

# Impulsive Noise Suppression of Images Using Adaptive Median Filter

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## Abstract

*In this paper, a new switch median filter is presented for suppression of impulsive noise in image. The proposed filter is Modified Adaptive Center Weighted Median (MACWM) filter with an adjustable central weight obtained by partitioning the observation vector space. Dominant points of the proposed approach are partitioning of observation vector space using clustering method, training procedure using LMS algorithm then freezing weights in each block are applied to test image. The proposed method includes fuzzy clustering part for clustering the observed vector of each pixel into one of  $M$  mutually exclusive blocks. In the training phase, Least Mean Square (LMS) algorithm use to train center weight in each block then obtained weights used in testing phase. Final results shows better performance in the impulse noise reduction over standard images relative the median (MED) filter, the switching scheme I (SWM-I) filter, the signal dependent rank order mean (SD-ROM) filter, the tristate median (TSM) filter, the fast peer group filter (FPGF), the fuzzy median (FM) filter, the PFM filter and the adaptive center weighted median (ACWM) filter.*

**Keywords:** *Impulsive noise; Median filter; FCM algorithm; Least mean square*

## 1. Introduction

During image acquisition or transmission, digital images are often contaminated by impulse noise due to a number of non-idealities in the imaging process. The noise usually corrupts images by replacing some of the pixels of the original image with new pixels having luminance values near or equal to the minimum or maximum of the allowable dynamic luminance range.

In the most applications, it is very important to remove impulse noise from image data, since the performances of subsequent image processing tasks are strictly dependent on the success of image noise removal operation. However, this is a difficult problem in any image processing system because the restoration filter must not distort the useful information in the image and preserve image details and texture while removing the noise.

A large number of methods have been proposed to remove impulse noise from digital images. The standard median filter [1] is a simple rank selection filter that attempts to remove

impulse noise by changing the luminance value of the center pixel of the filtering window with the median of the luminance values of the pixels contained within the window. Although the median filter is simple and provides a reasonable noise removal performance, it removes thin lines and blurs image details even at low noise densities. The weighted median filter [3] and the center-weighted median filter [4] are modified median filters that give more weight to the appropriate pixels of the filtering window. These filters have been proposed to avoid the inherent drawbacks of the standard median filter by controlling the tradeoff between the noise suppression and detail preservation. The switching median filter, which is obtained by combining the median filter with an impulse detector. In this approach, the impulse detector aims to determine whether the center pixel of a given filtering window is corrupted or not. If the center pixel is identified by the detector as a corrupted pixel, then it is replaced with the output of the median filter, otherwise, it is left unchanged.

Some extensions of the basic switching median filter including multiple median-based filters in the structure have also been proposed. The tristate median filter [9] is an improved switching median filter that is obtained by adding a center-weighted median filter into the basic switching median filter structure. The multistate median filter (MSMF) [10] is a further extended version of the tristate median filter, including multiple center-weighted median filters. These two filters exhibit enhanced filtering performance at the expense of increased computational complexity.

The progressive switching median filter (PSMF) [11] is a derivative of the basic switching median filter. In this filtering approach, detection and removal of impulse noise are iteratively done in two separate stages. The filter provides more improved filtering performance than many other median based filters, but it has a very high computational complexity due to its iterative nature. Signal-dependent rank-ordered mean filter (SDROMF) [12] is another switching filter utilizing rank-order information for impulse noise removal. The structure of the filter is the same as a switching median filter except that the median filter is replaced with a rank-ordered mean filter.

Adaptive center weighted median (ACWM) [13] filter that avoids the drawbacks of the CWM filters and switching median filters and input data will be clustered by scalar quantization (SQ) method, that is resulted in fix threshold for all of images, but modified adaptive center weighted median (MACWM) filter will be used from FCM method, then bound between clusters for any image achieved by information of same image, as a result, clustering of input data to M block would be done better.

This paper is organized as follows: In Section 2, the basic idea of an adaptive center weighted median filter is introduced. The design of the proposed MACWM filter and clustering the observed vector of each pixel into one of  $M$  mutually exclusive blocks are presented in Section 3,4. In Section 5, our experimental results are provided to demonstrate the performance of the proposed filter. Finally, the conclusions are in Section 6.

