A Detection and Parameter Estimation Algorithm of Multicomponent Chirp Signals Based on Generalized S-Transform

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Abstract

The target echo of downward-looking radar can be simulated by a multicomponent Chirp signal with noise, which makes signal detection and parameter estimation a very important processing. Aiming at the problems of error detection caused by the cross-terms in WVD-Hough methods and other modified WVD distribution methods, and low precision due to poor time-frequency concentration in S-Hough transform, this paper proposed a signal detection and parameters estimation algorithm of multicomponent Chirp signals based on generalized S-transform. Firstly calculate the time-frequency distribution of multicomponent Chirp signals by generalized S-transform, then execute Hough transform to the generalized S-transform TFD on the time-frequency plane to implement signal detection and parameter estimation to the multicomponent Chirp signals. The simulation results show that this algorithm has good performance in detection the multicomponent Chirp signals in low SNR case.

Keywords: *Signal Detection, parameter estimation, multicomponent Chirp Signals, generalized S-transform, Hough Transform*

1. Introduction

The target echo of downward-looking radar can be simulated by a multicomponent Chirp signal with noise, which makes signal detection a very important technology in multicomponent Chirp signal processing in noisy case [1]. For the Wigner-Ville distribution (WVD) of a Chirp signal has very good time-frequency (TF) concentration and its time-frequency distribution (TFD) expresses as a line characteristic, then the processes of signal detection can be accomplished through the technology of lines detection on the TF plane based on Hough transform [2-5]. The commonly used methods of Chirp signals detection are focused on combining the WVD and Hough transform, for instance the Wigner-Hough transform (WHT) [6], the RSPWVD-Hough distribution [7], the Radon-Ambiguity transform [8], etc. Whereas the WVD of multicomponent Chirp signals is disturbed by the cross-terms seriously, even its improved distributions, for instance the Pseudo WVD (PWVD), the Smooth Pseudo WVD (SPWVD), etc. all can not free from cross-term entirely, which limits further application of this type of methods. By combining the Short Time Fourier Transform and continuous wavelet, Stockwell R. G. et al. proposed the S-transform in [9], which has variable resolution by using Gaussian window functions and its width related with frequency; and the S-transform is a linear invertible transform and has no cross-term meanwhile, it has been a very important tool to time-frequency analysis of nonstationary signal processing.

Reference [10] presented a multicomponent LFM signals detection method based on S-transform and Hough transform, nevertheless, the TF concentration of S-transform of LFM signal is not ideal, particularly in low SNR case, the S-transform TFD of LFM signal has not line characteristic which cause low precision in signal detection and parameters estimation. In order to enhance the TF concentration of S-transform, a generalized S-transform has been proposed which has most of the properties of S-transform and has better TF resolution [11]. Based on studying the characteristic of Chirp signal and its generalized S-transform TFD, this paper proposed a detection method of multicomponent Chirp signals based on generalized S-transform and Hough transform, through using the generalized S-transform to replace the S-transform directly, then accomplish Chirp signal detection processes by Hough transform to the generalized S-transform TFD, which could improve the precision of signals detection in noisy signals. The simulation results show that this method has good performance in detection the multicomponent Chirp signals in low SNR case.

2. Background

2.1. Theory of Generalized S-transform

By combining the STFT and wavelet, S-transform of a time series derived by Stockwell, et al. in [9] is defined as:

$$\mathbf{S}(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|f|}{\sqrt{2\pi}} \mathrm{e}^{\frac{-f^2(\tau-t)^2}{2}} \mathrm{e}^{-j2\pi ft}\, dt \tag{1}$$

where $S(\tau, f)$ is S-transform of $h(t)$, f is frequency, τ is the center of time window. From equation 1 we can know that the S-transform can be treated as STFT with adaptive Gaussian window function or a continuous wavelet (CWT) with special mother wavelet function. Whereas the width of window function of S-transform is changing with frequency f therefore it has variable resolution; at the same time the basic wavelet of S-transform do not have to meet admissibility condition, which is different to the CWT. The S-transform is a new linear time-frequency transform and has superiority characteristic of free from cross-term, so it can be applied to the analysis and processing of nonstationary signals. The window function of S-transform should meet normalized condition as

