Discrete Curvelet Transform Based Super-resolution using Sub-pixel Image Registration

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Abstract

All the time, there is a demand of High-Resolution (HR) images in electronic imaging applications. Super-Resolution (SR) is an approach used to restore High-Resolution (HR) image from one or more Low-Resolution (LR) images. The goal of SR is to extract the independent information from each LR image in that set and combine the information into a single high resolution (HR) image. The quality of reconstructed SR image obtained from a set of LR images depends upon the registration accuracy of LR images. In this paper SR reconstruction using a sub-pixel shift image registration and Fast Discrete Curvelet transform (FDCT) for image interpolation is proposed. The Curvelet transform is a multiscale pyramid with many directions and positions at each scale. Image interpolation is performed at the finest level in Curvelet domain. Experimentation based results have shown appropriate improvements in PSNR and MSE. Also, it is experimentally verified that the computational complexity of the SR algorithm is reduced.

Keywords: *Image registration, sub-pixel shift, super-resolution, interpolation, curvelet transform*

1. Introduction

Most of imaging applications require high resolution (HR) images. HR images are having high pixel density so they offer more details. But, while capturing the images through CCD (Charge-couple device) sensors, images get blurred and it introduces aliases and noise to the image. Hence, captured image quality gets degraded, which is a Low-Resolution (LR) image. The resolution of an image can be enhanced spatially either by increasing the number of pixels per unit area or by increasing the chip size. Both techniques impose limitations by introducing shot noise and increase in capacitance. However, high cost for sensors is not appropriate. Image Super-Resolution (SR) is a technique by which one can enhance the spatial resolution of an image, more than the resolution of camera, in which degradations are removed. In this process the LR image is upsampled from one or several Low-Resolution observations to obtain a SR image. Any given set of Low-Resolution (LR) images only captures a finite amount of information from a scene. These LR images signify different information of the same scene. The aim of SR is to extract the independent information from each LR image in that set and combine the information into a single High-Resolution (HR) image. These LR images mapped onto a common reference plane should give sub-pixel shift from pixels of other images. This requires accurate image registration. Spatial resolution enhancement of single Low-Resolution image observation also can be achieved by

conventional interpolation techniques such as linear, bicubic and spline. But it does not attempt to remove aliases and blur, as it does not recover high frequency components lost during LR sampling process. Existing SR algorithms use multiscale wavelet transform for interpolation. It has been observed that classical wavelet transform have limited directionality and isotropic scaling, which is not capable to capture fine details of decomposed image along smooth curves. A Stationary Wavelet Transform (SWT) is close to classical wavelet transform. SWT for a given signal can be obtained by convolving the signal with appropriate filters as in the classical wavelet transform but without downsampling. The redundancy of SWT can be used for Super-resolution. In this paper, a Low-Resolution image pixel interpolation is performed in Fast Discrete Curvelet transform (FDCT) domain by sub-pixel shift image registration. FDCT is multi-scale geometric transform, which is a multi-scale pyramid with many directions and positions at each length scale. The directional decomposition with anisotropic scale enables FDCT to capture the smoothness along the curves. Performance of the proposed algorithm shows improvement in PSNR and MSE.

The organization of this paper is as follows; section two explains related work, proposed algorithm is explained in section three, section four shows the analysis of performance of the proposed algorithm, section five gives conclusion.

