Integral Images Compression using Discrete Wavelets and PCA

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Abstract

A technique for integral image compression is presented. The proposed technique relies on applying principle component analysis, PCA, on the wavelet coefficients of the elemental images to improve the quality of the recovered 3D image while achieving high compression ratio. The wavelet coefficients of the individual elemental images are stacked and rearranged before applying PCA compression. The PCA compression is applied to each sub-band individually to enhance the compression ratio. The quality of the reconstructed 3D images and received elemental images are calculated. Results show high compression ratio compared to PCA alone compression while maintaining the recovered 3D image quality. PSNR is used to measure the reconstructed 3D image quality.

Keywords: *Integral Imaging, compression, wavelet, PCA*

1. Introduction

Integral Imaging,(II), or Integral photography (IP) is proposed by Lippmann at 1908 [1].The idea of II simply is to collect a set of 2D images for the same object using microlens array. It gives images in continuous viewing angles nor different directions. So, it gives a true 3D image of the object [2]. II doesn't cause eye fatigue or need special glasses like the stereoscopic method. Because incoherent light is used in II, there is no speckle problem as holographic imaging. Because of these benefits, II has been studied for 3D TV, video, and movies [3]-[5].

The main drawback of II method is the narrow viewing angle and depth limitation. Also, II suffers from the low resolution of the reconstructed 3D image because the reconstructed 3D scene resolution depends on the number of lenses on the used lenslet array. So, increasing the number of lenses enhances the resolution of the 3D image. But it makes the transmitted elemental image so huge in size. In practical applications, the recorded images need to be stored and transmitted, which involve considerable storage capacity and large transmission bandwidth, thus promoting the need for high efficiency compression techniques for II images [6].

In general, the elemental images are very similar and there is a lot of redundancy between neighboring elemental images. So, II images exhibit high spatial correlation between adjacent elemental images. Thus, several approaches to effectively reduce the transmitted II images size by applying the conventional image compression techniques have been reported. MPEG2 is used to compress II images by rearranging the elemental images as the consecutive frames in a moving picture [7]. An improved compression algorithm based on a hybrid technique implementing a four-dimensional transform combining the discrete wavelet transform and the discrete cosine transform is presented in [8]. Jang et al. employed the Karhunen – Loeve transform (KLT) algorithm for compression of elemental array of images [9],[10]. Even though, these methods are simple, they have acceptable results.

This paper presents a technique that relies on using DWT and PCA. The elemental images are arranged to subsequent frames and the PCA compression is applied to the different DWT sub-bands coefficients on an individual basis. The DWT is applied to the different each elemental subimages then the PCA compression is applied to the DWT subband coefficients. Peak Signal to Noise Ratio, PSNR, is used to evaluate the quality of the reconstructed images.

The paper is arranged as follows. Section 1 is an introduction. Review of the Integral Imaging, discrete wavelet transform and PCA transform are presented in sections 2 ,3 and 4 respectively. The proposed technique is presented in section5. Experimental results and simulations are presented in section 6, and the conclusion is followed in section 7.

2. Integral Imaging

A general integral imaging system consists of two processes, pickup and reconstruction. In the pickup process, as shown in Fig. 1, a lenslet array is used to capture the 3D object. Each of the lenslets provides different perspective views of the 3D object, which results in a collection of de-magnified 2D images, known as an elemental image array. To store the elemental image array, a 2D image sensor such as a charge coupled device (CCD) sensor is used. In order to reconstruct the 3D image, rays are reversely propagated through the elemental image array and a similar lenslet array is used as in the pickup process. There are also computerized methods used to reconstruct the image [11],[12],[13]. These methods make it possible to improve qualities of the image such as contrast, brightness, and resolution by numerical techniques. Also, these methods eliminate the need for special purpose optical equipment such as high-quality liquid-crystal display and micro-optics components to display the 3-D images. In this paper, the reconstruction method explained in [11] is used. In this method an elemental image array of the 3-D object is formed by a microlens array and recorded by a CCD camera. Then we reconstruct 3-D images by extracting pixels periodically from the elemental image array, using a computer. Images viewed from an arbitrary angle can be retrieved by shifting the points to be extracted. By reconstruction of the 3-D image numerically with a computer, the quality of the image can be improved, and a wide variety of digital image processing techniques can be applied.