$$\int_{-\infty}^{\infty} w(\tau - t, f) d\tau = \int_{-\infty}^{\infty} \frac{|f|}{\sqrt{2\pi}} e^{\frac{-f^2(\tau-t)^2}{2}} d\tau = 1 \tag{2}$$

therefore S-transform and Fourier transform (FT) have relations as

$$\int_{-\infty}^{\infty} S(\tau, f) dt = \int_{-\infty}^{\infty} h(t) w(\tau - t, f) \mathrm{e}^{-2\pi ift}\, d\tau dt = \int_{-\infty}^{\infty} h(t) \mathrm{e}^{-2\pi ift}\, dt = H(f) \tag{3}$$

where $H(f)$ is the FT of $h(t)$. It is clear from equations 3 that the S-transform is nondestructive reversible which ensure it has no information loss in the interconversion between time domain and frequency domain. In consideration of the relation between S-transform and FT, the S-transform and its invert transform can be calculated by Fast Fourier transform (FFT), which enhance the calculation speed greatly and make S-transform has more extensive application prospect.

To Improve the S-transform by introduced two adjust parameters and reconstruct the Gaussian window function, the generalized S-transform is formulated [11]. This allows the transform adaptively verifies the window functions base on the distribution features of frequency so it is more effective in practice. The generalized S-transform is defined as:

$$GS(\tau, f) = \int_{-\infty}^{\infty} h(t) \frac{|\lambda||f|^p}{\sqrt{2\pi}} e^{\frac{-\lambda^2 f^{2p}(\tau-t)^2}{2}} e^{-i2\pi ft} dt \quad \lambda, p \in [1/2, 3/2] \tag{4}$$

where λ and p are adjustment parameters. If p is defined, the window width of the generalized S-transform broadened when λ increasing. In order to get high time resolution, narrower window function is chosen but the frequency resolution will be decreased at the same time due to the Heisenberg uncertainty. If $\lambda = p = 1$ it converts to the S-transform. So the generalized S-transform adapts to the TF analysis and filter to non-stationary signals effectively by choosing suitable parameters of λ and p.

For a multicomponent Chirp signal $u(t)$ have n components as $u_1(t), u_2(t), \cdots u_n(t)$ and its expression as

$$u(t) = \sum_{i=1}^{n} u_i(t) = \sum_{i=1}^{n} A_i(t) \exp\left[j\varphi_i(t) \right] \tag{5}$$

where $A_i(t)$ and $\varphi_i(t)$ are amplitude and instantaneous phase of $u_i(t)$ respectively. According to the definition and linear characteristic of generalized S-transform we can get the generalized S-transform TFD of $u(t)$ as:

$$\begin{aligned} GS_u(\tau, f) &= \int_{-\infty}^{\infty} u(t) w'(\tau - t, f) e^{-i2\pi ft} dt = \int_{-\infty}^{\infty} \left\{ \sum_{i=1}^{n} u_i(t) \right\} w'(\tau - t, f) e^{-i2\pi ft} dt \\ &= \int_{-\infty}^{\infty} u_1(t) w'(\tau - t, f) e^{-i2\pi ft} dt + \int_{-\infty}^{\infty} u_2(t) w'(\tau - t, f) e^{-i2\pi ft} dt + \cdots = \sum_{i=1}^{n} GS_{ui}(\tau, f) \end{aligned} \tag{6}$$

where $GS_{ui}(\tau, f)$ is the S-transform of $u_i(t)$. Figure.1 and figure. 2 are the TFD of WVD and S-transform of a three-component Chirp signal.

