Synthesis of QMF Bank Using a New Window Family

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Abstract

A new combinational window family for the design of prototype FIR filter of two-channel Quadrature Mirror Filter (QMF) bank is introduced. One variable window, viz., Kaiser Window is also used to design prototype filters. The design equations of variable window function based filter banks is also given in this article. Reconstruction error, which is used as an objective function, is minimized by optimizing the cutoff frequency of designed prototype filters. The Gradient based iterative optimization algorithm is used. The performances of filter banks designed with these window functions are compared on the basis of reconstruction error. The proposed combinational window provides the QMF bank with better reconstruction error.

Keywords: *Quadrature Mirror Filter (QMF), Filter Bank, Variable Window, Combinational Window*

1. Introduction

Digital filter banks are used in a number of communication applications such as sub-band coders for speech signals, frequency domain speech scramblers and image coding. The theory and design of QMF bank was first introduced by Johnston [1]. These filter banks find wide applications in many signal processing fields such as trans-multiplexers [2]-[3], equalization of wireless communication channel [4], sub-band coding of speech and image signals [5]-[8], sub-band acoustic echo cancellation [9]-[12]. The wide application of these configurations drew lot of attention for efficient design of such filter bank [10]-[11].

In QMF bank the input signal x(n) splits into two sub-band signals having equal bandwidth using the low pass and high pass analysis filter H_0 (z) and H_1 (z) respectively. These sub-band signals are down sampled by factor of two to reduce processing complexity. At the output, corresponding synthesis bank has two-fold interpolator for both sub-band signals, followed by $G_0(z)$ and $G_1(z)$ synthesis filters. The outputs of the synthesis filters are combined to obtain the reconstructed signal y(n). This reconstruction of signal at output is not perfect replica of the input signal x(n), due to three types of errors: aliasing error, amplitude error and phase error [12]-[13]. Since inception of the QMF banks most of the researchers giving main stress on the elimination or minimization of these errors and obtain near perfect reconstruction (NPR). In several design methods [14]-[18] aliasing and phase distortion has been eliminated completely by designing all the analysis and synthesis FIR linear phase filters by a single low pass prototype even order symmetric FIR linear phase filter. Amplitude distortion is not possible to eliminate completely, but can be minimized using optimization techniques [12]-[13]. Figure-1 shows the two - channel quadrature mirror filter bank designed by Johnston [1] in which Hanning window was used to design low pass prototype FIR filter and nonlinear optimization technique to minimize reconstruction error was employed.

This paper uses the algorithm as proposed in Creusere and Mitra [6] with certain modifications to optimize the objective function. The combinational window functions [19]-[21] with large SLFOR have been devised and used for designing FIR prototype filters. Due to the closed form expressions of the window functions, the optimization procedure gets simplified. Kaiser window [22] has also been used. Finally, a comparative evaluation has been done with reconstruction error and far-end attenuation being selected as the main figure of merit.

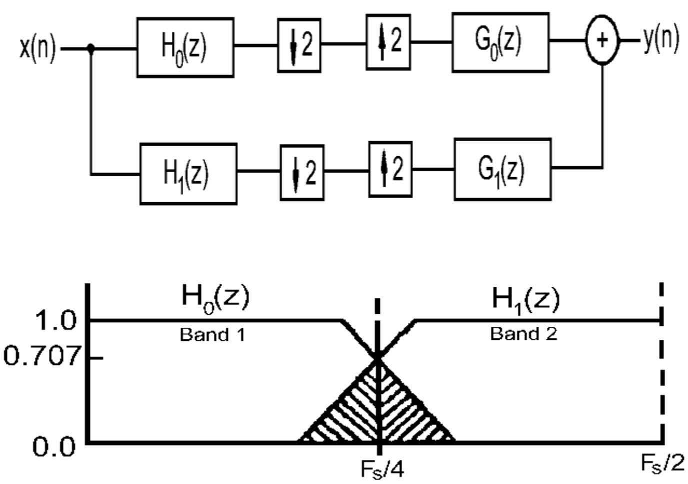


Figure 1. Two - channel quadrature mirror filter bank

2. FIR Filter Design Using Window Technique

The impulse response of the ideal low pass filter with cutoff frequency ω_c *is* given as-

$$h_{id}(n) = \frac{\sin(\omega_C n)}{\pi n}, \qquad -\infty < n < \infty \tag{1}$$