## 2. Adaptive center-weighted median filtering

Let  $k = \{(k_1, k_2) | 1 \leq k_1 \leq H, 1 \leq k_2 \leq W\}$  denote the pixel coordinates of the noisy image corrupted by impulsive noise, where  $H$  and  $W$  are the image height and width, respectively. Let  $x(k)$  represent the input pixel value of the noisy image at location  $k \in K$ . At each location  $k$ , the observed filter window  $L\{k\}$  whose size is  $N = 2n + 1$  ( $n$  is a non-negative integer) is defined in terms of the coordinates symmetrically surrounding the input pixel  $x(k)$ .

$$L\{k\} = \{x_s(k) : s = 1, 2, \dots, n, n+1, \dots, N\} \quad (1)$$

Where the input pixel  $x(k) = x_{n+1}(k)$  is the center pixel. For example, Fig. 1 shows a  $3 \times 3$  filter window which will be used throughout this work.

$x_1(k)$	$x_2(k)$	$x_3(k)$
$x_4(k)$	$x_5(k)$	$x_6(k)$
$x_7(k)$	$x_8(k)$	$x_9(k)$

Fig. 1. The filter window about  $x(k)=x_5(k)$ .

## 3. Structure of MACWM filter

The framework of the MACWM filter is illustrated in Fig. 2 It is composed of four parts: a median filter, a set of threshold by FCM, training the center weight each block by LMS algorithm, and a decision as to whether noises exist or not.

At first, according to observation vector space (input image), median of input image will be calculated. We propose FCM algorithm to partition observation vector space to  $M$  block, and related weights to any blocks will be trained by using the LMS algorithm. The output value  $y(k)$  of the MACWM filter at the processed pixel  $x(k)$  is obtained as follows:

$$y(k) = (1 - w(k))x(k) + y(k)m(k) \quad (2)$$

That the usual output value from a median filter is denoted as  $m(k)$  at location  $k$  in a filter window of size  $2n + 1$  as follows:

$$m(k) = MED\{x_1(k), \dots, x_N(k)\} \quad (3)$$

Where MED means the median operation.

The MACWM filter achieves its effect through the linear combinations of the weighted output of the median filter and the related weighted input signal. Here,  $w(k)$  denotes the membership function indicating to what extent an impulsive noise is considered to be located at the pixel  $x(k)$ . If  $w(k) = 1$ , an impulsive noise is considered to be located at pixel  $x(k)$ , and the output value of the filter is equal to the output value of the median filter. If  $w(k) = 0$ , an impulsive noise is considered not to be located at pixel  $x(k)$ , and the output value is the same as the input  $x(k)$ ; that is, the pixel  $x(k)$  is left unchanged. To judge whether an impulsive noise exists or not at the input pixel  $x(k)$ , the membership function  $w(k)$  should take a continuous value from 0 to 1. Therefore, the major concern of the MACWM filter is how to decide the value of the membership function  $w(k)$  at the pixel  $x(k)$ .