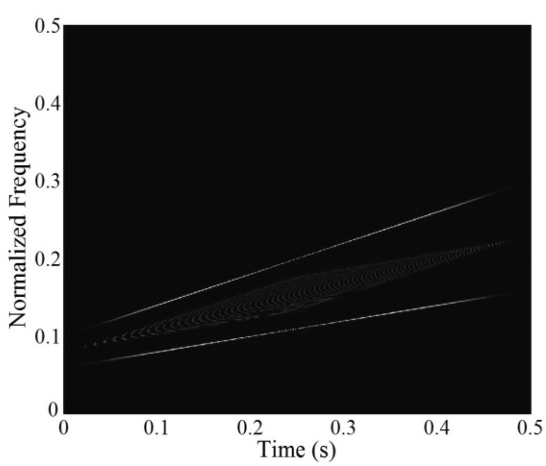

Figure 1. The WVD of a Chirp signal

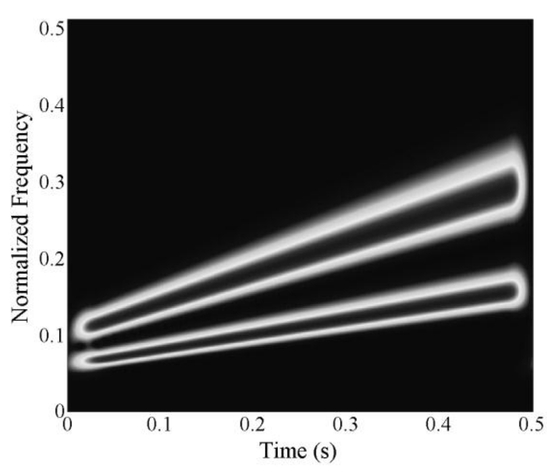

Figure 2. The Generalized S-transform of the Chirp Signal

Figure 1 shows that the to the multicomponent Chirp signals the TFD of WVD has the best TF concentration, but it disturbed by the cross-term so seriously to reduce the readability. Figure 2 indicates that the generalized S-transform is free from cross-term. Figure 3 shows that with adding suitable adjustment parameters to windows function of S-transform, the generalized S-transform has the same properties as S-transform but has better TF concentration than S-transform at the same time, which provides it unique advantages in multicomponent Chirp signals detection and parameters estimation.

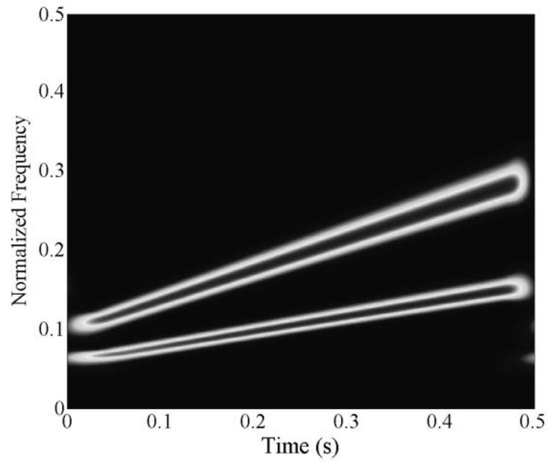

Figure 3. The Generalized S-transform of the Chirp Signal ($\lambda = 0.85, p = 1.25$ **)**

In addition, it can be seen from the comparison of figure 1 to figure 3 that to a multicomopnent Chirp signal, its WVD on the TF plane is composed of fine lines of effective components (auto-term) and the cross-term; the generalized S-transform TFD is well identical with the fit lines of auto-term of WVD, which indicate that the generalized S-transform of multicomponent Chirp signals can serve as effective characterization of TFD of this type of signals, because of that, the detection and parameter estimation of multicomopnent Chirp signals can be accomplished by execute Hough transform to the generalized S-transform.

2.2. Detection and Parameter Estimation Method of Chirp Signal based on Hough Transform [1]

The Hough transform is a straight lines detection method in transform domain and its fundamental is the duality of points and lines. The Hough transform maps straight lines in image space to a point on the Hough plane, which transform the straight lines detection processes in the image space to point's estimation operations in the parameter space [12]. The TFD of multicomponent Chirp signal on the TF plane is straight lines, which the intercepts of lines are initial frequency f_0 of Chirp signal and the slopes are the modulation slopes α, therefore through Hough transform can detect the multicomponent Chirp signal and estimate the parameters of each component. However when the slope α is reaching to 90^0, the calculation of Hough transform is too large, so the lines are more suitable to expressed as polar coordinates as