$h_{id}(n)$ is doubly infinite , not absolutely summable and therefore unrealizable [15]. Hence shifted impulse response of $h_{id}(n)$ will be-

$$h_{id}(n) = \frac{\sin(\omega_C (n-0.5N))}{\pi(n-0.5N)}, \qquad 0 \le n \le N-1 \tag{2}$$

For making a causal filter, direct truncation of infinite-duration impulse response of a filter results in large pass band and stop band ripples near transition band. These undesired effects are well known Gibbs phenomenon. However, these effects can be significantly reduced by appropriate choice of smoothing function $w(n)$. Hence, a filter $p(n)$ of order N is of the form [15-17]-

$$p(n) = h_{id}(n)\, w(n) \tag{3}$$

Where, $w(n)$ is the time domain weighting function or window function. Window functions are of limited duration in time domain, while approximates band limited function in frequency domain. Window functions are broadly categorized as fixed and variable windows.

In fixed window, the window length N governs main-lobe width. Variable window has two or more independent parameters that control the window's frequency response characteristics [24].

3. Variable Window Functions

The variable window functions used in designing the prototype FIR filter for the QMF banks are given in Table-1.The Table-1includes the expressions of variable window functions, expressions of variables (β, δ: which are defining the window families) and expressions of window shape parameters (D) of Kaiser and proposed window. The proposed window family is developed by taking the combination of Papoulis (Lag-Window) and three term cosine window functions (Data-Window). It has been observed that the resultant window function with δ = 0.207 gives the best result which is comparable to other such window functions or families. The time domain and frequency domain details of these window functions are given in Figure-2.

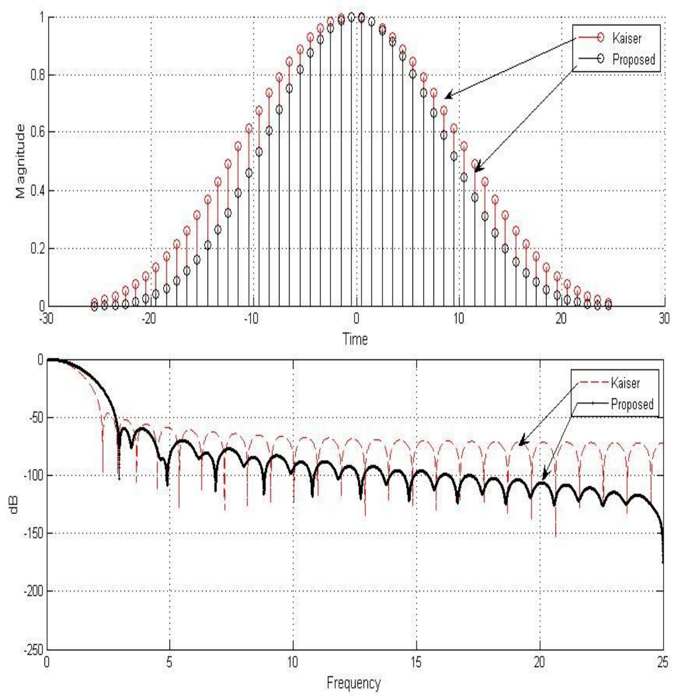


Figure 2. Time & Frequency Plot of Different Window Functions

The filter designed using one of the above window functions is specified by three parameters cut-off frequency (f_c), filter order (N), and window shape parameter (either β or δ respectively). For desired stop band attenuation (θ) and transition bandwidth, the order of the filter (N) can be estimated by