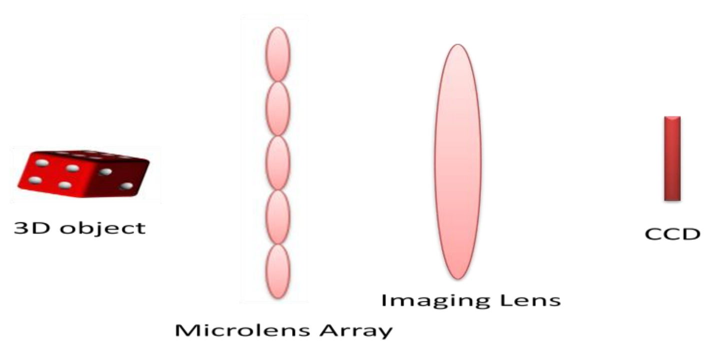


Fig. 1. Pickup of the Integral Image.

Basically, the resolution of the reconstructed 3D images is highly dependent on the number of picked-up elemental image array as well as the resolution of each elemental image. Therefore, resolution of the reconstructed 3D images can be improved as the number of elemental image array increases and the resolution of each elemental image is enhanced, but it simultaneously causes an increase of image data to be processed, stored and transmitted in this integral imaging system.

3. Discrete Wavelet Transform

Discrete Wavelet Transform DWT [14] is a multi-resolution de-compositions that can be used to analyze signals and images. It describes a signal by the power at each scale and position. DWT has been proved to be a very useful tool for image compression in the recent years [15]. Wavelet transform exploits both the spatial and frequency correlation of data by dilations (or contractions) and translations of mother wavelet on the input data. It supports the multiresolution analysis of data i.e. it can be applied to different scales according to the details required, which allows progressive transmission and zooming of the image without the need of extra storage. Another encouraging feature of wavelet transform is its symmetric nature that is both the forward and the inverse transform has the same complexity, building fast compression and decompression routines. Its characteristics well suited for image compression include the ability to take into account of Human Visual System's (HVS) characteristics, very good energy compaction capabilities, robustness under transmission, high compression ratio etc.

The implementation of wavelet compression scheme is very similar to that of subband coding scheme: The first stage of the DWT converts an image into four sub-bands by applying low-pass and high-pass filters to the image followed by down-sampling by a factor of two. the resulting coefficients grouped into four zones, where H symbolizes high frequency data and L symbolizes low frequency data.

The advantage of wavelet transform, it is that divides the information of an image into decomposing images to approximate subsignals (LL) and detail subsignals (LH, HL, HH) parts as shown in Fig.2 . This enables to isolate and manipulate the data with specific properties. With this, it is possible to determine whether to preserve more specific details. For instance, keeping more vertical detail instead of keeping all the horizontal(LH), vertical details (HL) and diagonal (HH) of an image that has more vertical aspects. This would allow the image to lose a certain amount of horizontal and diagonal details, but would not affect the image in human perception.

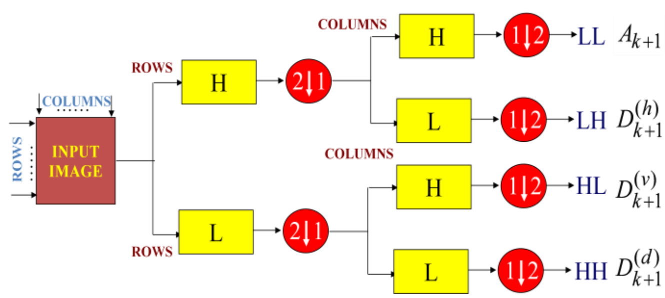


Fig. 2. The Two-level Sub-band Decomposition Used in the DWT.

There are a lot of wavelet filters like, Daubechies wavelets, Coiflets, biorthogonal wavelets, and Symlets. These various transforms are different in mathematical properties such as symmetry, number of vanishing moments and orthogonality.

4. Principle Component Analysis

PCA (Principle Component Analysis) [16] is widely used in image processing, especially in image compression. Also, it is called the Karhunen-Leove transform KLT or the Hotelling transform. Principal components analysis PCA is a statistical procedure that allows finding a reduced number of dimensions that account for the maximum possible amount of variance in the data matrix. The PCA basis vectors are the eigenvector of the covariance matrix of the input data. This is useful for exploratory analysis of multivariate data as the new dimensions called principal components PCs. A reduced dimension can be formed by choosing the PCs associated with the highest eigenvalues. So, we can consider KLT as a unique transform which decorellates its input.

Calculating a principal components analysis is relatively simple and depends on some characteristics associated with matrices eigenvalues and eigenvectors. To calculate the PCA we first estimate the correlation matrix or covariance matrix of the image array. The next step is to calculate the eigenvalues of the matrix. Each eigenvalue can be interpreted as the variance associated with a single vector. The next step is to calculate the eigenvectors associated with each eigenvalue. Each eigenvector represents the factor loading associated with a specific eigenvalue. By multiplying the eigenvector by the square root of the eigenvalue. This is all the information we need to begin to apply PCA. Finally, we need to select the number of eigenvectors needed to explain the majority data of the image. We simply select the eigenvectors associated with the largest eigenvalues to represent a sufficient amount of the image data.