2. Related Work

The idea of super-resolution was first introduced in 1984 by Tsai and Hung [1] for multi-frame image restoration of band limited signals. A good overview of the algorithms of super-resolution was compiled by S.C.Park, M.K.Park and M.Kang [2], which states that non-uniform interpolation approach has low computational complexity and is applicable for real time applications. The degradation model of non-uniform interpolation approach has limitation that they are applicable only when blur and noise characteristics are same for all LR images. Interpolation errors are ignored by restoration hence SR images obtained are blurred. Frequency domain methods are limited to global motion models. However aliasing is much easier to describe and to handle in frequency domain than in spatial domain. There is no correlation between the data in frequency domain. So it is difficult to apply spatial domain a priori knowledge for regularization. The deterministic regularization approach [3] for SR reconstruction is proposed. Constraint Least Square method which generates the optimum value of the regularization using the L-curve method is used in the proposed algorithm. Stochastic SR [4] has robustness and flexibility in modeling noise characteristic and a priori knowledge. Therefore efficient gradient descent methods can be used to estimate the HR images. The projection onto convex set [5] approach is simple and it utilizes spatial domain observation model and a priori knowledge can be included. The computational cost of method is high and has a slow conversion with no unique solution. Maximum likelihood-Projection onto Convex Sets [6] hybrid reconstruction approach has advantage that a priori knowledge is combined to get single optimal solution. Iterative back-projection approach [7] has no unique solution and it is difficult to apply a priori constraints. Motionless SR reconstruction approach shows that SR image reconstruction is possible from differently blurred images without relative motion. Blind SR image reconstruction proposed by Elad and Hel-Or [8] separates fusion and deblurring. The proposed fusion method is achieved through a non-iterative algorithm preserving its optimality in the ML sense. D.Keren, S.Peleg and R.Brada [9] suggested image sequence enhancement using sub-pixel displacement. Sequences of images taken from moving camera are registered with sub-pixel accuracy in respect to translation and rotation. This enables image enhancement in respect to improved resolution and noise cleaning. M.Elad and A. Feuer [10] suggested restoration of single super-resolution

image from several blurred, noisy and under sampled images. N.Nguyun and P.Milanfar [11] suggested a wavelet based interpolation restoration method for super-resolution. They used a special type of non-uniform sampling called interlaced sampling. The wavelet interpolation technique takes the advantage of the regularity and structure inherent in interlaced data which reduces computational burden. D.L.Ward [12] proposed a redundant discrete wavelet transform based super-resolution using sub-pixel image registration. The algorithm takes advantage of redundancy to interpolate the image keeping original pixel unchanged. They used Stationary wavelet transform as redundant wavelet transform. P.Vandewalle, S.Susstrunk and M.Vetterli [13] proposed a frequency domain technique to precisely register a set of aliased images, based on their low-frequency, aliasing-free part. Aliasing-free high resolution image is reconstructed using cubic interpolation. W.L.Lee, C.Yang, H.Wu and M.Chen [14] proposed wavelet based interpolation scheme for resolution enhancement of medical images. This method takes low resolution image to be interpolated as the LL subband in wavelet domain. Other three HF subbands are fulfilled by applying the filters. The filters are designed from the lifting scheme version of Haar transform. The interpolated image has high contrast and sharpness. The wavelet transform captures the image features in vertical, horizontal and diagonal directions with isotropic scaling in each scale decomposition. The Wavelet based methods have the limitation of directionality. Fast Discrete Curvelet Transform (FDCT) [16] is multiscale pyramid with many directions and positions at each length scale and needle-shaped elements at fine scales. FDCT provides optimally sparse representation of objects which display smoothness except for discontinuity along a general curve. Such representation is far sparser than the wavelet decomposition of the object. The approximation error with FDCT is optimal. This digital transformation improves upon earlier first generation of Curvelets.

3. Proposed Algorithm

The Super-resolution algorithm developed in this paper uses a Fast Discrete Curvelet transform based interpolation scheme to create one high-resolution image from two adjacent low-resolution frames. Two low-resolution frames, I_1, I_2 are obtained from the original image by a half-pixel shift in x and y directions. These frames are mapped on quincunx grid. Algorithm determines pixel-shift between adjacent frames at sub-pixel level. The high resolution grid is rotated at 45-degree to exploit pixel correlation between two frames. Rotated grid upsamples two frames and create space for interpolation. This created space denotes the missing pixel in Super-resolution image. FDCT is applied to rotated high-resolution grid and the interpolation is performed at finest scale. Inverse FDCT is applied to interpolated high-resolution grid. The high-resolution grid is re-rotated back to its original orientation to obtain Super-resolved image.