$$\rho = x\cos(\theta) + y\sin(\theta) = \sqrt{x^2 + y^2}\,\sin(\theta + \arctan\frac{x}{y}) \quad (7)$$

where ρ is the normal distance from line to origin point, θ is the included angle between the normal line and the X-axis. From equation (7) we can see that to a point (x_0, y_0) in the image space, then point (x_0, y_0) can be transformed to a line as $\rho = x_0\cos\theta + y_0\sin\theta$ in the parameter space, then n points on the same line correspond with n lines in the parameter space of (ρ, θ). It can be known form equation (7) that all these n lines pass through the same point (ρ_0, θ_0), the corresponding line in the image space can be determined by finding the point in the parameter space. After getting the TFD of generalized S-transform of multicomponent Chirp signals, we can get the image with TFD lines of each Chirp signal component, and then the detection and parameters estimation processes of multicomponent Chirp signals can be transformed to the searching and coordinate determination operations of local maximum value points in the Hough transform parameter space. Refer to [6] we compare the local maximum value points of Hough transform of generalized S-transform TFD on the TF plane and the amount of effective local maximum value points correspond to the number of LFM component in the signals. According to the coordinate of the i th local maximum value point (ρ_i, θ_i), the initial frequency f_{0i} and modulation slop k_i of i th Chirp signal component can be deduced from

$$f_{0i} = \rho_i / \sin\theta_i, \quad k_i = -1/\tan\theta_i \tag{8}$$

3. The Detection and Parameter Estimation Algorithm Flow

Suppose a N component Chirp signal, the main steps of Chirp signal detection algorithm based on generalized S-transform TFD and Hough transform proposed in this paper as:

Step 1 Get the TFD $GS_u(t, f)$ of multicomponent Chirp signal $u(t)$ by generalized S-transform.

Step 2 Regularize the generalized S-transform TFD as a digital image and the TFD of each Chirp signal has line characteristic on the digital image.

Step 3 Detect lines $L_u(k, f_0)$ on the TFD plane by Hough transform then get points Hough transform $P_u(\rho, \theta)$ in the parameter space.

Step 4 Count the amount of $P_u(\rho, \theta)$ which corresponding to the number of Chirp component, then the initial frequency f_0 and modulation slop of each Chirp component can be deduced from the coordinate ρ and θ of each point $P_u(\rho, \theta)$ by equation (8).

4. Simulation Results and Analysis

To illustrate the validity of the method proposed in this paper, a 3-component Chirp signal is designed with the duration time is 1s and the number of sampling points is 1000. The initial instantaneous frequencies of each component are 20, 65 and 200, the final instantaneous frequencies are 50, 90 and 180 respectively. The relative amplitudes of each component are equal. In order to verify the performance of noise immunity of this method, a zero-mean Gaussian white noise is added to the simulation signal and the signal to noise ratio (SNR) is set to -5 by a suitable variance is selected to the Gaussian white noise. To the simulation 3-component Chirp signal, the detection and parameter estimation results of comparative analysis of this method with the WHT and S-Hough is shown in Figure 4.

Figure 4(a) and Figure 4(b) are the detection and parameters estimation results of the 3-component Chirp signal by the methods of Wigner-Hough transform. It is indicate from Figure 4(a) and Figure 4(b) that the TFD of the 3-component Chirp signal is disturbed by noise and cross-term seriously, there are a mass of local maximum values caused by cross-term and noise in the detection result of WHT, which causing great difficulty to detection and parameters estimation processes. Figure 4(c) and Figure 4(d) are the TFD of S-transform and the result of S-Hough transform. It is clear from Figure 4(c) and Figure 4(d) that the detection result of the Chirp signal is distorted due to the TF concentration of S-transform is not as good as the WVD's.