5. The Proposed DWT-PCA II Compression Algorithm

When wavelet transform is applied to an image, it produces as many coefficients as there are pixels in the image (i.e.: there is no compression yet since it is only a transform). These coefficients can then be compressed more easily because the information is statistically concentrated in just a few coefficients. Due to high correlation between elemental images, the sub-bands of the wavelet transform are highly correlated. So, applying PCA to the DWT sub-bands resulted in a good compression ratio.

5.1. Steps of the algorithm

This paper presents a technique that uses DWT and PCA to compress II images. The proposed technique consists of three steps as shown in Fig 3. The first step is to stack the II images into a sequence of frames that are composed of the elemental images. Scanning the 2D II images with any type has no effect on the resultant compression ratio. A 3D image is formed as shown in Fig.4 . In the second step, the II sub-images are decomposed into different scales using wavelet transform. The DWT coefficients are rearrange in an array form such that each column represents the same scale DWT coefficient of all the II images as shown in Fig.5. At the third step, principal component analysis is applied on wavelet sub-band of all subimages, as shown in Fig.6. Fig. 7 shows a flow chart for the proposed algorithm.

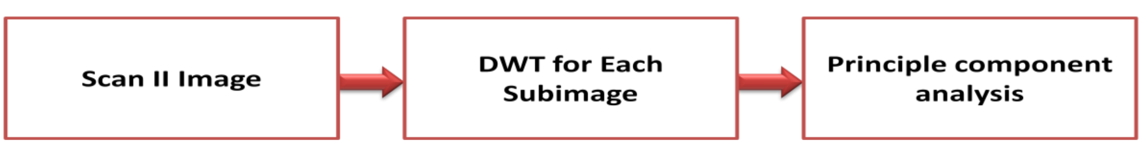


Fig. 3. The DWT Combined PCA Compression Algorithm

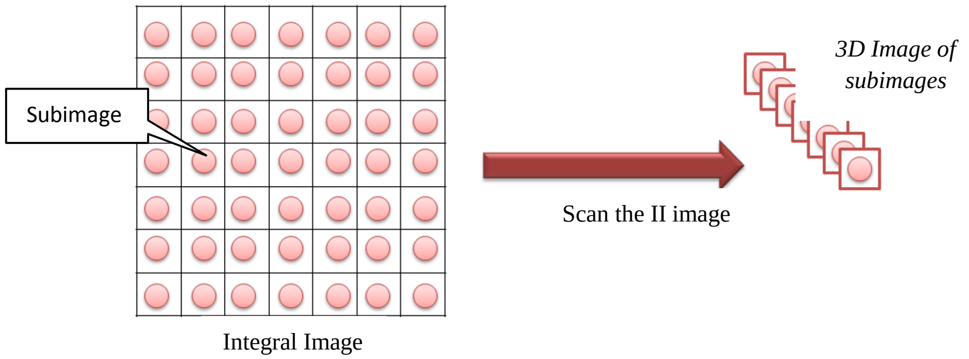


Fig. 4. Scan of the Integral Image.

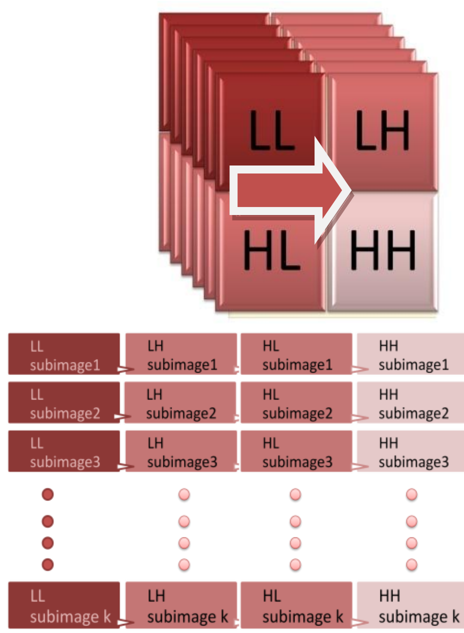


Fig. 5. Wavelet for Each Subimage.