3.1 Half Pixel Shift Using Quincunx Sampling:

The original image is sampled to obtain two low resolution frames I_1, I_2 which are, at a half pixel-shift in the x and y direction relative to one another. This shift gives sampling pattern called quincunx sampling. Quincunx sampling takes pixels in an image with purely even and odd valued indices. Frame I_1 contains only the odd indices pixels and frame I_2 contains even indices pixels. Shift in two frames simulate the diagonal motion of a camera over an area, assuming CCD array of the camera samples the area in this half-pixel

manner. In Super-resolution algorithms, one low-resolution pixel corresponds to a set of high-resolution pixels. Here each low-resolution pixel corresponds to only one high-resolution pixel [12]. Fig.1 illustrates quincunx sampling by splitting of 6x6 pixel image into two 3x3 pixel images at desired half-pixel shift. Zeros indicates odd indices pixels where as star indicates even indices pixels.

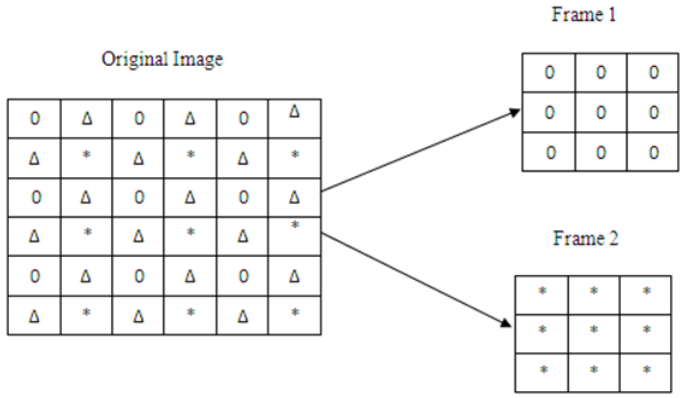

Fig. 1 A 6x6 Image is Split into 3x3 Pixel Images at Desired Half-Pixel Shift

3.2 Combining Frames on Quincunx sampling grid:

Low resolution frames, I_1 and I_2 obtained by half-pixel shift, are combined on a quincunx grid. Each low-resolution pixel of the frames I_1, I_2 corresponds to one high resolution pixel. Placing the low-resolution pixels into high resolution grid gives half-pixel shift in the x and y direction and creates blanks spaces. The high resolution grid is designated as $H(x_H, y_H)$ and the low resolution frames as $F_1(x_1, y_1)$ and $F_2(x_2, y_2)$ corresponding to two frames I_1, I_2. $T_r(\cdot)$ is a transformation for combining on a quincunx grid.

$$H(x_H, y_H) = T_r[F_1(x_1, y_1), F_2(x_2, y_2)] \tag{1}$$

$$x_1 = 2x_H - 1, \qquad y_1 = 2y_H - 1$$

$$x_2 = 2x_H \qquad y_2 = 2y_H$$

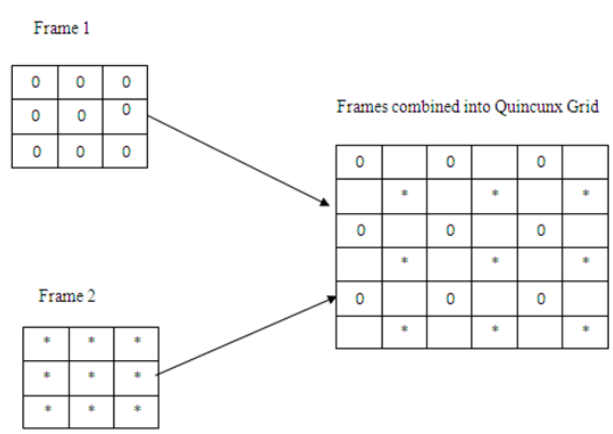

Fig.2 Combining Frames into High Resolution Grid

The resultant image is a quincunx sample of the desired high resolution image. Fig 2 illustrates combining of frames I_1 and I_2 into quincunx high resolution grid.

