t - \mu_{k,t})\right\} \quad (2)$$

with  $C = ((2\pi)^n |\Sigma|)^{-\frac{1}{2}}$

For simplicity we use  $\Sigma_{k,t} = \sigma_k^2 I$ , where  $I$  is the identity matrix. Next, we update the mixture of each pixel consecutively by reading a new pixel values and update each matching mixture component. We initialize the parameters  $W, \mu$  and  $\sigma$ . They are updated for each frame in the mixture. We calculate  $W_{k,t}$  for each  $k^{\text{th}}$  Gaussian component in the mixture. When the current pixel value matches none of the distributions, the least probable distribution is updated with the current pixel values. The parameters of the mixture are updated as follows:

$$W_{k,t} = (1 - \alpha) * W_{k,t-1} + \alpha (M_{k,t}) \quad (3)$$

where the constant  $\alpha$  is a learning rate to update the components weights,  $M_{k,t}$  is a dummy variable with value 1 for matching models and 0 otherwise. We introduce the following criterion for screening the preliminary Gaussian matching distribution with Mahalanobis distance less than a threshold  $T$ . Accordingly, the matching distribution is given by:

$$\min\left(\|X_t - \mu_{k,t}\|_{\Sigma_k^{-1}}^2\right) < T \quad (4)$$

with the updated recursive equations

$$\mu_{k,t} = (1 - p) * \mu_{k,t-1} + p * X_{k,t} \quad (5)$$

and

$$\sigma_{k,t}^2 = (1 - p) * \sigma_{k,t-1}^2 + p * \sum_{k=1}^K (X_{k,t} - \mu_{k,t})^T (X_{k,t} - \mu_{k,t}) \quad (6)$$

where  $p = \alpha * \eta(X_t | \mu_t, \sigma_t)$  indicates how to accelerate the update. Thus, we can find the foreground resulted from the background and then find the best segment of the image. The non active background Gaussian is treated as foreground. In the ordering, we have selected the ratios  $W_k / \sigma_k$ . The first  $B$  distributions are chosen under the expression:

$$B = \operatorname{argmin}_b (\sum_{k=1}^b (W_{k,t})^3 > T) \quad (7)$$

The current pixel threshold  $T$  was sentenced prior probability for the background value. Distributions which have high weights and low variances score large values according to segmentation. We have considered the pixel value of the match of each component movement caused by the background (such as tree branches shaking, surface fluctuations, etc.) were considered to segment for the foreground object.

### 3. Two Dimensional Principal Component Analysis (2DPCA)

The accurately in evaluating the covariance matrix for a given number of training images with ease and the computational efficiency in determining the eigenvectors are among the main advantages of the 2DPCA. The following steps summarize the process of 2DPCA:

Let  $X$  denotes a matrix of  $N$  frames,  $X = \{X_i\}$  of dimensions  $(m \times n)$ . We use the mean  $\mu_{k,i}$  of each frame in the model at each component  $k$ . The differences,  $A_{k,i}$  of the training data from the respective  $\mu_{i,k}$  are given by:

$$A_{k,i} = X_i - \mu_{k,i} \quad (8)$$

We calculate the covariance matrix  $\emptyset$  of dimension  $(n \times n)$  by:

$$\emptyset_{k,i} = A_{k,i}^T \cdot A_{k,i} \quad (9)$$

We compute the eigenvectors and the corresponding eigenvalues using Singular Value Decomposition (SVD) method.

We obtain  $r$  eigenvectors related to the largest eigenvalues  $\lambda_i$  of covariance matrix  $w = [w_1, w_2, \dots, w_r]$  of size  $(n \times r)$ , where  $w$  is used to obtain the feature of each training frame  $X_i$  as the follows:

$$\emptyset_{k,i} = \lambda_{k,i} w_{k,i} \quad (10)$$

We have projected each frame  $A_i$  matrix onto the eigenvectors matrix to find the new matrix of dimension  $(m \times r)$ , given by:

$$S_{k,i} = A_i \cdot w_{k,i} \quad (11)$$

Next we define the estimated data  $\hat{X}_i$  as follows:

$$\hat{X}_i = S_{k,i} \cdot w_{k,i}^T + \mu_{k,i} \quad (12)$$

### 4. The 2DPCA-GMM Integrated Algorithm

To improve on the illumination changes and noise as the Gaussian mixture model does not vary greatly in this respect, we propose an algorithm based on linking 2DPCA method with GMM.