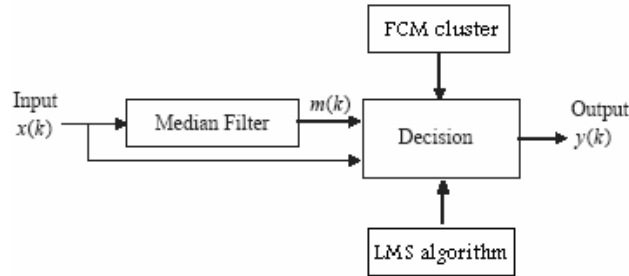


Fig. 2. The basic structure of the MACWM filter

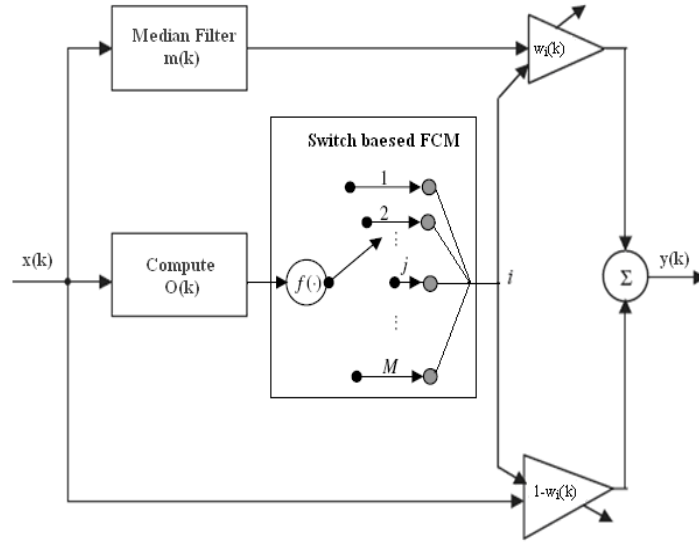
#### 4. Partitioning of observation vector space

In general, the amplitudes of most impulses are more prominent than the fine changes of signals [14]. Thus, the following two variables can be defined to generate the observation vector [17].

**Definition 1.** The variable  $p(k)$  denotes the absolute difference between the input  $x(k)$  and the median value of  $L\{k\}$  as follows [14]:

$$p(k) = |x(k) - MED(L\{k\})|. \quad (4)$$

A large  $p(k)$  value indicates that the input  $x(k)$  is dissimilar to the median value of the filter window  $L\{k\}$ . Note that  $p(k)$  is a measure for detecting the possibility of whether the input  $x(k)$  is contaminated.



**Fig. 3.** The structure of the MACWM filter.

**Definition 2.**

$$q(k) = \frac{|x(k) - x_{c1}(k)| + |x(k) - x_{c2}(k)|}{2} \quad (5)$$

Where  $|x(k) - x_{c1}(k)| \leq |x(k) - x_{c2}(k)| \leq |x(k) - x_i(k)|$ ,  $1 \leq i \leq 2n + 1$ ,  $i$  is not equal to  $n+1$ ,  $c_1, c_2$ . Notably, the values of  $x_{c1}(k)$  and  $x_{c2}(k)$  are selected to be the two closest values of  $x(k)$  in the filter window  $L\{k\}$ . If only  $p(k)$  is considered, then line component in the filter window will be identified as noise, as shown in Fig. 4. However, if the variable  $q(k)$  is also applied, then the input  $x(k)$  will not be identified as noise just because  $q(k)$  is small.

The switch, shown in Fig. 3 determines that the  $R^2$  observation vector space is partitioned into  $M$  mutually exclusive blocks  $\{i = 1, 2, \dots, M\}$  based on the observation vector given by

$$O(k) = (p(k), q(k)) \in R^2 \quad (6)$$

Where the classifier  $f(\cdot)$  is now defined as a function of the observation vector  $O(k)$ . In other words, it determines the partitioning of the observation vector space  $R^2$  into  $M$  nonoverlapping blocks according to the value of the observation vector  $O(k) \in R^2$ . As a result, the  $M$  blocks,  $i = 1, 2, \dots, M$ , satisfy.

10	232	230
230	10	230
230	232	10

**Fig. 4.** The pixel  $x(k)$  with value 10 on the line.

After the calculation of  $p(k)$  and  $q(k)$ , each scalar component  $O_d(k) \in \{p(k), q(k)\}$ ,  $d = 1, 2$  of  $O(k)$  can be classified independently by using FCM algorithm. At first, by using FCM algorithm  $p(k)$  and  $q(k)$  will be clustered to  $B$  clusters, separately. Then the representative values  $s$  and  $n$  of the components  $p(k)$  and  $q(k)$  of  $O(k)$  are obtained respectively, the vector  $(s, n)$  represents the block  $i$  ( $i = (s - 1) \times B + n$ ), ( $B$  denotes the number of class of  $p(k)$  and  $q(k)$ ). That is, the partitioning can be described as  $f(O(k)) = i$ ,  $i \in \{1, 2, \dots, M\}$ . Each unique vector  $(s, n)$  defines a distinct block, and thus the observation vector space consists of  $M$  exclusive blocks.

The FCM help to MACWM which can adapt over wide range of image. As aforementioned in introduction ACWM [13] and PFM [17] filter proposed fix threshold based on scalar quantization (SQ) and fuzzy rules, respectively for each images. Dominant points in the proposed scheme MACWM is adaptive threshold. The advantage of clustering in this case is that, the bound between clusters for any image achieved by information of same image. as a result, clustering of input data to  $M$  block would be done better.