3.3 Quincunx Image Rotation:

A 45 degree rotation exploit the relation between pixels of two frames I_1 and I_2. As the high-resolution grid is rotated the indices of both frames get changed. The pixels of each frame get arranged alternately row and column wise which is helpful for interpolation process to be carried out for the proposed Super-resolution algorithm. This rotation changes the index values of each pixel form x_H, y_H to x_{rot}, y_{rot}. These indices of each pixel have to be transformed from the quincunx high resolution grid to the rotated high resolution grid. Let R_{ot} denote the 45-degree rotation transform applied to high resolution grid to get H_{rot}. H_{rot} denote the rotated grid expressed as

$$H_{rot}(x_{rot}, y_{rot}) = R_{ot}[H(x_H, y_H)] \tag{2}$$

3.4 Up-Sampling:

The high-resolution grid get upsampled by a factor of two when rotated, creating space for each missing pixel. Upsampled grid indicates missing pixel locations. Such missing pixels are missing coefficients in transform domain. Such missing coefficients are interpolated in curvelet domain. H_R denote the up-sampled grid expressed as

$$H_R(x_R, y_R) = UP[H_{rot}(x_{rot}, y_{rot})] \tag{3}$$

$$x_R = 2x_{rot} \qquad y_R = 2y_{rot}$$

3.5 Fast Discrete Curvelet Domain Operation:

Fast Discrete Curvelet transform (FDCT) gives different frequency components locally for analysis and synthesis of digital image in multi-resolution analysis. FDCT is multi-scale geometric transform, which is a multi-scale pyramid with many directions and positions at each length scale. FDCT is basically 2D anisotropic extension to classical wavelet transform that has main direction associated with it. Analogous to wavelet, FDCT can be translated and dilated. The dilation is given by a scale index that controls the frequency content of the curvelet with the indexed position and direction can be changed through a rotation. This rotation is indexed by an angular index. Curvelet satisfy anisotropic scaling relation, which is generally referred as parabolic scaling. This anisotropic scaling relation associated with curvelet is a key ingredient to the proof that curvelet provides sparse representation of the C^2 function away from edges along piecewise smooth curves. FDCT is constructed by a radial window W and angular window V. The radial window W is expressed as

$$\tilde{W}_j(w) = \sqrt{\phi_{j+1}^2(w) - \phi_j^2(w)} \quad , \quad j \geq 0 \tag{4}$$

Where, ϕ is defined as the product of low-pass one dimensional window. The angular window V is defined as

$$V_j(w) = V\left(2^{\lfloor j/2 \rfloor} w_2 / w_1\right) \tag{5}$$

where, $W1$ and $W2$ are low pass one dimensional windows. The Cartesian window $\tilde{U}_{j,l}$ is constructed as

$$\tilde{U}_{j,l}(w) = W_j(w) V_j(S_{\theta l} w) \tag{6}$$

Where, S_θ is shear matrix, $S_\theta = \begin{pmatrix} 1 & 0 \\ \tan\theta 1 & \end{pmatrix}$. Shear matrix S_θ is used to maintain the symmetry around the origin and rotation by $\pm \prod / 2$ radiance. The family $\tilde{U}_{j,l}$ implies a concentric tiling whose geometry is pictured in Fig.3

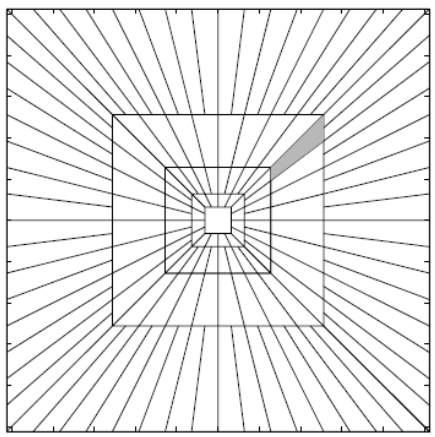

Fig.3 Basic Digital Tiling.