$$N = \left[\frac{D}{\Delta F_S}\right] + 1, \tag{4}$$

Table 1. Window Functions with Filter Design Equations

Sr.No.	Name of window	Expression for		
		Expression for Window function	Window variable	Window shape parameter
1	Kaiser window	$w(n)=\frac{I_0\left\{\beta\sqrt{1-(n/M)^2}\right\}}{I_0(\beta)}, \quad \text{for } -M<n<M$	$\beta=\begin{cases}0, & \text{for } \theta<21\\ 0.5842(\theta-21)+0.07886(\theta-21) & \text{for } 21\le\theta\le 50\\ 0.1102(\theta-8.7) & \text{for } \theta>50\end{cases}$	$D=\begin{cases}0.9222 & \text{for } \theta\le 21\\ \frac{(\theta-7.95)}{14.36} & \text{for } \theta>21\end{cases}$
2	Proposed combinational window	$w_{Proposed}=\begin{cases}\delta[l(n)]+(1-\delta)[d(n)] & \lvert n\rvert\le N/2 \quad 0\le\delta\le 1.315\\ 0, & \lvert n\rvert>N/2\end{cases}$ $l(n)=\frac{1}{\pi}\left\lvert\sin\left(\frac{2\pi n}{N}\right)\right\rvert+\left(1-2\frac{\lvert n\rvert}{N}\right)\cos\left(\frac{2\pi n}{N}\right), \quad \lvert n\rvert\le N/2$ $d(n)=0.4+0.5\cos\left(\frac{n\pi}{N}\right)+0.1\cos\left(\frac{2n\pi}{N}\right), \quad \lvert n\rvert\le N/2$	$\delta=a+b(\theta)+c(\theta^2)+d(\theta^3)+e(\theta^4)$ for $22\le\theta\le 74$ a = -47.551, b = -2.3814, c = 0.04524, d = -0.0003841, e = 0.0000012265	$D=a(\theta^2)+b(\theta^3)+c(\theta^4)+d(\theta^5)$ for $22<\theta\le 26$ a= -0.48548, b = 0.057101, c = -0.0022059, d = 0.000028274 $D=a(\theta^6)+b(\theta^7)+c(\theta^8)+d(\theta^{10})+e(\theta^{12})$ for $26<\theta\le 37$ a=-0.0000058038, b=0.00000073426, c=-10.00000003459, d=0.000000007426, e=0.000000000055982 $D=a(\theta^2)+b(\theta^3)+c(\theta^4)+d(\theta^5)$ for $37<\theta\le 49$ a =3607.3, b=-335.13, c=11.662, d=-0.18009, e=0.0010417 $D=a+b(\theta)+c(\theta^2)+d(\theta^3)$ for $49<\theta\le 62.8$ a = 255.1, b = -13.531, c = 0.23899, d = -0.013889 $D=a+b(\theta)+c(\theta^2)+d(\theta^3)+e(\theta^4)+f(\theta^5)$ for $62.8<\theta\le 74$ a= 257450, b=-18972, c=558.74, d = -8.2203, e = 0.060417, f=-0.00017747

Where, D is window shape parameter, ΔF_s, the normalized transition width=$(f_s-f_p)/F_s$, and F_s is the sampling frequency in Hertz. The window shape parameter can be determined by the desired stop band attenuation.

4. Optimization Algorithm

The amplitude distortion in reconstructed signal can be minimized by optimization techniques. The gradient based iterative optimization algorithm is described in this section.

4.1. Objective Function

To get the high-quality reconstructed output $y(n)$, the frequency response of low pass prototype filter, $H(e^{j2\pi f})$, must satisfy the following [13]-

$$\left|H\left(e^{j(2\pi f)}\right)\right|^2 + \left|H\left(e^{j\left(2\pi f - F_S/4\right)}\right)\right|^2 = 1 \qquad for \quad 0 < f < F_S/4 \tag{5}$$

$$\left|H\left(e^{j(2\pi f)}\right)\right| = 0 \qquad for \quad f > F_S/4 \tag{6}$$

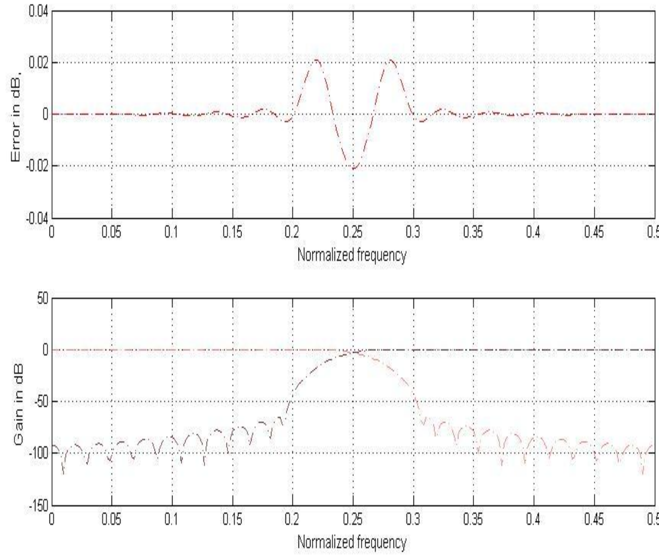
by assuming that filters have even number of coefficients.