In the next stage with the use of training image, the suitable weight of  $w_i(k)$  which is related to each block will be determined. For example, we can assume Fig.5b as a training image and Fig.5a as a desired, then  $w_i(k)$  will be trained by carrying out the LMS algorithm. Method of weights calculation will be discussed in appendage.

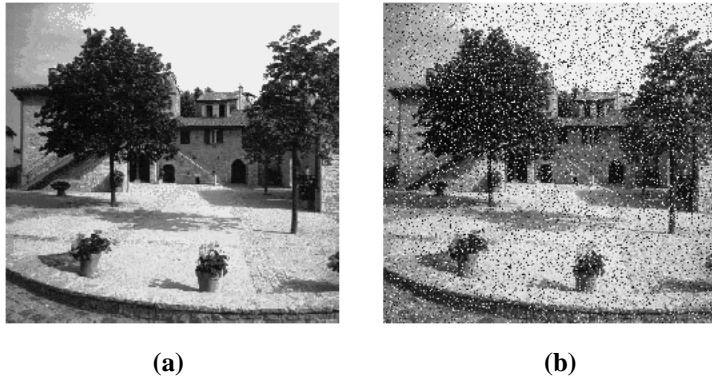


Fig. 5. The original training 'Couple' image.

## 5. Experimental results

The proposed filter is experimented upon to see how well it can remove the impulsive noises and enhance the image restoration performance for signal processing. These extensive experiments have been conducted on a variety of  $512 \times 512$  test images. The peak signal-to-

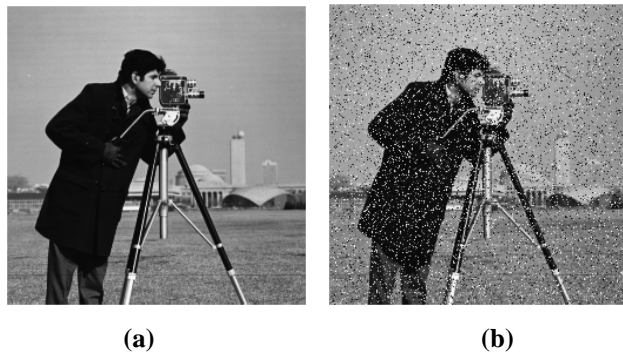
noise ratio (PSNR) criterion is adopted to measure the restoration performance quantitatively, which is defined as

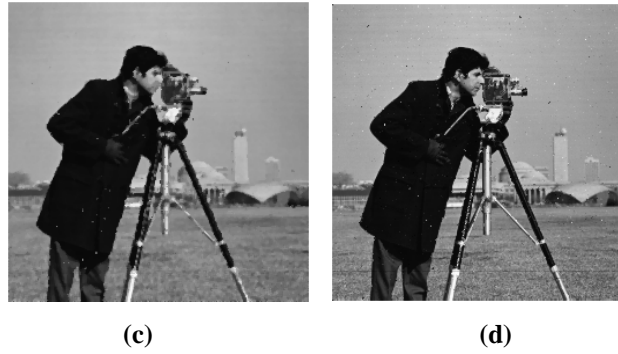
$$PSNR = 10 \log_{10} \left( \frac{\sum_k 255^2}{\sum_k (d(k) - y(k))^2} \right) \text{dB}, \quad (7)$$

Where 255 is the peak gray-level of the image,  $d(k)$  represents the value of the desired output, and  $y(k)$  represents the value of the physical output.