The frequency domain definition of digital curvelet is,

$$\overline{\varphi^D_{j,l,k}}[t1,t2] = \hat{U}_j[t1,t2] e^{-i2\Pi[k1t1+k2t2]} \tag{7}$$

where, $\hat{U}_j[t1,t2]$ is Cartesian window. Discrete Curvelet transform is expressed as

$$c^D(j,l,k) = \sum_{0 \le t_1, t_2 < n} f[t_1,t_2] \overline{\varphi^D_{j,l,k}}[t_1,t_2] \tag{8}$$

where, $c^D(j,l,k)$ represents curvelet coefficients with j as scale parameter, l as orientation parameter and k as position parameter. $f[t1,t2]$ is an input of Cartesian arrays [16]. This transform is also invertible. The classical wavelet transform captures the image features only in vertical, horizontal and diagonal directions with isotropic scaling. Wavelets do well for point singularities and not for singularities along curves. Wavelets are not well adapted to edges because of its isotropic scaling [15].

FDCT is applied to a rotated and upsampled high-resolution grid. The high- resolution grid is decomposed at three levels in curvelet domain. In order to interpolate the missing pixels their locations must be determined in each subband. In curvelet domain missing pixels corresponds to missing coefficients of each subband. The missing coefficients are interpolated at finest scale. Inverse curvelet transform reconstructs the original high-resolution grid.

3.6 Interpolation:

The simple averaging technique is intended for interpolation. The missing coefficient value is calculated using the four coefficients closest to the missing coefficient location. Based on the clustering property the coefficients nearest to the missing coefficient, are the best approximate of its value. Missing coefficients are designated with M , known closest coefficients are denoted as C_l where l varies from 1 to 4.The missing coefficient M is interpolated as an average of the four surrounding coefficients of C_l

$$M = \frac{C_1 + C_2 + C_3 + C_4}{4} \tag{9}$$

This process is repeated for each missing coefficient location in each subband. After interpolating all missing coefficients inverse FDCT is applied. The interpolated new high-resolution grid is re-rotated back to its original orientation to obtain high-resolved image.

3.7 Post processing step:

Some of the pixel values in the high-resolved image may be outside the possible gray scale range, due to interpolation error. These pixel values are truncated to 0 to 255 gray levels. Filter is used to smooth out this interpolation error. Daubechies (db9) wavelet filter works better on edges so it is used to smooth out the interpolation error.

3.8 Proposed Algorithm:

The complete algorithm is summrised in steps as below

Step1: Generate two low-resolution frames of size MxM from original image of size 2Mx2M by quincunx sampling.
Step2: Map two low-resolution frames on quincunx grid of size 2Mx2M.
Step3: Rotate high-resolution grid at 45-degree.
Step4: Perform three levels Curvelet decomposition on the rotated high-resolution grid.
Step5: Find missing coefficient location in each subband.
Step6: Interpolate the missing coefficients in each subband.
Step7: Apply inverse curvelet transform to interpolated high-resolution grid.
Step8: Re-rotate the high-resolution grid to its original orientation.
Step9: Perform post processing.

4. Experimental Result

The performance of the proposed super-resolution algorithm is tested on five different grayscale images viz Cameraman of size 256 x 256, Lena of size 512 x 512, Boat of size 512 X 512, Barbara of size 512 X 512 and Castle size 128 X 128. Two Low-resolution frames are generated by half-pixel shift in x and y directions from the original images. Experimental results using FDCT tabulated in Table 1 shows, MSE, PSNR in db.