To extend the Gaussian mixture of observing pixel data  $X_i$  from (1) and (2), given the data  $X = \{X_t\}$ ,  $t=1, 2, N$ , then the formula of  $p(X_t)$  is now:

$$p(X_t) = \sum_{k=1}^K W_{k,t} * \eta(X_t, \mu_{k,t}, \Sigma_{k,t}) \quad (13)$$

with

$$\eta(X_t, \mu_{k,t}, \Sigma_{k,t}) \approx \eta(\hat{X}_t, \hat{\mu}_{k,t}, \hat{\Sigma}_{k,t}) = \frac{1}{(2\pi)^{\frac{n}{2}} |\hat{\Sigma}|^{\frac{1}{2}}} \exp \left\{ -\frac{1}{2} (\hat{X}_t - \hat{\mu}_{k,t})^T \hat{\Sigma}_{k,t}^{-1} (\hat{X}_t - \hat{\mu}_{k,t}) \right\} \quad (14)$$

In linking 2DPCA-GMM we insert the estimates of the mean  $\hat{\mu}_{i,t}$  and covariance  $\hat{\Sigma}_{k,t}$  of each component of 2DPCA into GMM, its physical structure, contains the maximum amount

of information or has the minimum squared errors as compared to GMM, in handling the limitation and shadow that contribute to noise in the adaptive Gaussian model, and use  $\hat{X}_t$  from 2DPCA method form (12).

To update each mixture of each pixel entails reading the new pixel values and select the matching components, Accordingly, we update the mean and covariance from (5, 6) as follows:

$$\hat{\mu}_{k,t} = (1 - p) * \hat{\mu}_{k,t-1} + p * \hat{X}_{k,t} \quad (14)$$

and

$$\hat{\sigma}_{k,t}^2 = (1 - p) * \sigma_{k,t-1}^2 + p * \sum_{k=1}^K (\hat{X}_{k,t} - \hat{\mu}_{k,t})^T (\hat{X}_{k,t} - \hat{\mu}_{k,t}) \quad (15)$$

And we update the covariance in each frame as follows:

$$\phi_{k,t} = (1 - \alpha) * \phi_{k,t-1} + \alpha * \left( (X_k - \mu_{k,t})^T (X_t - \mu_{k,t}) \right) \quad (16)$$

Where  $p = \alpha * \eta (\hat{X}_t | \hat{\mu}_t, \hat{\sigma}_t)$  indicates how to accelerate the update. Again in the update, using the integrating method, we insert the estimates of the mean  $\hat{\mu}_{k,t}$  and covariance  $\hat{\Sigma}_{k,t}$  of each component in 2DPCA into GMM respectively to select the components with the largest weights. The re-estimating of (3) can be obtained from:

$$\hat{W}_{k,t} = (1 - \alpha) * \hat{W}_{k,t-1} + \alpha * (\hat{M}_{k,t}) \quad (17)$$

And their Gaussian matching parameters will then take the form:

$$\min \left( \left\| \hat{X}_{k,t} - \hat{\mu}_{k,t} \right\|_{\hat{\Sigma}_{k,t}^{-1}}^2 \right) < T \quad (18)$$

and

$$\hat{B} = \operatorname{argmin}_b \left( \sum_{k=1}^b (\hat{W}_{k,t})^3 > T \right) \quad (19)$$

For estimating the distribution of data by integrating 2DPCA-GMM, we need to perform both the appropriate partitioning of the components and the estimation of model parameters.

## 5. Kalman Filtering

A Kalman Filter (KF) is applied to estimate the state of a linear system where the state is assumed to be Gaussian distributed. This filter not only provides an efficient computational solution to sequential systems but also provides an optimal solution for the discrete data for linear filtering problem [22]. On tracking objects from frame to frame in long sequences of images, the continuity of the motion of the observed scene, allows the prediction of the image, at any instant, based on their previous trajectories. Because the moving state changes little in

the neighboring consecutive frames, we model the system as linear Gaussian with the state parameters of Kalman Filter given by the object location, its velocity, and its size of the object respectively. A discrete-time dynamic equation state is given by:

$$X_{(t+1)} = \Phi_t \cdot X_t + s_t \quad (20)$$

where the state vector  $X = [x, y, u, v, \omega, \Delta t]^T$ ,  $\Phi = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ ,