By satisfying exactly (5) the aliasing error is eliminated between nonadjacent bands. Similarly, the amplitude distortion is eliminated by satisfying (6) [11]. Phase distortion is removed by selecting even-order FIR prototype filter [1, 4]. Constraints (5) and (6) cannot be satisfied exactly for finite length filter order, so it is necessary to design a filter which approximately satisfies (5) and (6). Johnston [1] combined the pass band ripple energy and out-of-band energies into a single cost function having nonlinear nature and then minimized it using Hooke and Jeaves algorithm [23]. Creusere and Mitra [11] designed filters using Parks–McClellan algorithm that approximately satisfied (5) and (6). The filter length, relative error weighting, and stop band edge were fixed before optimization procedure started, while the pass-band edge was adjusted to minimize the objective function ε.

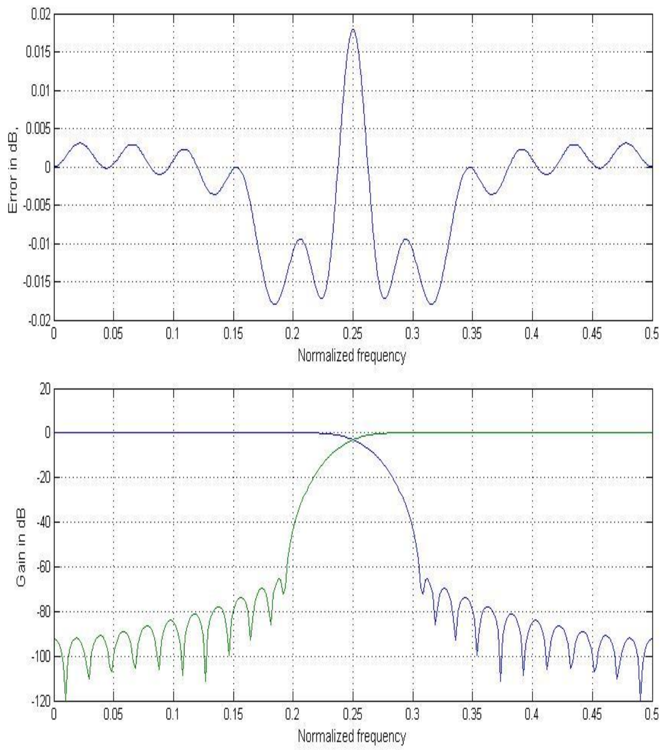
$$\varepsilon = \max imum\left(\left|H^{j2\pi f}\right|^2 + \left|H^{j\left(2\pi f - F_S/4\right)}\right|^2 - 1\right), \quad for \quad 0 < f < F_S/4 \tag{7}$$

4.2. Algorithm

A gradient based linear optimization algorithm (as given in Annexure-1) is used to adjust the cutoff frequency. Filter design parameters and optimization control parameters like step size (*step*), target error (*terror*), direction (*dir*) and previous error (*prev-error*) are initialized. Prototype filter is designed using windowing technique. With each iteration, f_c of p(n) and reconstruction error (*error*) is computed, which is also the objective function. If the error increases in comparison to previous iteration (*prev-error*), step size (*step*) is halved and the search direction (*dir*) is reversed. This step size and direction is used to re-compute f_c for new prototype filter. The optimization process is halted when the error of the current iteration is within the specified tolerance (depicted as *t-error*), which is initialized before the optimization process begins or when *prev-error* equals error [24].



(a)



(b)

Figure 3. QMF filter and reconstruction error using combinational and variable window for N = 45. (a)Kaiser Window (b) Proposed Window

5. Performance Analysis of QMF

QMF banks were designed by using window functions described in Table-1 and optimization algorithm in Annexure-1. In these design examples the stop-band edge frequency and pass-band edge frequency are taken as $F_s/4$ and $F_s/6$ respectively. In Table 2, the value of stop band attenuation was kept at 50 dB, resulting in different filter orders for different window functions, which clearly indicates the improvement in reconstruction error is obtained with proposed window function.