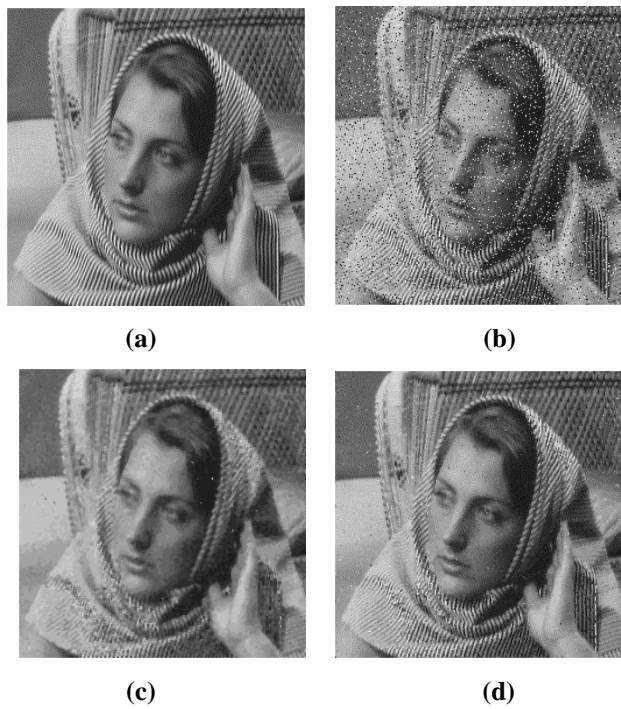
The noise removal capability of the proposed MACWM filter was extensively tested. The experimental results were compared with many other median-based filters, namely the median (MED) filter, the switching scheme I (SWM-I) filter [6], the signal dependent rank order mean (SD-ROM) filter [15], the tristate median (TSM) filter [9], the fast peer group filter (FPGF) [19], the fuzzy median (FM) filter [14] and the PFM filter [17]. The parameters of each tested filtering method were tuned exhaustively to obtain the best possible result. Note that, to demonstrate the generalization capability of the MACWM filter, the optimized weights are used to restore an image outside the training set, where the ‘Couple’ image, as shown in Fig. 5 is used as a training reference image.

To assess the effectiveness of the new filter for different images, and compare it with other median-based filters, That is, the MACWM filter could achieve better improvement than other filters for suppressing impulsive noises. Fig. 6 shows the image restoration results for the ‘cameraman’ image corrupted by 15% impulse. Fig. 7 shows the image restoration results for the ‘woman’ image corrupted by 20% impulse. Fig. 8 shows the average PSNR performance evaluation of different methods in filtering 10 images corrupted by impulsive noise. The MACWM filter’s output clearly has fewer spots and other artifacts, and provides a visually more pleasing image. The comparative PSNR performed better performance in noise attenuation.





**Fig. 6.** The 'cameraman' image degraded by 15% impulsive noise: (a) original image, (b) noisy image, and images filtered by (c) MED filter, (d) MACWM filter.



**Fig. 7.** The 'woman' image degraded by 20% impulsive noise: (a) original image, (b) noisy image, and images filtered by (c) MED filter, (d) MACWM filter.



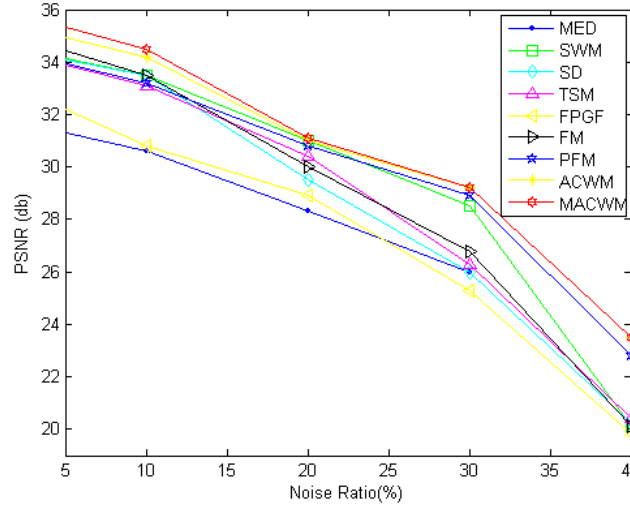


Fig. 8. Average PSNR performance evaluation of different methods.

## 6. Conclusion

In this paper, the MACWM filter, has been proposed for removing noise from corrupted images as an improvement on the median-based filters. With the filtering framework, the observed vector of each pixel  $x(k)$  is classified into one of  $M$  mutually exclusive blocks, and then the weights  $w_i(k)$  of membership function of impulsive corruption is indicated for the filtering operation. The classifier is based on FCM clustering that produce the observed vector space. Training the filter over a reference image with the constrained LMS algorithm derives the optimal weight coefficient of each block. The extensive experimental results included in the paper have demonstrated that the proposed MACWM filter is superior to a number of well-accepted median-based filters in the literature.