$x$  and  $y$  are the predicted coordinates of the object,  $u$  and  $v$  are velocities in respective direction,  $\omega$  represents the width of the object rectangle and  $\Delta t$  represents the change in time  $t$ .  $s_t$  is the white Gaussian noise with zero means and covariance matrix  $Q_t$ , that is  $s_t \sim N(0, Q_t)$ . The position obtained by 2DPCA-GMM algorithm will be the measurement vector  $Z_t$ . The measurement model will then takes the form:

$$Z_t = H_t \cdot X_t + q_t \quad (21)$$

The predicted coordinates  $(x, y)$  and dimensions  $\omega$  of the rectangle are used to locate the object in the present frame. Where  $q_t \sim N(0, R_t)$ ,  $q_t$  is white Gaussian noise,  $R_t$  is covariance matrix and  $H_t$  is the design matrix such that:

$$H_t = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The corresponding covariance matrices  $Q_{t-1}$  and  $R_{t-1}$  are the inputs to the Kalman filter. In the sequential image, if the dynamics of the moving object is known, prediction can be made about the positions of the objects in the current image. The Kalman filter state prediction  $\bar{X}_t$  and state covariance prediction  $\bar{P}_t$  are defined by:

$$\bar{X}_t = \Phi_{t-1} \hat{X}_{t-1} \quad (22)$$

$$\bar{P}_t = \Phi_{t-1} \hat{P}_{t-1} \Phi_{t-1}^T + Q_{t-1} \quad (23)$$

where  $\hat{X}_t$  and  $\hat{P}_t$  denotes the estimated state vector and error covariance matrix respectively at time  $t$ . Then the Kalman filter update steps are as follows:

$$K_t = \bar{P}_t H_t^T (H_t \bar{P}_t H_t^T + R_t)^{-1} \quad (24)$$

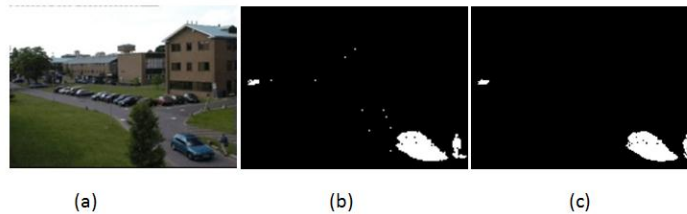
$$\hat{X}_t = \bar{X}_t + K_t (Z_t - H_t \bar{X}_t) \quad (25)$$

$$\hat{P}_t = (I - K_t H_t) \bar{P}_t \quad (26)$$

KF algorithm starts with initial conditions with  $K_0$  and  $\bar{P}_0$ .  $K_t$  is the Kalman gain, which defines the updating weights between the new measurements and the predictions from the dynamic model.

## 6. Experiment Results and Analysis

In our application, we first show how the 2DPCA-GMM out-performs the classical GMM algorithm. This relatively better performance of 2DPCA-GMM is mainly due to its ability to manipulate the variation to ease the segmentation of the foreground from the background. Moreover, the 2DPCA-GMM method is based on the minimum reconstruction error over the data needed to be estimated which contribute for the better segmentation.



**Figure 1:(a) Original Frame, (b) Segmentation of Moving Objects under Conventional GMM Algorithm, (c) Segmentation of Moving Object under 2DPCA-GMM Algorithm**

Based on this segmentation results (b) and (c) above, we are motivated to tackle the tracking problem based on 2DPCA-GMM rather than the GMM.

The tracking experimental results based on KF are arranged into two cases, tracking of a single moving object and tracking of multiple moving objects. The experiment resulted used two frame's sequences in the different backgrounds. The frame's dimensions were 125x166 in each sample.

The tracking algorithm represented by performing the following: first we apply the 2DPCA-GMM algorithm to extract the moving objects from background in each video frame, and then we predict the next position candidates in the next frame by predicting it form KF by equation (22) and (23). Sequentially the tracking results were based on the routine loop of detecting the objects by applying the data association and the add new hypotheses in KF algorithm respectively and finally we update the frame using the Kalman update function which in turn provides new input in the loop. The results are shown in the following two cases: (i) single moving object (Figure 2(a)). (ii) Multiple moving objects (Figure 2(b)).



**Figure 2. (a) Tracking of Single Moving Object based on 2DPCA-GMM and KF**



**Figure 2. (b) Tracking of Multiple Moving Objects was based on 2DPCA-GMM and KF**

In the Figure 2b the target can be tracked, though it is not clearly visible. The tracking exercise continues to capture it in addition of detecting new objects.

## 7. Conclusion

In this article, tracking dynamic objects under static and dynamic backgrounds is reported by utilizing the advantage of the new 2DPCA-GMM method. This method has shown to provide the projections that capture the most relevant pixels for segmentation of moving object within the background models relative to GMM. Based on this new method we address the tracking problem of moving objects by using KF algorithm. The results are illustrated by first considering a single moving object (people) where a complete tracking of this single object was successfully captured. Next, we apply KF to multiple moving objects (vehicle, people). Here KF algorithm enables us to capture moving objects in their dynamic tracking environment. In each case the tracking performance is based under sequence of images which are successfully generated by segmentation provided by 2DPCA-GMM method.

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