Table 1

TEST IMAGES	MSE			PSNR		
	Bicubic	SWT	CT	Bicubic	SWT	CT
CAMERAMAN	484	74.6845	38.1814	21.2823	29.4389	32.3123
LENA	144	19.3935	16.0829	26.5472	35.2542	36.0672
BOAT	15625	30.5301	17.8700	6.1926	33.4689	35.6096
BARBARA	3969	94.8460	24.3697	12.1440	28.3265	34.2623
CASTLE	1225	144.8346	71.6671	17.2494	26.5221	29.5776

Experimental result demonstrates that proposed algorithm gives better results. The directionality of Curvelet transform with parabolic scaling provides tight frames to capture fine details of the decomposed image whereas Stationary wavelet transform have limited orientations and have isotropic scaling. By using FDCT, the edges of images are well reconstructed at smooth curves as compared to SWT. Image shown in Fig.4 (a) is low-resolution Cameraman image, Fig 4(b) is upsampled by a factor of two using bicubic interpolation technique, Fig 4(c) is reconstructed by SWT and Fig. 4 (d) is reconstructed by the proposed algorithm. The features such as tripod, buildings at back ground appear blurred in the interpolated image Fig. 4(b). Fig. 4 (d) is well reconstructed with the details on the tripod and the background as compared to 4(c). The lines on the tripod are reconstructed without artifacts. Image shown in Fig. 5(a) is low-resolution Lena image, Fig. 5(b) is upsampled by a factor of two using bicubic interpolation technique, Fig. 5(c) is reconstructed by SWT and Fig. 5(d) is reconstructed by the proposed algorithm. The features such as eyeballs,curvature of the hat and heirs appear blurred in the interpolated image Fig. 5(b). A comparison of Fig. 5(c) and 5(d) shows, Lena image is reconstructed with fine details. The eyeballs, curvature of the hat and heirs are well reconstructed. Image shown in Fig. 6(a) is low-resolution Barbara image, Fig. 6(b) is upsampled by a factor of two using bicubic interpolation technique, Fig. 6(c) is reconstructed by SWT and Fig.6 (d) is reconstructed by the proposed algorithm. Fig.6 (d) shows the fine details on the scarf along with edges of the table and books. The features such as table edge, lines on the trouser are blurred in Fig. 6 (b) and 6(c). Fig. 7 (a) is low-resolution Boat image, Fig. 7(b) is upsampled by a factor of two using bicubic interpolation technique, Fig. 7(c) is reconstructed by SWT and Fig. 7(d) is reconstructed by the proposed algorithm. Boat image Fig.7(d) gives every detail with words on boat. Fig. 8(a) is low-resolution Castle image, Fig. 8(b) is upsampled by a factor of two using bicubic interpolation technique, Fig. 8(c) is reconstructed by SWT and Fig. 8(d) is reconstructed by the proposed algorithm. The image in Fig 8(b) appear blurred. Fig. 8(d) shows, more details of the grill of the windows, edges of roof and fine structure of the bricks as compared in Fig. 8(c).

Interpolation error gets exposed after applying inverse curvelet transform to interpolated high-resolution grid. Daubechies (db9) wavelet filter is used to smooth out the interpolation error. The proposed algorithm requires less computational time.

Fig 4 a) Original image b)bicubic interpolation c) Superresolved using SWT d) Superresolved using proposed algorithm.

Fig 5 a) Original image b)bicubic interpolation c) Superresolved using SWT d) Superresolved using proposed algorithm.

Fig 6 a) Original image b)bicubic interpolation c) Superresolved using SWT d) Superresolved using proposed algorithm.

Fig 7 a) Original image b)bicubic interpolation c) Superresolved using SWT d) Superresolved using proposed algorithm.

Fig 8 a) original image b)bicubic interpolation c) Superresolved using SWT d) Superresolved using proposed algorithm.

5. Conclusion

In this paper FDCT based image super-resolution using sub-pixel shift image registration is proposed. The super-resolved image is reconstructed by registering the image frames at sub-pixel shift and interpolating the missing pixel locations in curvelet domain where original pixels are kept unchanged. Experimentation based results have shown appropriate improvements in PSNR and MSE. The experimental results also demonstrate that a super-resolved image obtained by proposed algorithms provides fine details compared to conventional stationary wavelet transform. The performance of proposed algorithm can be experimented on multiple frames from the set of same image for better results. Computational complexity of the proposed algorithm is also reduced. The limitation of proposed algorithm is that it requires post processing to reduce interpolation errors.

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