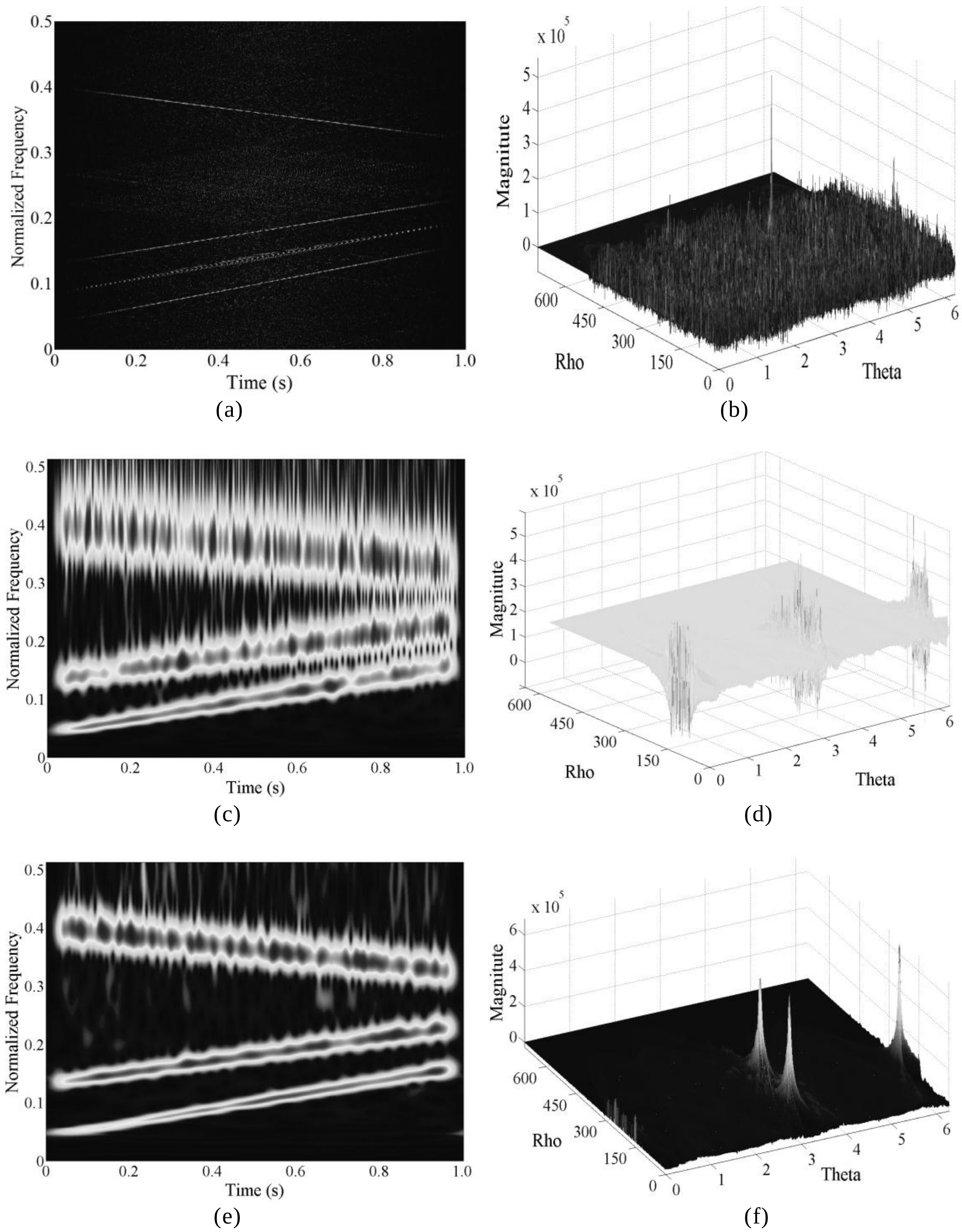

Figure. 4. Detection results of a three-component Chirp signal: (a) Signal detection result of WHT; (b) Parameters estimation result of WHT; (c) Signal detection result of S-Hough transform; (d) Parameters estimation result of S-Hough transform; (e) Signal detection result of generalized S-Hough transform; (f) The parameters estimation result of generalized S-transform and Hough transform

Figure 4(e) and Figure 4(f) is the detection result of generalized S-transform and parameters estimation result of generalized S-transform and Hough transform. It is shown from Figure 4(e) and Figure 4(f) that the method proposed in this paper not only overcome the shortcoming of poor TF concentration of S-transform but also has better performance of multicomponent Chirp signals detection in the Hough transform parameter space than the methods of WHT and S-Hough, which provide a novel way of thought and methods.