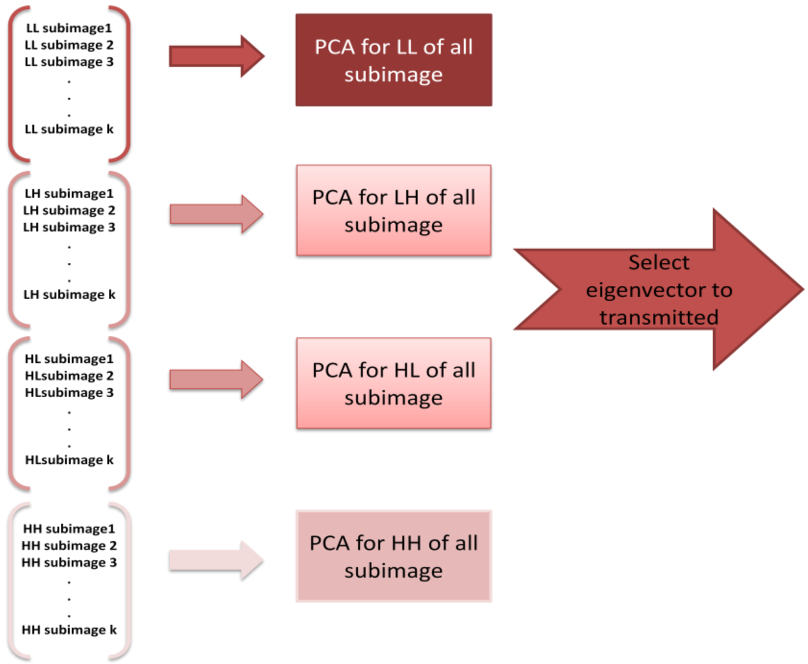


Fig. 6. PCA for Each Type of DWT Subfilter.

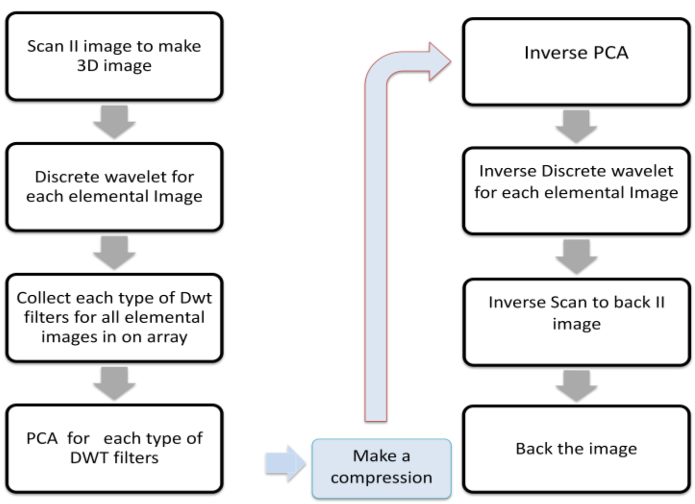


Fig.7. Flow Chart of the Proposed Algorithm.

5.2. Mathematical Analysis

Let the input 2D integral image called I where has a size of $N\times M$ pixels, the elemental image E_i size is $d\times d$ then the number of elemental images in one row r is given by $r = \frac{N}{d}$ the number of elemental images in a column is given by $c = \frac{M}{d}$ the total number of elemental images in the Integral image I is k where $k = r\times c$. Scanning the II image I and stacking the elemental images gives a volumetric 3D image E having a size $d\times d\times k$ pixels. Then, DWT is applied to each sub-image E_i. where $i = 0,1.....k$. DWT transformation splits the input image into a low pass component L and a high pass component H both of which are decimated (down sampled) by 2:1. Repeat this again to make 2D DWT. It converts an input series E_0, E_1,.. E_k into one high-pass wavelet coefficient series and one low-pass wavelet coefficient series (of length I/2 each) given by:

$$H_i = \sum_{m=0}^{j-1} E_{2i-m} \cdot HC_m$$

$$L_i = \sum_{m=0}^{j-1} E_{2i-m} \cdot LC_m \qquad (1)$$

Where HC_m and LC_m are the high coefficients and low coefficients of the used type of DWT filter called *wavelet filters*, j is the length of the filter, and $i=0, ..., [d/2]-1$. In practice, such transformation will be applied recursively on the low-pass series until the desired number of iterations is reached. X_{LL} the coefficient for the low low filter , X_{LH} is the low high filter coefficient, X_{HL} is the low high filter coefficient and X_{HH} is the high filter coefficient. In general, most of the energy in the image tends to be concentrated in the low frequency regions with the detail subbands contain the edge information.

Then the covariance matrix of the vector $\boldsymbol{X_s}$ **,**where s the sub-band type, generated using the DWT coefficients with similar allocation in all arranged sub-image of 3D-matrix, as show in Fig. 8 is calculated.