Table 2. Performance of QMF filter at 50 dB stop-band attenuation

Window function	Reconstruction error (dB)	Filter order (*N*)	Far-end attenuation (dB)
Kaiser window	0.3208	90	107
Proposed window	0.0975	29	123

Table 3. Optimum performance in terms of order

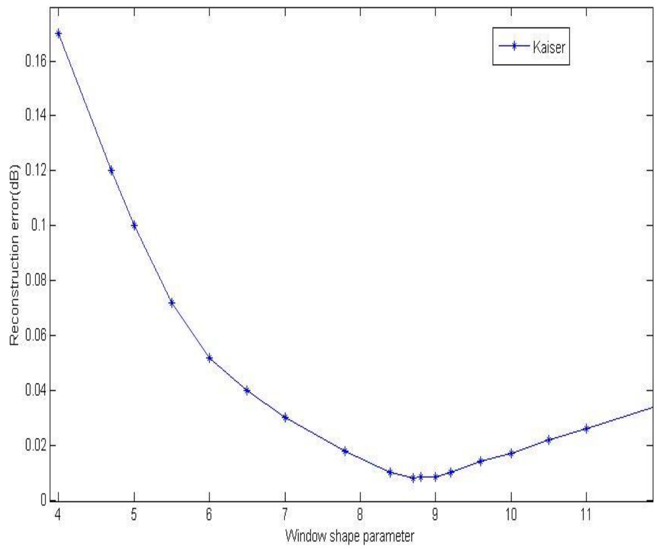
Window function	Reconstruction error (dB)	Stop-band attenuation (dB)	Filter order (*N*)	Far-end attenuation (dB)
Kaiser window	0.0097	88.00	90	107
Proposed window	0.0057	52.15	15	123

Table 4: Performance in terms of far-end attenuation

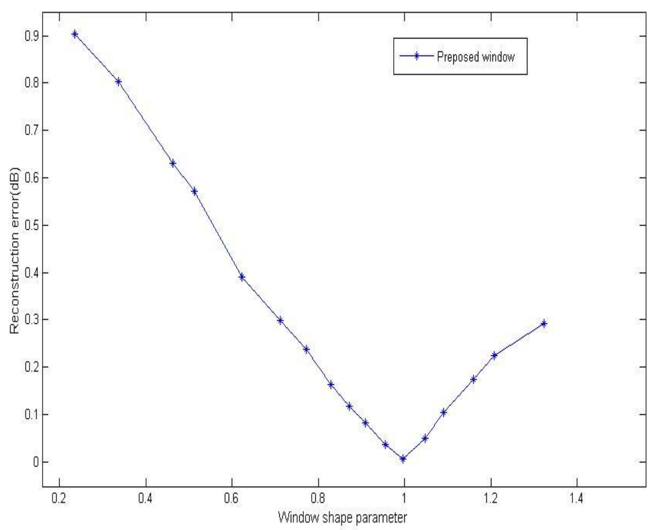
Window function	Reconstruction error (dB)	Stop-band attenuation (dB)	Filter order (*N*)	Far-end attenuation (dB)
Kaiser window	0.0473	52.168	37	66
Proposed window	0.00179	52.168	45	84

In Table-3 results corresponding to filter order are shown. In Table-4, a comparison is made of the optimum performance that can be attained with the two window functions. Apart from the reconstruction error, the far-end attenuation (amplitude of the last ripple in the stop band) is also selected as one of the figures of merits for the comparative study. This parameter is of significance when the signal to be filtered has great concentration of spectral energy. In a sub-band coding, the filter is intended to separate out various frequency bands for independent processing. In the case of speech, e.g. the far-end rejection of the energy in the stop band should be more so that the energy leak from one band to other will be minimum. From Tables-2 and Table-3, it is inferred that as the stop band attenuation increases the value of reconstruction error decreases. The proposed window-designed FIR filter gives better performance as compared to the other window functions.

Far-end attenuation is maximum for proposed window-based FIR filters. As from Table-4, the optimum performance in terms of reconstruction error has been obtained in proposed window function. Reconstruction error for the two prototype filters designed with N = 45 are shown in Figure-3. By practically observation it is concluded that the reconstruction error depends on the window shape parameter for Kaiser and proposed windows. Plots corresponding to these variations are shown in Fig. 4 for the two windows.



(a)



(b)

Figure 4. Variation of reconstruction error with window shape parameter (a)Kaiser Window (b) Proposed Window

6. Conclusion

A new combinational window has been proposed for designing the low-pass prototype filters for QMF banks. Linear gradient base optimization algorithm has been used to optimize the reconstruction error by varying the filter cut-off frequency. Prototype filters designed using high SLFOR combinational window, Kaiser-window have been compared. Reconstruction error was found to be dependent on the window shape parameter. The performance which was optimum with respect to reconstruction error, has observed for proposed and Kaiser window function. Combinational window provides better far-end rejection of the stop-band energy. This feature helps to reduce the aliasing energy leak into a sub-band from that of the signal in the other sub-band.

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Annexure 1

Flowchart for Gradient Based Optimization Algorithm