5. Conclusions and Suggestion

Aiming at the problems of detection and parameter estimation of multicomponent Chirp signals, this paper proposed a detection method based on generalized S-transform and Hough transform, which solve the detection performance decreasing problem of WHT method to multicomponent Chirp signals caused by interference of cross-term and low parameters estimation precision of S-Hough method owing to poor TF concentration of S-transform. Simulation results show that this method has very good performance of signal detection of multicomponent Chirp signals in low SNR, which provide a novel way of thought and method to signals detection and parameters estimation. Nevertheless, the TF concentration of generalized S-transform will be decreased if the SNR is too low. Future studies would be needed to add appropriately processes of denosing so that the signals detection performance in low SNR case could be improved greatly.

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References

[1] F. Q. Yu and J. L. Gao, “Detection for multi-target radar echo signals based on time-frequency reassignment-Hough transfoim”, Modern Radar, vol. 27, no. 40, **(2005)**.
[2] W. Niczyporuk, Editors, “LFM Radar Signal Detection in the Joint Time-Frequency Domain”, Proceedings of the 7th IEEE Conference on Signal Processing Algorithms. Architectures, Arrangements and Applications, (**2007**) September 7-7; New York, USA.
[3] J. Treetrong, “Fault Detection of Electric Motors Based on Frequency and Time-Frequency Analysis using Extended DFT”, International Journal of Control and Automation, vol. 4, no. 49, **(2011)**.
[4] C. K. Song, K. S. Kim, W. G. Park and J. Kim, “Performance Analysis of Signal feature detection using Spectral Correlation Function for Cognitive Radio Environments”, I International Journal of Multimedia and Ubiquitous Engineering, vol. 1, no. 32, **(2006)**.
[5] M. Khalil, A. Fahmy and N. Osman, “Correction of Left Ventricle Strain Signals Estimated from Tagged MR Images”, International Journal of Future Generation Communication and Networking, vol. 3, no. 21, **(2003)**.
[6] S. Barbarossa, “Analysis of Multicomponent Chirp signals by a Combined Wigner-Hough Transform”, IEEE Trans. on Signal Processing, vol. 43, no. 1511, **(1995)**.
[7] H. Liang, X. J. Hu and Y. Z. Zhu, “Multi-component Chirp signal Detection Based on Reassign-Smooth-Pseudo-Wigner-Ville Distribution”, J. System Simulation, vol. 19, no. 3030, **(2007)**.
[8] X. G. Tan, P. Wei and L. P. Li, “Combining Radon-ambiguity transform with second- order difference to improve detection probability of Chirp signals in low SNR”, J. Systems Engineering and Electronics, vol. 20, no. 13, **(2009)**.
[9] R. G. Stockwell, L. Mansinhal and R. P. Lowe, “Localization of the Complex Spectrum: the S-transform”, IEEE Trans. on Signal Processing, vol. 44, no. 998, **(1996)**.
[10] M. Niu, Z. Li, J. J. Li, L. Y. Hu and T. B. Wang, “LFM signal time frequency detection based on S-Hough transform”, China Computer and Network, vol. 11, no. 40, (**2009**).
[11] J. H. Gao, W. C. Chen, Y. M. Li and F. Tian, “Generalized S transform and seismic response analysis of thin interbeds”, Chinese J. of Geophysics, vol. 46, no. 526, (**2003**).

[12] Q. Guo, Y. J. Li, C. H. Wang and J. Wan, Editors, "A New Analysis Method for Multi-component Linear Frequency Modulated Emitter Signals", Proceedings of the 3rd International Symposium on Systems and Control in Aeronautics and Astronautics, **(2010)** June 8-10; New York.

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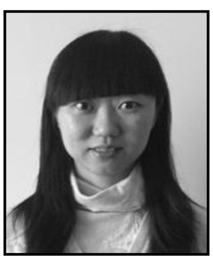

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