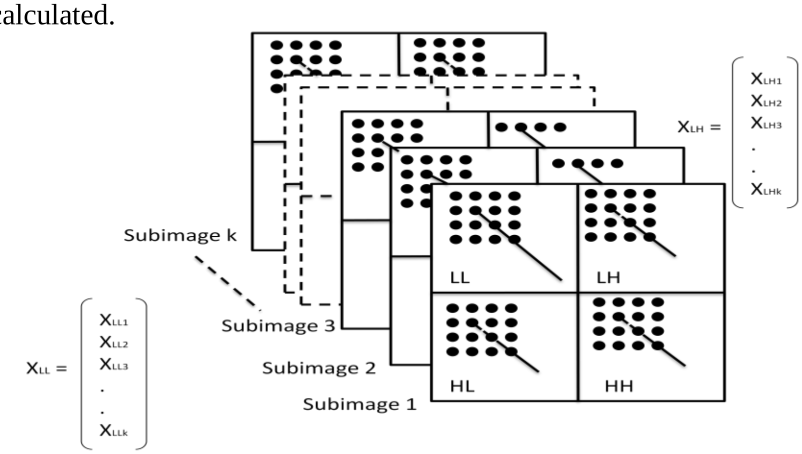


Fig. 8. PCA for all Subimages of Each Type of Sub-bands DWT Filters.

For each Sub-band let $X_s = (x_{s1}, x_{s2}, ... ,x_{sk})^T$, where s is the sub-band are LL, LH, HL or HH, T indicates the transpose and k is the number of subimages.
let $m_x = E\{x\}$ is the mean vector.

$$C_x = E\{(x - m_x)(x - m_x)^T\} \qquad (2)$$

Then

$$m_x = \frac{1}{k}\sum_{i=1}^{k} x_{si} \qquad (3)$$

Where rr = no. of row of X_s . note $rr= k$.

cc= no. of column of X_s. $cc=\frac{d^2}{4}$

From Eq^n (2)

$$C_x = \frac{1}{rr * cc} \sum_{l=1}^{rr*cc} (x_l - m_x)(x_l - m_x)^T \tag{4}$$

Then calculate the eigenvector of the covariance matrix indicated by the symbol **v**

The vectors **v** have the same direction as C_x. Then $C\mathbf{v}=\lambda\mathbf{v}$, λ is called the *eigenvalues* of C.

$C\mathbf{v}=\lambda\mathbf{v} \Leftrightarrow (C-\lambda I)\mathbf{v}=0$

The eigenvectors and eigen values are calculated as follows:

1. Calculate $det(C-\lambda I)$
2. Determine roots to $det(C-\lambda I)=0$, roots are eigenvalues λ.
3. Solve $(C- \lambda I)$ $\mathbf{v}=0$ for each λ to obtain eigenvectors **v.**

The extracted uncorrelated components are called *principal components (PCs)* which is estimated from the eigenvectors of the covariance matrix. The first few PC's contain most of the variations in the original dataset. For using PCA in image compression we perform the following:

1. Eigenvalues λ and eigenvectors **v** are sorted in descending order
2. The n component with highest λ is principal component is kept
3. *Feature vector* = (v_1, ... ,v_n) where v_i is a column oriented eigenvector contains chosen components.

5.3. Metric of Measure:

Many metrics could be used to measure the error between the original image and the compressed image. One of the important metrics is the peak signal to noise ratio *PSNR* where

$$PSNR = 10\log_{10}\left(\frac{P^2}{MSE(O,I)}\right) \tag{5}$$

Where P is the maximum possible pixel value, I is the original image, and O is the compressed image and the *MSE* given by

$$MSE = \frac{1}{XY}\sum_{x=0}^{X-1}\sum_{y=0}^{Y-1}\left[O(x,y) - I(x,y)\right]^2 \tag{6}$$

Where x and y are the spatial coordinates of images having a dimensions of $X \times Y$ pixels. The *PSNR* is calculated against the compression ratio where the compression ratio is defined as the ratio between the original image size to the transmitted compressed image size. The compression ratio, CR, of the proposed technique is calculated using the following formula:

$$CR = \frac{\text{The total size of the eigenvectors of the image}}{\text{The size of the transmitted eigenvectors}} \tag{7}$$

$$\text{The total size of the eigenvectors of the image} = N \times M \tag{8}$$

For our proposed algorithm DWT combined PCA :

$$\text{The size of the transmitted eigenvectors} = \left(t_{LL} + t_{LH} + t_{HL} + t_{HH}\right) \times \frac{d^2}{4} \tag{9}$$

Where t_{LL} is the number of eigenvectors transmitted from the LL filter, t_{LH} is the number of eigenvectors transmitted from the LH filter, t_{HL} is the number of eigenvectors transmitted from the HL filter and t_{HH} is the number of eigenvectors transmitted from the HH filter.

For compression using PCA algorithm only:

$$The\ size\ of\ the\ transmitted\ eigenvectors = t_1 \times d^2 \tag{10}$$

Where t_1 is the number of eigenvectors transmitted.