## Appendix

### a. Structure of FCM

One of the most widely used fuzzy clustering models is fuzzy c-means (FCM) [20]. The FCM algorithm assigns memberships to which are inversely related to the relative distance of to the point prototypes that are cluster centers in the FCM model. Some problems in FCM are as follows,

- a) Samples with equidistance to centers
- b) Measurement of distance to crisp centers
- c) Data's are crisp

Objective function in FCM is

$$J_m(U, V) = \sum_{k=1}^n \sum_{i=1}^c u_{ik}^m \|x_k - v_i\|^p \quad (8)$$

Where  $V = \{v_i, i = 1, 2, \dots, c\}, U = \{u_{ik}^m\}$  are centers and membership functions and are  $X = \{x_1, \dots, x_n\} \subset R^s$  data's.  $v_i \in R^s$  is center of  $i$ th centers.  $u_{ik} \in [0, 1]$  is membership of  $i$ th data to  $k^{\text{th}}$  centers.  $N$  samples are clustered to  $c$  cluster as following constraints are satisfied.

$$M_{fcm} = \left\{ U \in R^{c \times n} \left| \begin{array}{l} \forall i, k : 0 \leq u_{i,k} \leq 1; \\ \sum_{i=1}^c u_{ik} = 1, \sum_{k=1}^n u_{ik} > 0 \end{array} \right. \right\} \quad (9)$$

Optimization procedure gives,

$$v_i = \frac{\sum_{k=1}^n (u_{i,k})^m x_k}{\sum_{k=1}^n (u_{i,k})^m}, \quad (10)$$

$$u_{ik} = \sum_{l=1}^c \left( \frac{\|Y_k - v_i\|}{\|Y_k - v_l\|} \right)^{-2/(m-1)} \quad (11)$$

### b. Training the weight by LMS algorithm:

In this section with the use of training image, the suitable weight of  $w_i(k)$  which is related to each block will be determined. When we get the expected value  $f(O(k)) = i$ , the conditional mean square error is obtained by

$$\hat{\varepsilon}_i(k) = E[e^2(k) | f(O(k)) = i] = E[(d(k) - y(k))^2 | f(O(k)) = i], \quad (12)$$

Where  $E[.|\cdot]$  is the conditional expectation, and the error  $e(k)$  is the difference between the desired output  $d(k)$  and the physical output  $y(k)$ . Since the  $M$  blocks are mutually exclusive, the total minimum mean square error can be expressed as

$$\varepsilon = \sum_{i=1}^M [\hat{\varepsilon}_i(k) | f(O(k)) = i]. \quad (13)$$

The value of  $w_i(k)$  can be trained by carrying out the LMS algorithm that is capable of minimizing the error function  $\varepsilon_i(k)$  with respect to the  $i$ th block. The weights  $w_i(k)$  corresponding to  $i$ th block can be adjusted in an iterative fashion along with the error surface toward the optimal solution. As shown in [16], the iterative learning algorithm of  $w_i(k)$  is derived as

$$w_i^{(t+1)}(k) = \begin{cases} w_i^{(t)}(k) - \eta_i |e(k)| x(k) - d(k), & w_i^{(t+1)} \geq 0, \\ 0 & w_i^{(t+1)} < 0, \end{cases} \quad (14)$$

Here,  $\eta_i$  denotes a learning rate,  $w_i^{(0)}(k)$  denotes the initial weight and  $w_i^{(t)}(k)$  the weight after the  $t$ th iteration,  $t = 0, 1, \dots$ . For each  $x(k)$  associated with  $i$ th block, the value of  $w_i(k)$  is updated iteratively in a gradient way by using Eq. (14). Note that for each new observation vector  $O(k)$ , only one  $w_i(k)$  is adapted. Based on the assumptions presented in [16] for the derivation of LMS convergence, the following condition are sufficient for the convergence in the mean and mean square.

$$0 < \eta_i < \frac{2}{E[(x(k) - d(k))^2 | f(O(k)) = i]} \quad i=1, 2, \dots, M \quad (15)$$

Even though many of the necessary assumptions for the convergence do not necessarily hold [16], it is shown experimentally that the learning algorithm can converge toward the solution.

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