From eqn. 8, 9 in 7 the compression ratio CR for DWT combined PCA algorithm:

$$CR = \frac{N \times M}{(t_{LL} + t_{LH} + t_{HL} + t_{HH}) \times \frac{d^2}{4}} \tag{11}$$

From eqn 8, 10 in 7 the the compression ratio CR for PCA algorithm :

$$CR = \frac{N \times M}{t_1 \times d^2} \tag{12}$$

The number of the used eigenvectors depends on the desired *MSE*. So, the needed number of eignvectors is calculated by solving equation 6 on an iterative basis. Westart from a low threshold value and ignoreall the eigenvectors with eigenvalues lower than this threshold; this will be repeated for the eigenvectors of all of the four subands. Then, the remaing eigrnvectors are used to calculate *O(x,y)*. If the calculated MSE, using equation 6, is lower than the desired one the threshold is increased and the process is repeated until we reach the desired MSE.

5.4. The Proposed Algorithm can be Summarized as Follows:

As summary, The proposed algorithm divided into two parts encoder and decoder .

5.4.1. Encoder

1. Divides the 2D II image into sub-images. Each sub-image consists of one elemental image with size d^2 pixels.
2. Sort the *k* elemental images using row scan to form 3D image with size $k \times n \times n$ pixels.
3. DWT is applied to each sub-image.
4. PCA transform is applied to each sub-band coefficient of DWT.
5. Select the eigenvectors associated the largest eigenvalues for each sub-band and transmit.

5.4.2. Decoder

To reconstruct the image back again

1- Apply inverse PCA to each sub-band individually.
2- Construct the DWT of the 3D II images
3- Apply inverse DWT to each elemental image.
4- Inverse Scan II image to reconstruct the II image.
5- Evaluate the PSNR.
6- Reconstruct the 3D image.
7- Evaluate the PSNR for the reconstructed image.

6. Experimental Results

Here, we present the experimental results for II compression using the proposed technique.

The used micro lens array has a total size of 10 cm by 10 cm and each lens has a diameter of 1mm and a focal length of 5.2mm. In our experiment, two different 3D objects are used to evaluate the proposed algorithm performance. The first object is a 10mm× 10mm× 10mm die which is placed at a distance of 80 mm from the microlens array. While the second object is a home toy with an average size of 15cm × 15 cm × 10cm.

For the die object, A total of 54× 54 elemental images are used in the experiments, the total image size is 1836× 1836 pixels in which each elemental image consisted of 34× 34 pixels. This image data is stored in a TIFF (tagged image file format). In the home image, A total of 68× 92 elemental images are used in the experiments, the total image size is 1836× 2484 pixels in which each elemental image consisted of 27× 27 pixels.Fig.9 and Fig.10 show the used 3D objects and the corresponding elemental images arrays for the used die and home toy respectively.

In our experiment five types of wavelet are used. These are the Haar wavelet, Daubechies 8, Coiflets 1, Symlets 2, and the Biorthogonal 3.3. In the following experiments the Daubechies 8 wavelet is used unless other thing is stated.

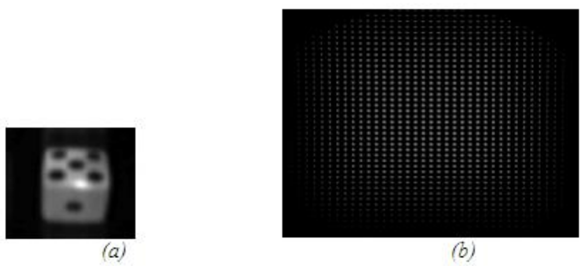


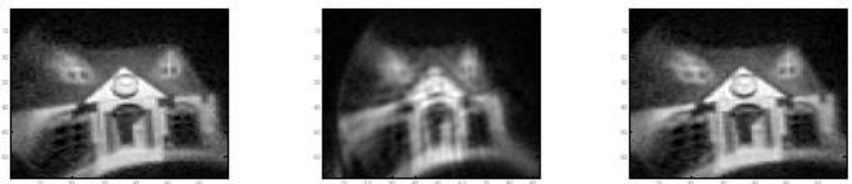
Fig.9 (a) an Image for the Used Die (b) The Die Elemental Images Array.

Fig.10 (a) an Image for the Used Home Toy. (b) The Corresponding Elemental Images Array.

In the first experiment the performance of the proposed PCA combined DWT compression algorithm is compared to the regular PCA compression where the *PSNR* is calculated for the elemental images array rather than the 3D object that is because the elemental II images are the transmitted ones. The eigenvectors corresponding to the highest eigenvalues are chosen to be transmitted. The number of chosen eigenvectors depends on the desired compression ratio. The number of transmitted eigenvectors is changed and both the *PSNR* and the compression ratio are computed and plotted. Fig. 11 shows the comparison between the two algorithms for the elemental images array of die object shown in Figs. 9-b.

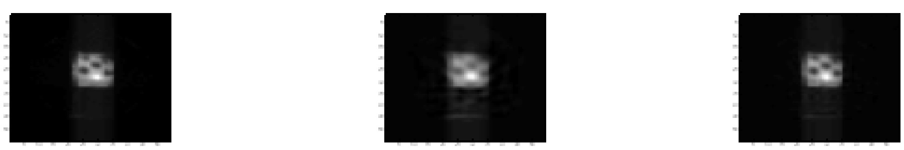
In the next experiment the performance of the proposed algorithm for the 3D reconstructed object is compared to the elemental images array. Fig. 11, 12 compares reconstructed 3D image of the toy home image and the die image consequently with PCA alone and the proposed algorithm at compression ratio of 150 for the home image and compression ratio of 70 for die image which shows that the DWT combined PCA algorithm give better result.

As Fig. 13 shows, the proposed compression algorithm is superior compared to the regular PCA by around 8 dB for a compression ratio of 50. The *PSNR* of the proposed algorithm is better by 2 dB at a high compression ratio of 500 compared to regular PCA compression. The next experiment computes the *PSNR* at different compression ratios for the proposed compression algorithm and the regular PCA compression for the elemental image array of the home toy object shown in Fig. 10-b. as Fig. 14 shows the *PSNR* is better by 11 dB compared to the regular PCA at low compression ratio of 50 and around 2 dB at high compression ratio of 500. Also, Fig. 14 shows that the result for the proposed algorithm gives better PSNR than the PCA algorithm in the home toy object . This relatively better results compared to the die object.



(a) (b) (c)

Fig. 11 a) The reconstructed toy home image. b) The decompressed reconstructed toy home image using PCA at compression ratio150. c) The decompressed reconstructed toy home image using DWT combined PCA at compression ratio150.



(a) (b) (c)

Fig. 12 a) The reconstructed Die image. b) The decompressed reconstructed die image using PCA at compression ratio 70. c) The decompressed reconstructed die image using DWT combined PCA at compression ratio70.

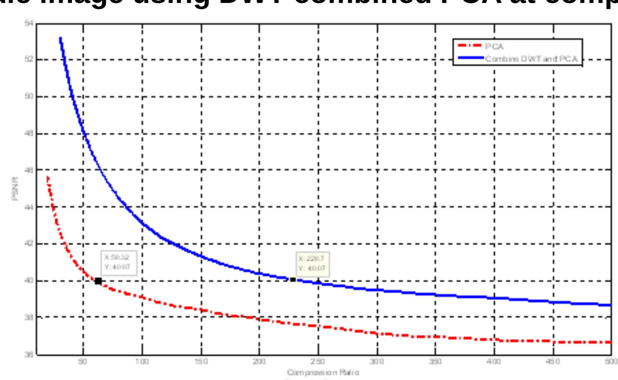


Fig. 13 PSNR versus compression ratio for the elemental images array of the die object for the proposed compression algorithm compared to regular PCA compression.

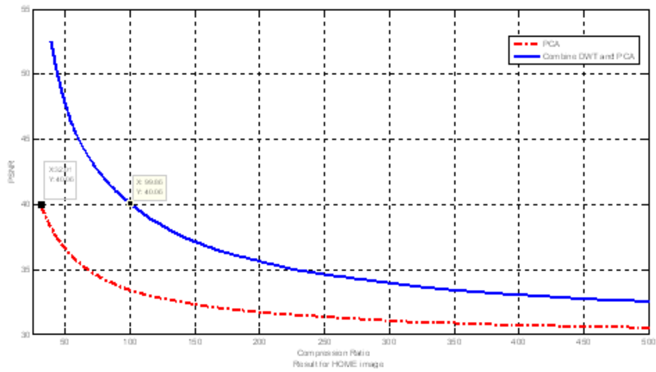


Fig. 14 PSNR versus compression ratio for the elemental images array of the home toy object for the proposed compression algorithm compared to regular PCA compression.

As figure 13 shows, at PSNR=40 the corresponding CR=228 for PCA algorithm and CR=58 for the proposed algorithm combined DWT with PCA.

As figure 14 shows, at PSNR=40 the corresponding CR=99 for PCA algorithm and CR=b32 for the proposed algorithm combined DWT with PCA.

Fig. 15 shows the *PSNR* of the reconstructed image compared to the elemental images array for different compression ratios for the die object shown in Fig. 9. As the figure shows the reconstructed image has a better *PSNR* that is because the field integration process at the reconstruction phase that could average the error. Fig. 16 shows the *PSNR* of the reconstructed image compared to the elemental images array for different compression ratios for the home toy object shown in Fig. 10.

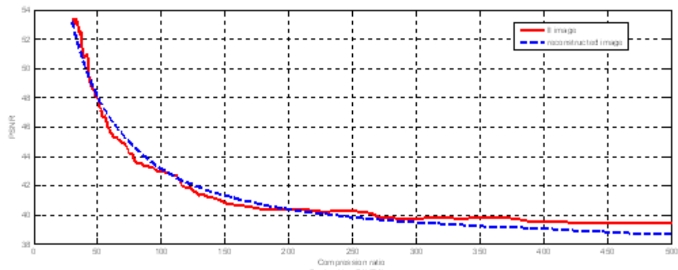


Fig. 15 PSNR versus compression ratio for the elemental images array compared to the reconstructed 3D object for the die object for the proposed compression algorithm.

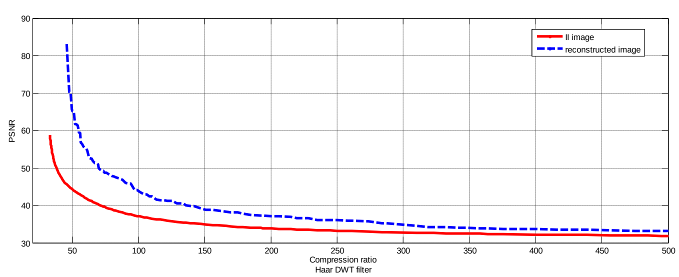


Fig. 16 PSNR versus compression ratio for the elemental images array compared to the reconstructed 3D object for the home toy object for the proposed compression algorithm.

In the last experiment different wavelets are used. Fig. 17 shows the PSNR versus the compression ratio for the die object and Fig. 18 shows the *PSNR* versus the compression ratio for the home toy object. As the figures show the *PSNR* changes according to the wavelet types. We can conclude that the Haar wavelet gives the worst results. Also, Daubechies wavelet db8 has a slightly better compression ratio compared to the wavelet families.

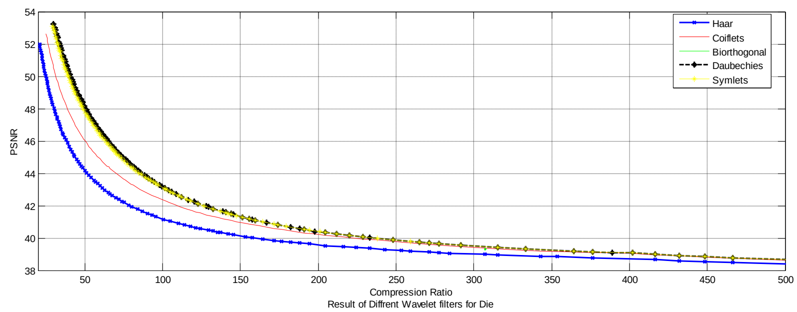


Fig. 17 PSNR versus compression ratio for the elemental images array for the die object for different wavelets

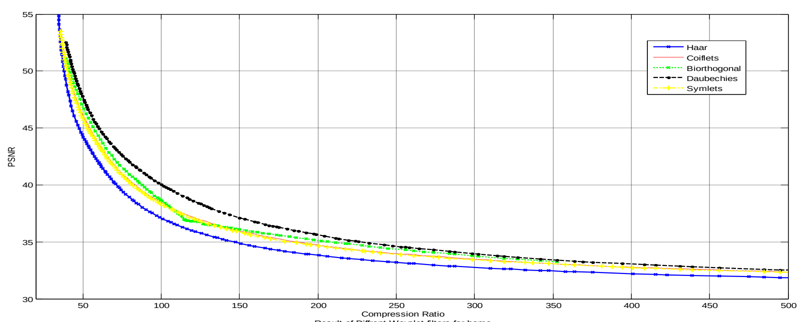


Fig. 18 PSNR versus compression ratio for the elemental images array for the home toy object for different wavelets

7. Conclusion

A new compression scheme has been developed for 3-D images captured using II technique. The proposed compression scheme combines discrete wavelet transform DWT and principle component analysis PCA. The DWT is calculated for each elemental image and the elemental images are stacked. PCA is applied to each of the sub-bands individually. The image quality obtained with the presented technique is compared with that of PCA based compression scheme at the same compression ratio. Two different 3D objects are used. The *PSNR* is calculated for both elemental images array and the reconstructed 3D object. The results show that the proposed algorithm gives better performance compared with the other type of compression. The performance enhancement is better at low compression ratio compared to higher compression ratio. As the results show, the *PSNR* of the reconstructed object using the proposed algorithm is better than the elemental image array *PSNR* for the same compression ratio. Different wavelet types are used, Daubechies wavelet db8 gives better result compared to the other wavelet families.

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